CHAPTER 4

DYNAMIC MATRIX CONTROL

4.1 Introduction

This chapter is introduced a dynamic matrix control (DMC) that selected to control the acetylene concentration in ethylene product. DMC is a control approach based on convolution model of process(Seborg D. E., Edgar T. F., and Mellichamp D. A., 1989).

4.2 The dynamic matrix control

The general control objectives are :

1. To reduce the effect of the external disturbance
2. To keep the process always in the stability condition
3. To increase the quality of production such as product quality, minimum cost of production, or maximum profit.

One general approach to model predictive control, where the process model is used to predict the future outputs over a long time period and minimize the future error between the set point and the future outputs is the dynamic matrix control (DMC).

The DMC is a control approach based on the convolution model or the discrete impulse response model. The advantage of the convolution model is that the model coefficients which many number as many as 50, can be obtain directly from the experimental step response without assuming a model structure and the convolution model also provides a convenient way to design a controller based on the used of optimization theory like DMC.

4.2.1 Step response model

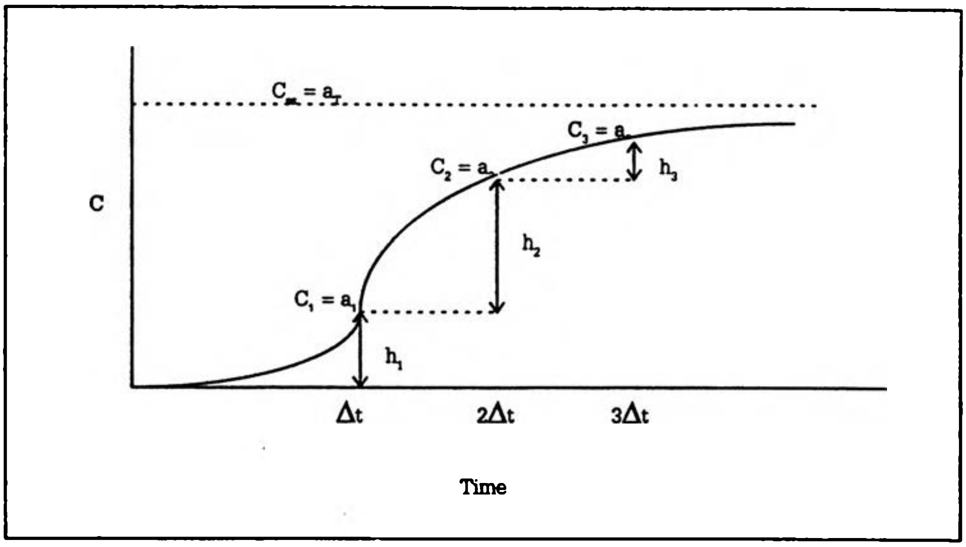

Figure 4.1 Identification of coefficients of step response (a_i) and convolution model (h_i)

To illustrate how a convolution model is developed, consider the typical open-loop step response shown in Figure 4.1. The values of the unit step response are given by a_0, a_1, a_2,..., a_T using the sampling period Δt. Let define $a_i = 0$ for $i \le 0$. $T\Delta t$ may be taken to be the setting time of the process (the time for the open-loop step response to reach 99% completion) and integer T is called the model horizon.

Now consider the step response resulting from a change Δm in the input. Let $\tilde{c}_n$ be the predicted value of the output variable and m_n the value of the manipulated variable at the nth sampling instant. Can also define c_n as the actual output ; thus $\tilde{c}_n = c_n$ if there is no modeling error and no disturbances. Both c and m are expressed as deviation variables. Denoting $\Delta m_i = m_i - m_{i-1}$, the convolution model is

$$\tilde{c}_{n+1} = c_0 + \sum_{i=1}^{T} a_i \Delta m_{n+1-i} \tag{4.1}$$

4.2.2 Impulse response model

The impulse response can be expressed as the first derivative of the step response. For digital system with a zero order hold the impulse response can be found by taking the first backward difference of the step response. The unit impulse response coefficients of the process, $h_1, h_2, ..., h_T$, then are given by

$$h_i = a_i - a_{i-1} \qquad i = 1, 2, ..., T \tag{4.2}$$
$$h_0 = 0$$

and the discrete convolution model using the impulse response coefficients is

$$\tilde{c}_{n+1} = c_0 + \sum_{i=1}^{T} h_i \Delta m_{n+1-i} \tag{4.3}$$

4.2.3 Matrix forms for predictive models

In this section will generalize the convolution model to include an arbitrary number of predictions. A central idea in predictive control is the use of horizons. The two other horizons will define in this section, namely

