

## CHAPTER 3

### THEORETICAL CONSIDERATIONS

The section discusses how the Layered Stabilized Flow Model is developed. All relevant equations are presented and the derivation of the solution is fully described. The information required to build the model is enumerated and the calculation method is clearly established.

#### 3.1 The Layered Stabilized Flow Model

El-Banbi and Wattenbarger<sup>[1,2]</sup> proposed the Layered Stabilized Flow Model (LSFM) as an alternative procedure to estimate the original gas in place (OGIP) in a commingled gas reservoir. The authors wrote two papers on the analysis of commingled gas reservoirs and were published in 1996 and 1997. The first paper basically presents the approach using simulated and actual data for tight gas reservoirs (reservoirs with permeability ranging from 0.1 to 10 md) producing against a constant flowing bottomhole pressure. Also, the model ignored the effect of non-Darcy flow. The second paper is an extension of the first study whereby the effects of variation in the flowing bottomhole pressure and non-Darcy flow were considered. The results from the model showed good OGIP estimate of each component layer of the commingled reservoir.

The technique combines the material balance equation and the productivity index equation (stabilized flow equation) to come up with a model rate equation for each layer in the commingled reservoir system. When stabilized flow is reached in a commingled system, each layer can be characterized by its OGIP and its flow coefficient,  $J_g$ . The LSFM basically calculates the “model production rate” for each layer and sums up these layer rates to calculate the “total model production rate” for the commingled reservoir at a given time step. This total model production rate over

time (model forecast) is then matched to the actual production rate history using multi-variable non-linear regression analysis.

The method is simple and requires only the flow rate and flowing bottom hole pressure ( $p_{wf}$ ) history as well as the initial reservoir pressure and gas properties of a given well. The method shows excellent results for moderate to high permeability reservoirs even with long shut-ins and considerable variations in the  $p_{wf}$  (the model accounts for cross-flow between layers during shut-in periods).

### 3.1.1 The Layered Stabilized Flow Model for Single-Layered Reservoir

In order to formulate the model equations for multi-layered reservoirs, a single layer reservoir is examined first. In this section, the equations to model a single-layer reservoir are developed.

The two equations describing stabilized flow for single-layer gas reservoirs are the material balance (MB) equation and the productivity index equation. These two equations were used by El-Banbi and Wattenbarger to arrive at the layered stabilized flow equation.

The material balance equation for a single-layer gas reservoir is given by:

$$\left(\frac{p}{z}\right) = \left(\frac{p}{z}\right)_i \left(1 - \frac{G_p}{G}\right) \quad (3.1)$$

The gas productivity index equation or the gas flow solution is given by:

$$q_g = J_g (m(p) - m(p_{wf})) \quad (3.2)$$

The real gas pseudo-pressure is defined by:

$$m(p) = 2 \int_{p_0}^p \frac{p}{\mu z} dp \quad (3.3)$$

Due to the large magnitude and the awkward unit of pseudo-pressures, the use of normalized pseudo-pressures is preferred to simplify the flow equation. Normalized pseudo-pressure<sup>[10]</sup> is defined as:

$$p_{pn} = p_i + \frac{\mu_i z_i}{p_i} \int_{p_i}^p \frac{p}{\mu z} dp \quad (3.4)$$

The advantage of using the normalized pseudo-pressure is that it has the unit of pressure and that liquid well test equations can be used for gas flow equations.

Hence, the simplified gas flow equation (Eq 3.2) becomes:

$$q_g = J_g (p_{pn} - p_{pnwf}) \quad (3.5)$$

In addition, the relation between the gas flow rate and cumulative gas production is given by:

$$G_p = \int_0^t q_g(t) dt \quad (3.6a)$$

Or in finite terms:

$$G_p = \sum_{l=1}^j q_{gl} \Delta t_l \quad (3.6b)$$

where the subscripts  $l$  and  $j$  are time indices.

The simplified form of the gas flow equation (Eq. 3.5) requires the pressure terms to be expressed as normalized pseudo-pressures. Therefore, we need to convert the pressure terms from the material balance equation (Eq. 3.1) to normalized pseudo-pressure terms so we can directly use them in Eq. 3.5.

Solving for  $p$  from Eq. 3.1

$$p = \frac{z p_i}{z_i} - \frac{z p_i G_p}{z_i G} \quad (3.7)$$

From observations, the plot of the normalized pseudo-pressure versus pressure shows a linear relationship at high pressure values.

$$p_{pn} = kp \quad (3.8)$$

where  $k$  is a proportionality constant.

Substituting  $p_{pn}$  for  $p$  in Eq. 3.7:

$$\frac{p_{pn}}{k} = \frac{zp_{pni}}{z_i k} - \frac{zp_{pni} G_p}{z_i k G} \quad (3.9a)$$

All the  $k$  terms cancel out and Eq. 3.9a can be simplified into

$$p_{pn} = \frac{zp_{pni}}{z_i} - \frac{zp_{pni} G_p}{z_i G} \quad (3.9b)$$

By combining Eqs. 3.5, 3.6b and 3.9b, the change of production rate through time can be related to the change of the average reservoir pressure through time. If the initial reservoir pressure, the initial gas properties, and the production and pressure history are known, the three equations can be solved simultaneously to determine the original gas in place  $G$  and the productivity index  $J_g$ .

