CHAPTER V

CONCLUSION AND DISCUSSION

By using the first - order semiclassical approximation in the Baker - Hausdorff lemma, one can get the propagator for the potential $V(x) = ax^2 + bx^3$

$$K(x,\xi,t)=\sqrt{\frac{m\omega}{2\pi i\hbar\,\sin(\omega t)}}\left(\cos\left(\frac{\omega t}{2}\right)\right)^{-\frac{4b}{m\omega^2}(2x+\xi)}\exp\left[\frac{im\omega}{2\hbar\,\sin(\omega t)}\left\{\left(x^2+\xi^2\right)\cos(\omega t)-2x\xi\right\}\right.$$

$$+b\left\{\frac{x}{m\omega^2}\left(3-4\sin^2\left(\frac{\omega t}{2}\right)\right)-\frac{3i\xi}{m\omega^2}+\frac{ix^2}{\hbar}(2x+3\xi)t\right.$$

$$\left.\left.-\frac{ix^3}{2\hbar\omega}\left(\sin(\omega t)+\tan\left(\frac{\omega t}{2}\right)\cos(\omega t)+7\tan\left(\frac{\omega t}{2}\right)\right)-\frac{ix\xi}{\hbar\omega}(5x+2\xi)\tan\left(\frac{\omega t}{2}\right)\right\}\right]. \qquad (5.1.1)$$

It is reduced to the well known propagator for the SHO. in eq. (4.3.33)

$$K(x,\xi,t)=\sqrt{\frac{m\omega}{2\pi i\hbar\,\sin(\omega t)}}\exp\left[\frac{im\omega}{2\hbar\,\sin(\omega t)}\left\{\left(x^2+\xi^2\right)\cos(\omega t)-2x\xi\right\}\right] \qquad (5.1.2)$$

when b is zero

$$K(x,\xi,t) = \sqrt{\frac{m}{2\pi\hbar}}\exp\left[\frac{im}{2\hbar t}(x-\xi)^2\right] \tag{5.1.3}$$

which corresponds to the free - particle propagator in eq. (4.3.25). Obviously we can see how the arbitrary constant b in eq. (5.1.1) affects anharmonic part of the propagator.

From equations of motion (3.2.11) and (3.2.14), one can show the first - order approximation should give the second harmonic behavior . If the anharmonic part is rearranged to get the expected result as following:

$$\exp\left[b\left(\left(\frac{x}{m\omega^2}-\frac{3i\xi}{m\omega^2}\right)+\left(\frac{2ix^3}{\hbar}+\frac{3ix^2\xi}{\hbar}\right)t+\frac{2x\cos(\omega t)}{m\omega^2}-\frac{\sin(\omega t)}{2\hbar m\omega(\cos(\omega t)+1)}\right.\right.$$

$$\left.\left.\times\left(8imx^3+2i\hbar x\xi(5x+2\xi)\right)-\frac{ix^3}{2\hbar\omega(\cos(\omega t)+1)}\sin(2\omega t)\right)\right]. \tag{5.1.4}$$

It is an interesting to expect in general that the nth order approximation will give the $(n+1)$ th harmonic term $((n+1)\omega t)$. This means that the exact propagator will be composed of all harmonic. This is the answer why the propagator is very complicated, too complicated to be solved by the path integral technique.