1.the control horizon U

2.the prediction horizon V

The control horizon U is the number of control action (or control moves) that are calculated in order to affect the predicted outputs over the prediction horizon V, i.e., over the next sampling periods. Thus at time step n the next U values of m are calculated (m_n, m_{n+1}, ..., m_{n+U-1}) as well as the next V output predictions ($\tilde{c}_{n+1}, \tilde{c}_{n+2}, ..., \tilde{c}_{n+V}$) over V future time steps.

Consider the general case of an arbitrary sequence of U input changes, Δm_0, $\Delta m_1, ..., \Delta m_{U-1}$ and an initial steady state $c_0 = 0$. The response can be calculated using the following matrix equation, which based on equation (4.1) and a prediction horizon of V sampling periods :

$$\begin{bmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \cdot \\ \cdot \\ \tilde{c}_{n+V} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0. & \cdots & 0 \\ a_2 & a_1 & 0.. & \cdots & 0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ a_V & a_{V-1} & a_{V-2} & \cdots & a_{V-U+1} \end{bmatrix} \begin{bmatrix} \Delta m_0 \\ \Delta m_1 \\ \cdot \\ \cdot \\ \Delta m_{U-1} \end{bmatrix} \tag{4.4}$$

Both U and V are control design parameters.

Multistep predictions

The major advantage of predictive control is that it incorporation for a number of future time step. This strategy enables the model-based control system to anticipate where the process is heading. The prediction horizon V is a design parameter that influences control system performance. A V-step predictor can be expressed in terms of incremental changes in the manipulated variable :

$$\tilde{c}_{n+j} = c_{n+j-1} + \sum_{i=1}^{T} h_i \Delta m_{n+j-i} \qquad (j = 1, 2, ..., V) \tag{4.5}$$

Equation (4.5) can be applied sequentially to obtain $\tilde{c}_{n+V}$. The recursive version of (4.5) is

$$c'_{n+j} = \tilde{c}_{n+j} + (c'_{n+j-1} - \tilde{c}_{n+j-1}) \tag{4.6}$$

Equation (4.6) uses the difference between c' and $\tilde{c}$ to update the new value of c' using the model prediction at $t = (n+j)\Delta t$, assuming a constant future prediction error. To obtain the solution of equation (4.6) into the future can first set $c'_n = \tilde{c}_n$, the current measured value.

Substituting (4.5) into (4.6) yeilds

$$c'_{n+j} = c'_{n+j-1} + \sum_{i=1}^{T} h_i \Delta m_{n+j-i} \tag{4.7}$$

for $j = 1, 2, ..., V$ and $c'_n = c_n$.

The equation (4.7) can be written in more convenient vector-matrix from by taking the future incremental input changes Δm_{n+j} out of the summations and rearranging. For a prediction horizon V and control horizon U where $U \leq V$, equation (4.7) is equivalent to

$$\begin{bmatrix} c'_{n+1} \\ c'_{n+2} \\ \cdot \\ \cdot \\ c'_{n+V} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0. & \cdots & 0 \\ a_2 & a_1 & 0.. & \cdots & 0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ a_V & a_{V-1} & a_{V-2} & \cdots & a_{V-U+1} \end{bmatrix} \begin{bmatrix} \Delta m_n \\ \Delta m_{n+1} \\ \cdot \\ \cdot \\ \Delta m_{n+U-1} \end{bmatrix} + \begin{bmatrix} c_n + P_1 \\ c_n + P_2 \\ \cdot \\ \cdot \\ c_n + P_V \end{bmatrix} \tag{4.8}$$

where the $\{a_i\}$ are the same step response coefficients defined earlier, namely

$$a_i = \sum_{j=1}^{i} h_j \tag{4.9}$$

and

$$P_i = \sum_{j=1}^{i} S_j \qquad \text{for } i = 1,2,\ldots,V \tag{4.10}$$

$$S_j = \sum_{i=j+1}^{T} h_i \, \Delta m_{n+j-I} \qquad \text{for } j = 1,2,\ldots,V \tag{4.11}$$

The P_i terms are elements of the projection vector, which essentially includes future predictions of c based on previously implemented input changes.