Substituting the value of  $p_{pn}$  from Eq. 3.9b and  $G_p$  from Eq. 3.6b to Eq. 3.5, Eq. 3.5 becomes:

$$q_{gl} = J_g \left( \frac{p_{pni} z_j}{z_i} - \frac{p_{pni} z_j \left( \sum_{l=1}^j q_{gl} \Delta t_l \right)}{z_i G} - p_{pnwff} \right) \quad (3.10)$$

Simplifying Eq. 3.10, we obtain:

$$q_{gj} \left( 1 + \frac{J_g p_{pmi} z_j \Delta t_j}{z_i G} \right) = \frac{J_g p_{pmi} z_j}{z_i} - \frac{J_g p_{pmi} z_j \left( \sum_{l=1}^{j-1} q_{gl} \Delta t_l \right)}{z_i G} - J_g p_{pnwff} \quad (3.10a)$$

$$q_{gj} = \frac{J_g p_{pmi} z_i z_j G - J_g p_{pmi} z_i z_j \left( \sum_{l=1}^{j-1} q_{gl} \Delta t_l \right) - J_g p_{pnwff} z_i^2 G}{z_i^2 G + J_g p_{pmi} z_i z_j \Delta t_j} \quad (3.11)$$

Eq. 3.11 represents the “model production rate” for the stabilized flow in a single-layer reservoir. All pressure terms are expressed as normalized pseudo-pressures. Note that the derived equation does not have the average reservoir pressure term  $p_j$ . Elimination of this variable is the advantage of using the LSFM model equation. In most cases, measurement of the average reservoir pressure can be estimated from production logging but these type of surveys are not practically done in a frequent interval to get a close estimate of the average layer pressure. Also, the average reservoir pressure that can be measured in pressure surveys may not represent the true value because of crossflow between layers as discussed previously.

By initially assuming  $G$  and  $J_g$ , the model production rate can now be calculated from Eq. 3.11 and subsequently history matched or “calibrated” with the actual production of the well. Matching is done through curve fitting using non-linear least square regression analysis.

### 3.1.2 The Layered Stabilized Flow Model for Multi-Layered Reservoirs

The individual layer performance from the single layer model can now be used to compute for the total reservoir performance of a multi-layered system. After solving each layer’s model production rate at time  $t_j$ , the total model production rate for the whole commingled reservoir is simply the sum of the individual layer rate at time  $t_j$ .

$$q_{Tj} = \sum_{k=1}^n q'_{gjk} \quad (3.12)$$

In equation form:

$$q_{Tjk} = \sum_{k=1}^n \left( \frac{J_{gk} p_{pnik} z_{ik} z_{jk} G_k - J_{gk} p_{pnik} z_{ik} z_{jk} \left( \sum_{l=1}^{j-1} q_{glk} \Delta t_l \right) - J_{gj} p_{pmwfj} z_{ik}^2 G_k}{z_{ik}^2 G_k + J_{gk} p_{pnik} z_{ik} z_{jk} \Delta t_j} \right) \quad (3.13)$$

where the subscript  $k$  refers to the layer index.

For commingled reservoirs of two or more layers, Eq. 3.13 introduces an additional unknown parameter to solve the model production rate. This is  $G_{pk}'$ , or the cumulative production of each layer before time  $t_j$  (i.e. the second term in the equation) defined as:

$$G_{pk}' = \sum_{l=1}^{j-1} q_{glk} \Delta t_l \quad (3.14)$$

$G_{pk}'$  is the cumulative production for each layer from the start of production until the start of the calibration period. For a single-layered reservoir,  $G_p'$  is known, i.e., the cumulative production from the layer. This is not the case for multi-layered reservoirs. Although the total  $G_p'$  for the commingled reservoir is known, the contribution from each sand layer is unknown. Hence,  $G_{pk}'$  is an additional unknown parameter in the calculation of the model production rate.

Thus,  $G_{pk}'$ , like  $G$  and  $J_g$ , is also initially assumed for each layer. Once an estimate of  $G_{pk}'$  is made for each layer, the cumulative production through each time step in the calibration period can now be calculated. To illustrate how to calculate  $G_{pk}'$ , consider Layer 1 in a multi-layered reservoir:

For Layer 1:

1. Assume  $G_{p11}$  at  $j = 1$  (cumulative production from time 0 until start of calibration period)
2. At  $j = 2$  (next time step):

$$G_{p12} = G_{p11} + q_{g11} \Delta t_2 \quad (3.15)$$

where

$$\Delta t_2 = t_2 - t_1 \quad (3.16)$$

and

$$q_{g11} = \frac{J_{g1} p_{pni1} z_{i1} z_{11} G_1 - J_g p_{pni1} z_{i1} z_{11} G_{p10} - J_{g1} p_{pnwf11} z_{i1}^2 G_1}{z_{i1}^2 G_1 + J_g p_{pni1} z_{i1} z_{11} \Delta t_1} \quad (3.17)$$

3. At  $j = 3$ :

$$G_{p13} = G_{p12} + q_{g21} \Delta t_3 \quad (3.18)$$

where

$$\Delta t_2 = t_3 - t_2 \quad (3.19)$$

and

$$q_{g21} = \frac{J_{g1} p_{pni1} z_{i1} z_{21} G_1 - J_g p_{pni1} z_{i1} z_{21} G_{p11} - J_{g1} p_{pnwf21} z_{i1}^2 G_1}{z_{i1}^2 G_1 + J_g p_{pni1} z_{i1} z_{21} \Delta t_2} \quad (3.20)$$

4. Do the same until  $j = n$ .

Thus  $G_p$ ,  $G$  and  $J_g$  of each layer are the unknown parameters that are assumed and used in the model production rate to match the actual production rate for reservoirs with more than one layer. The number of unknown parameters is, therefore, equal to  $3n$ .

### 3.2 LSFM Data Requirements

The information required in order to build and run the model are outlined below:

1. Initial static reservoir pressure of the individual reservoirs in the well. Possible sources of these pressure measurements include:
  - (a) known pore pressure gradients for the gas field;
  - (b) Repeat Formation Tester (RFT) pressure measurements made after drilling but before completion of the well
2. Temperature of the individual reservoirs in the well
3. Gas PVT properties such as gas specific gravity ( $\gamma$ ), gas density ( $\rho$ ), real gas deviation factor ( $z$ ), formation volume factor ( $B_g$ ) and gas viscosity ( $\mu_g$ ) for calculating normalized pseudo-pressures
4. Production history of the well, which includes:
  - (a) flowrate of gas, water and condensate versus time
  - (b) flowing bottom hole pressure measurements versus time
  - (c) In the absence of flowing bottom hole pressure measurements, flowing tubing head pressure measurements versus time and the following well completion information for multi-phase flow correlation:
    - tubing size and estimated roughness
    - components of the well completion and their depths
    - well trajectory including measured depth of the well versus true vertical depth
  - (d) record of periods during which the well was shut-in
5. Layer information such as depth of each sand and their perforation intervals



6. Other information such as estimates of reservoir areas from seismic data, porosity, permeability and water saturation, which will be of use for validating the OGIP calculated by the model

### **3.3 LSFM Assumptions and Limitations**

In applying the Layered Stabilized Flow Model, the following assumptions were made:

- (a) Average fluid properties are used for the calculation of the normalized pseudo-pressures of the layers of the commingled reservoir system unless the PVT properties of each layer are defined.
- (b) The productivity index  $J_g$  is constant during the production period.
- (c) There is no pressure loss in the tubing occurring between layers.

### **3.4 LSFM Calculation Method**

From the required data and information outlined above, the Layered Stabilized Flow Model (LSFM) Program is constructed for modeling multi-layered gas reservoirs. The program is a spreadsheet program created in Microsoft Excel. The model has three main elements or modules: the production data table, the normalized pseudo-pressures table and the LSFM program.

#### **3.4.1 Production Data Table**

The production data input table contains all the pertinent information about the well production history: gas, water and condensate rates, flowing pressures and temperatures, and the absolute and relative flow periods of the well. A “calibration period” is normally selected from the production history where the model is matched. This calibration period should exclude transient production rate data.

### 3.4.2 The Normalized Pseudo-pressure Table

The normalized pseudo-pressure table module in the LSFM program allows calculation and conversion of gas pressures to normalized pseudo-pressures. This is done through interpolation of the normalized pseudo-pressure values provided in the table.

As given in Eq 3.4, the normalized pseudo-pressure is:

$$p_{pm} = p_i + \frac{\mu_i z_i}{p_i} \int_{p_i}^p \frac{p}{\mu z} dp \quad (3.4)$$

where the subscripts  $i$  on  $\mu$  and  $z$  refers to the evaluation of these parameters at the initial pressure  $p_i$ , the references pressure.

Integration of Eq. 3.4 can be done numerically through a spreadsheet table calculation. Using numerical integration, equation (3.4) can be simplified into:

$$p_{pn} = \frac{\text{integral}}{\left(\frac{p}{\mu z}\right)_i} + p_i \quad (3.21)$$

where the “integral” is defined as:

$$\text{integral} = \left( \sum_{j=2}^n \frac{1}{2} \left[ \left(\frac{p}{\mu z}\right)_{j-1} + \left(\frac{p}{\mu z}\right)_j \right] \right) (p_j - p_{j-1}) \quad (3.22)$$

From Eq. 3.4, Eq. 3.21 and Eq. 3.22, the calculation of the normalized pseudo-pressures requires the gas deviation factor  $z$  and the gas viscosity  $\mu$  at each pressure. Therefore, the normalized pseudo-pressure component of the LSFM program has two other sub-components: the  $z$ -factor calculator and the fluid property calculator.