4.3 Controller design method

Controller design in model predictive control is based on the predicted behavior of the process over the prediction horizon. Values of the manipulated variables are computed to ensure that the predicted response has certain desirable characteristics. One sampling period after the application of the current control action, the predicted response is compared with the actual response. Using corrective feedback action for any errors between actual and predicted responses, the entire sequence of calculations is then repeated at each sampling instant.

The control objective is to have the corrected predictions c'_{n+j} approach the set point as closely as possible. Denote the set point trajectory, that is , the desired values of the set point V time steps into the future, as r_{n+j}, $j = 1, 2, \ldots, V$, and define the following vectors :

$$E = \begin{bmatrix} r_{n+1} - c'_{n+1} \\ r_{n+2} - c'_{n+2} \\ \cdot \\ \cdot \\ r_{n+V} - c'_{n+V} \end{bmatrix} \tag{4.12}$$

$$E' = \begin{bmatrix} E_n - P_1 \\ E_n - P_2 \\ \cdot \\ \cdot \\ E_n - P_V \end{bmatrix} \tag{4.13}$$

E' is the predicted value of the process error, *r-c*, at *V* future sampling instants based on manipulated variable(input) changes up to time $(n-1)\Delta t$ and the current error signal, $E_n = r_n - c_n$. P_i , an element of the projected output vector, is defined in equation (4.13)

Note that both E and E' are vectors of prediction errors. E' is an open-loop prediction because it is based only on part control actions. It dose not include the current and future control actions (Δm_{n+j} *for* $j \geq 0$). By contrast, E is referred to as a close-loop prediction because it is based on the current and future control actions.

With the above definitions equation (4.9) can be written as follows :

$$E = -A\Delta m + E' \tag{4.14}$$

where A is the $V*U$ triangular matrix given in equation (4.10), Δm is the $U*1$ vector of future control moves. If a perfect match between the predicted output trajectory of close-loop system and the desired trajectory is required, then $E = 0$ and, from equation (4.14),

$$0 = -A\Delta m + E' \tag{4.15}$$

If the number of control moves and the number of predicted outputs are equal $(U=V)$, then the solution to equation (4.15) is given by

$$\Delta m = A^{-1} E' \tag{4.16}$$

Dynamic matrix control requires that $U < V$ so that the resulting system of equations is overdetermined. Thus, only U future control actions(Δm) are calculated and A is the $V*U$ dynamic matrix. However, to obtain the best solution in the least-squares by minimizing the performance index

$$J[\Delta m] = E^T E \tag{4.17}$$

The optimal solution is

$$\Delta m = (A^TA)^{-1}A^TE' = K_cE' \qquad (4.18)$$

One difficult with the control law of equation (4.17) is that it can result in excessively large changes in the manipulated variable. This undesired phenomenon occurs when the matrix A^TA is ill-conditioned or singular. Can over came this problem by multiplying the diagonal elements of A^TA by a number greater than one before performing the matrix inversion, thus, the performance index is

$$J[\Delta m] = E^TW_1E + \Delta m^TW_2\Delta m \qquad (4.19)$$

where W_1 and W_2 are positive-definite weighting matrices (usually diagonal matrices with positive elements). The resulting control law that minimizes J is

$$\Delta m = (A^TW_1A+W_2)^{-1}A^TW_1E' = K_cE' \qquad (4.20)$$

4.4 Summary

The Dynamic Matrix Control is the selected control approach for this thesis. The basic theories that are presented in this chapter will apply to controller design for the acetylene hydrogenation process. The designed controlier and the simulation results are shown later in chapter 8. The next chapter is the first section in an area of the application. The reviews and the detail of an acetylene hydrogenation process are presented in it.