To calculate gas deviation factors, the Dranchuk and Abu-Kassem correlation<sup>[11]</sup> was used. The Dranchuk and Abu-Kassem correlation is an empirical equation for determining the z-factor. It was chosen for ease of calculation since it can be incorporated readily in the LSFM program. The correlation is an 11-constant empirical equation given by:

$$\begin{aligned}
 z = & \left( A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5} \right) \rho_r \\
 & + \left( A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \right) \rho_r^2 - A_9 \left( \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \right) \rho_r^3 \\
 & + A_{10} \left( 1 + A_{11} \rho_r^2 \right) \frac{\rho_r^2}{T_{pr}^3} \exp(-A_{11} \rho_r^2) + 1.0
 \end{aligned} \tag{3.23}$$

where  $\rho_r$  is expressed as:

$$\rho_r = \frac{0.27 p_{pr}}{z T_{pr}} \tag{3.24}$$

The pseudo-reduced and pseudo-critical pressures and temperatures are defined as follows:

$$p_{pr} = \frac{p}{p_{pc}} \tag{3.25}$$

$$T_{pr} = \frac{T}{T_{pc}} \tag{3.26}$$

$$p_{pc} = 706 - 51.7\gamma_g - 11.1\gamma_g^2 \tag{3.27}$$

$$T_{pc} = 187 + 330\gamma_g - 7.15\gamma_g^2 \tag{3.28}$$

Eq. 3.23 is implicit in  $z$  and must be solved iteratively by the Newton-Rhapson technique. The 11-empirical constants were determined from non-linear regression on 1,500 data points from the Standing and Katz  $z$ -factor chart<sup>[12]</sup>.

The 11 constants in the Dranchuk and Abu-Kassem Correlation are:

$$\begin{aligned} A_1 &= 0.3265, \\ A_2 &= -1.0700, \\ A_3 &= -0.5339, \\ A_4 &= 0.01569, \\ A_5 &= -0.05165, \\ A_6 &= 0.5475, \\ A_7 &= -0.7361, \\ A_8 &= 0.1844, \\ A_9 &= 0.1056, \\ A_{10} &= 0.6134, \text{ and} \\ A_{11} &= 0.7210 \end{aligned}$$

To calculate gas viscosities, the Lee *et al.*<sup>[13]</sup> correlation was used. The Lee *et al.* correlation is a semi-empirical equation for calculating gas viscosity. The viscosity equation is given by:

$$\mu_g = 10^{-4} K \exp \left[ X \left( \frac{\rho_g}{62.4} \right)^Y \right] \quad (3.29)$$

where

$$K = \frac{(9.4 + 0.02M_g)T^{1.5}}{(209 + 19M_g + T)} \quad (3.30)$$

$$Y = 2.4 - 0.2X \quad (3.31)$$

$$X = 3.5 + \left(\frac{986}{T}\right) + 0.01M_g \quad (3.32)$$

$$M_g = 28.97\gamma_g \quad (3.33)$$

### 3.4.3 The Layered Stabilized Flow Model Program

The Layered Stabilized Flow Model Program calculates each layer's flow rate using Eq. 3.9 by initially assuming values of  $G_{pk}$ ,  $G_k$  and  $J_{gk}$  for each layer and then calculating the total flow rate by summing up the layer rates of the entire commingled system (refer to Eq. 3.13). Calculation of the total flow rate is done for each time step.

In order to get the correct values of  $G_{pk}$ ,  $G_k$  and  $J_{gk}$ , the model rates are history matched with the actual production rates using non-linear least square regression analysis. Non-linear least square regression analysis is a multivariable optimization technique commonly used to infer unknown parameters of a given model function by comparing and minimizing the difference between the model function and the actual measurements.

#### 3.4.3.1 Non-linear Regression Analysis

In general terms, non-linear regression is a mathematical procedure whereby model parameters are determined through minimization of the difference between the calculated model response and the actual measurement. In the least squares approach, an objective function  $E$  is set such that the sum of the squares of the difference between the actual measurement (total actual rate) and the model function (total model rate) is minimized:

$$E = \sum_{i=1}^n [y_i - f(\theta, x_i)]^2 \quad (3.34)$$

where

- $E$  = objective function
- $f$  = model function
- $x$  = independent variable
- $y$  = dependent variable
- $\theta$  = model parameters

The objective function  $E$  can be approximated as a quadratic relationship by taking a 2<sup>nd</sup> order Taylor Series expansion of the function. This quadratic approximation,  $E^*$ , is defined as:

$$E^* \approx E|_{\theta^0} + \sum_{j=1}^m (\theta_j - \theta_j^0) \left. \frac{\partial E}{\partial \theta_j} \right|_{\theta^0} + \frac{1}{2} \sum_{j=1}^m \sum_{k=1}^m (\theta_j - \theta_j^0) (\theta_k - \theta_k^0) \left. \frac{\partial^2 E}{\partial \theta_j \partial \theta_k} \right|_{\theta^0} \quad (3.35)$$

where the subscripts  $j$  and  $k$  refer to the unknown parameters and the superscript  $0$  in  $\theta$  refers to the initial guesses for the unknown parameters. The use of the 2<sup>nd</sup> order approximation or Eq. 3.35 is known as the Newton's Method.

If we define a vector of 1<sup>st</sup> order partial derivatives as,

$$g \equiv \{g_j\} = \left\{ \left. \frac{\partial E}{\partial \theta_j} \right|_{\theta^0} \right\} \quad (3.36)$$

a vector of increments in the independent variables as  $\delta\theta$ ,

$$\delta\theta = \theta - \theta^0 \quad (3.37)$$

and a matrix of 2<sup>nd</sup> order partial derivatives of the function as,

$$H \equiv \{h_{jk}\} = \left\{ \left. \frac{\partial^2 E}{\partial \theta_j \partial \theta_k} \right|_{\theta^0} \right\} \quad (3.38)$$

this allows Eq. 3.35 to be written in the following matrix form as:

$$E^* = E|_{\theta^0} + (\delta\theta)^T g + \frac{1}{2} (\delta\theta)^T H (\delta\theta) \quad (3.39)$$

where  $g$  is the gradient of the objective function (known as the Jacobian) and  $H$  is the Hessian matrix.

Taking the first derivative of  $E$  with respect to the unknown parameters, we obtain from Eq 3.34:

$$\left. \frac{\partial E}{\partial \theta_j} \right|_{\theta^0} = -2 \sum_{i=1}^n [y_i - f(\theta^0, x_i)] \left. \frac{\partial f}{\partial \theta_j} \right|_{\theta^0} \quad (3.40)$$

And the second derivative is defined by:

$$\left. \frac{\partial^2 E}{\partial \theta_j \partial \theta_k} \right|_{\theta^0} = -2 \sum_{i=1}^n \left[ - \left. \frac{\partial f}{\partial \theta_j} \right|_{\theta^0} \left. \frac{\partial f}{\partial \theta_k} \right|_{\theta^0} + [y_i - f(\theta^0, x_i)] \left. \frac{\partial^2 f}{\partial \theta_j \partial \theta_k} \right|_{\theta^0} \right] \quad (3.41)$$

In order to minimize  $E^*$ , its derivative with respect to  $\delta\theta$  must be zero:

$$\frac{\partial E^*}{\partial(\delta\theta)} = 0 \quad (3.42)$$

From Eq 3.39, we obtain

$$\frac{\partial E^*}{\partial(\delta\theta)} = g + H(\delta\theta) \quad (3.43)$$

Thus, by equating Eq. 3.42 and Eq. 3.43, we obtain:

$$\delta\theta = -H^{-1}g \quad (3.44)$$

Note that  $\delta\theta$  obtained from Eq 3.44 is from  $E^*$  which is used to approximate the true objective function  $E$ . Therefore,  $\delta\theta$  should be solved iteratively until  $E^*$  is very close to  $E$  or  $\theta^0$  is approximately equal to  $\theta$ .

In summary, the objective is to find  $\delta\theta$  that ensures  $E$  is a minimum value. This will allow the determination of  $\theta$  (model parameters) which for a quadratic surface will be the optimum values  $\theta^*$ . For general non-linear function, the minimum of  $E$  will not be obtained in one iteration, hence solving for  $\theta$  is normally modified as a recursive formula where:

$$\delta\theta = \theta^{k+1} - \theta^k \quad (3.45)$$

And the new solution to the unknown model parameters is

$$\theta^{k+1} = \theta^k + \delta\theta \quad (3.46)$$

Here  $\theta^{k+1}$  represents the new solution vector to the unknown parameters and the final (or optimum) solutions are obtained when a set of termination or convergence criteria is satisfied.

The Newton's method requires the Hessian matrix to be positive definite, i.e., all the diagonal terms in the Hessian matrix are positive. In order to ensure positive definiteness of the Hessian matrix, Gauss-Newton method can be used. In this



method, the second derivatives of the function are set to zero. However, even with the application of the Gauss-Newton method, the Hessian matrix may still be ill conditioned. Levenberg and Marquardt<sup>[14]</sup> proposed an algorithm that forces the Hessian matrix to be positive-definite by introducing a small constant to the diagonal elements of the Hessian matrix. This method is known as the Gauss-Marquardt Algorithm.

In general, the Gauss-Marquardt algorithm is summarized in Figure 3.1:

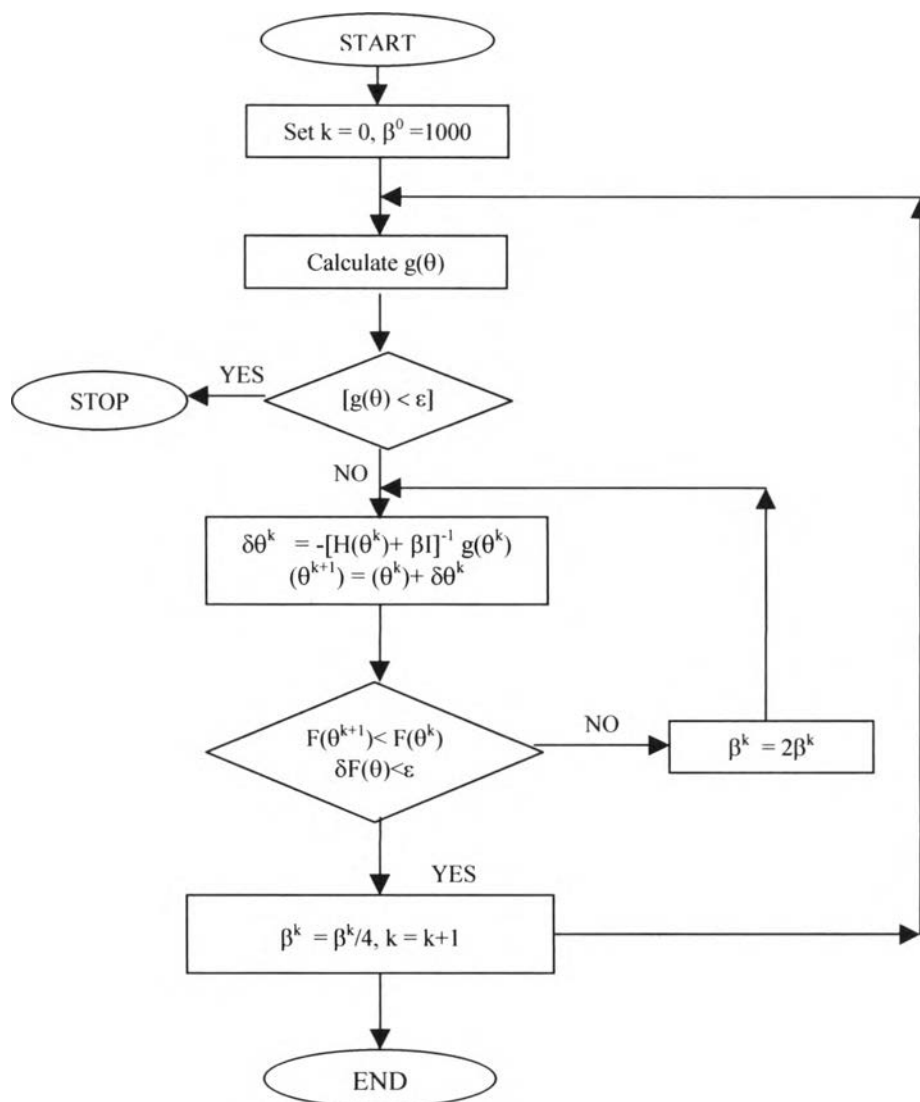


Figure 3.1: Gauss-Marquardt algorithm.

The algorithm starts by introducing a positive constant  $\beta$  large enough to make the Hessian matrix positive definite when it is not. For the first iteration ( $k=0$ ), an initial value of 1000 is used for  $\beta$  and is multiplied by an identity matrix  $I$  to modify the Hessian matrix. The vector of increments of the model parameters  $\delta\theta$  is then solved using Eq.3.44 using the modified Hessian matrix. The new solution to the unknown model parameters  $\theta^{k+1}$  is then determined. The first condition to be satisfied is that the model function with the new solution vector should be smaller than the previous one. If so, the value of  $\beta$  can be reduced (by a quarter). This new  $\beta$  can be used in the new iteration following the same procedure until the second condition, the termination or convergence criteria for the Jacobian  $g(\theta)$  is satisfied. If the first condition is not satisfied, the initially assumed  $\beta$  is doubled and  $\delta\theta$  is again solved until both conditions for the model function and the termination criteria are achieved.

Initially, the values of the convergence and termination criteria,  $dG$ ,  $dJ_g$  and  $dG_p'$  (the difference between the values of the guessed parameters in the two successive iterations), are set to be 5% of the guessed values for  $G$ ,  $J_g$  and  $G_p'$ . Further refinement is done by reducing the values of  $dG$ ,  $dJ_g$  and  $dG_p'$  to less than 5% depending on the actual data used to match the model equation.

### 3.4.3.2 Non-linear Regression Equation for the Layered Stabilized Flow Model

Applying the principle of non-linear least squares regression analysis to the Layered Stabilized Flow Model, the objective function  $E$  can be represented as:

$$E = \sum_{j=1}^n (q_{Tj} - f(\theta, t_j))^2 \quad (3.47)$$

The model function  $f$  as defined earlier in Eq. 3.13 is:

$$f(\theta, t_j) = q_{Tj} = \sum_{k=1}^n \left( \frac{J_{gk} p_{ik} z_{ik} z_{jk} G_k - J_{gk} p_{ik} z_{ik} z_{jk} \left( \sum_{l=1}^{j-1} q_{glk} \Delta t_l \right) - J_{gj} p_{wfj} z_{ik}^2 G_k}{z_{ik}^2 G_k + J_{gk} p_{ik} z_{ik} z_{jk} \Delta t_j} \right) \quad (3.48)$$

The gradient vector (Jacobian) is:

$$\left. \left\{ \frac{\partial E}{\partial \theta_x} \right\} \right|_{\theta^0} = -2 \left[ \begin{array}{c} \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_1} \right) \Big|_{\theta^0} \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_2} \right) \Big|_{\theta^0} \\ \dots \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_k} \right) \Big|_{\theta^0} \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial J_1} \right) \Big|_{\theta^0} \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial J_2} \right) \Big|_{\theta^0} \\ \dots \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial J_k} \right) \Big|_{\theta^0} \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_{p1}} \right) \Big|_{\theta^0} \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_{p2}} \right) \Big|_{\theta^0} \\ \dots \\ \left\{ Q_{Tj} - \sum_{j=1}^n f(\theta^0, t_j) \right\} \left( \frac{\partial f}{\partial G_{pk}} \right) \Big|_{\theta^0} \end{array} \right] \quad (3.49)$$

And the Hessian matrix is:

$$H = \left\{ \frac{\partial^2 E}{\partial \theta_x \partial \theta_y} \Big|_{\theta^0} \right\} = -2 \sum_{j=1}^n \left[ - \frac{\partial f}{\partial \theta_x} \Big|_{\theta^0} \frac{\partial f}{\partial \theta_y} \Big|_{\theta^0} + [Q_{t_j} - f(\theta^0, t_j)] \frac{\partial^2 f}{\partial \theta_x \partial \theta_y} \Big|_{\theta^0} \right] \quad (3.50)$$

Simplifying with the Gauss-Newton Method,

$$\frac{\partial^2 f}{\partial \theta_x \partial \theta_y} \Big|_{\theta^0} = 0 \quad (3.51)$$

The final form of the Hessian matrix is given in Eq. 3.50:

$$H = 2 \begin{bmatrix} \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial G_1} & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial G_k} & \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial J_1} & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial J_k} & \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial G_{p1}} & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_1} \frac{\partial f}{\partial G_{pk}} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \sum_{j=1}^n \frac{\partial f}{\partial G_k} \frac{\partial f}{\partial G_1} & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_k} \frac{\partial f}{\partial G_k} & \dots & \dots & \dots & \dots & \dots & \dots \\ \sum_{j=1}^n \frac{\partial f}{\partial J_1} \frac{\partial f}{\partial G_1} & \dots & \dots & \sum_{j=1}^n \frac{\partial f}{\partial J_1} \frac{\partial f}{\partial J_1} & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \sum_{j=1}^n \frac{\partial f}{\partial J_k} \frac{\partial f}{\partial G_1} & \dots & \dots & \sum_{j=1}^n \frac{\partial f}{\partial J_k} \frac{\partial f}{\partial J_k} & \dots & \dots & \dots & \dots & \dots \\ \sum_{j=1}^n \frac{\partial f}{\partial G_{p1}} \frac{\partial f}{\partial G_1} & \dots & \dots & \dots & \dots & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_{p1}} \frac{\partial f}{\partial G_{p1}} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \sum_{j=1}^n \frac{\partial f}{\partial G_{pk}} \frac{\partial f}{\partial G_k} & \dots & \dots & \dots & \dots & \dots & \dots & \sum_{j=1}^n \frac{\partial f}{\partial G_{pk}} \frac{\partial f}{\partial G_{pk}} & \dots \end{bmatrix} \Big|_{\theta^0} \quad (3.52)$$

All the partial derivatives are solved using numerical differentiation (finite-difference approximation), e.g., for the partial derivative of  $f$  with respect to  $G_1$ ,

$$\frac{\partial f}{\partial G_1} \Big|_{\theta^0} = \frac{f(G_1^2) - f(G_1^0)}{G_1^0 - G_1^2} \quad (3.53)$$

where

$f(G_1^0)$  = model function evaluated at 1% less of the guessed value of  $G_1$  (other unknown parameters held constant at their guessed value)

$f(G_1^2)$  = model function evaluated at 1% more of the guessed value of  $G_1$  (other unknown parameters held constant at their guessed value)

$G_1^0$  =  $G_1$  decreased by 1% from its guessed value

$G_1^2$  =  $G_1$  increased by 1% from its guessed value

### 3.5 LSFM Procedure for Calculating Original Gas In Place

The algorithm for calculating the OGIP is outlined below:

**1. Determine the following reservoir and fluid properties for each layer:**

- a) initial reservoir pressure
- b) initial reservoir temperature
- c) gas specific gravity

**2. Setup the production history table with the following required information:**

- a) Date when initial production starts

The production start date is an important information in order to get the correct total cumulative production from all layers from the start of production to the start of the calibration period. From this total initial cumulative production, the initial cumulative production from each layer can then be estimated.

- b) Production data at each time step that provides the following information:

- Date of measurement

The date at which the production data is taken gives the duration or time period in which the well flows at that given rate. This information is used for the calculation of cumulative production from each layer at each time step in the calibration period.

- Choke setting
- Flowing temperature at the surface

- Flowing pressures, either tubing head or bottomhole flowing pressures. If flowing pressures are given in terms of surface pressures, any multi-phase flow calculator can be used for pressure loss calculation to get the flowing bottomhole pressure
- Gas rates
- Condensate and/or water rates

In the absence of measured flowing bottomhole pressure, the condensate and/or water rates together with the flowing temperature and tubing head pressure are used to calculate the flowing bottomhole pressure using an appropriate multi-phase flow correlation. For the bottomhole pressure calculation, this research used the “Prosper Module” of the Petroleum Experts Program Suite for the calculation of the flowing bottomhole pressures.

- 3. Setup the normalized pseudo-pressure table for each layer. This will require the calculation of the z-factors and viscosities at each pressure.**
- 4. Perform the history matching using the Layered Stabilized Flow Model Program.**

The algorithm for history matching using the Layered Stabilized Flow Model Program is presented in the schematic flow chart in Fig. 3.2.

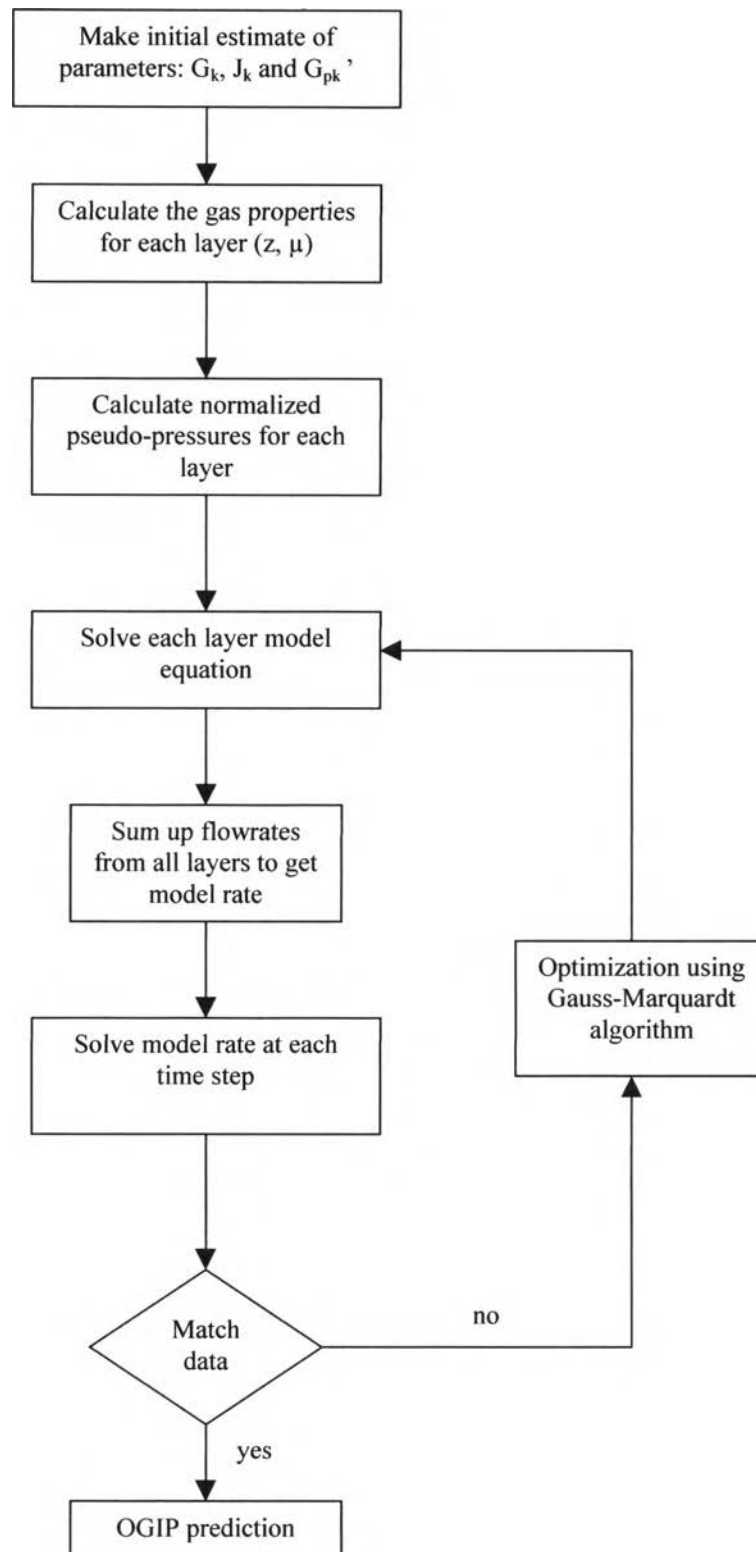


Figure 3.2: Algorithm for the layered stabilized flow model program.