CHAPTER 3
THEORITICAL CONSIDERATIONS

This chapter reviews some theoretical work carried out in the area of maintenance. The review of literature of this thesis can be separated into five sections.

1. Probability distribution
2. Failure data analysis techniques
3. Bath tub curve concept
4. Maintenance costs
5. Maintenance system

3.1 Probability distribution

In maintenance studies, we are interested in the probability of a failure before some specific time, say *t*. This probability can be obtained from the relevant probability density function as followings:

$$\text{Probability of failure before time } t = \int_{-\infty}^{t} f(t)\,dt = F(t) \qquad \mathbf{3.1}$$

Reliability function is defined as the probability that equipment will survive at least some specific time. The reliability function is denoted by R(t) and is defined as:

$$R(t) = \int_{t}^{\infty} f(t)\,dt \text{, and so}$$

$$R(t) = 1 - F(t) \qquad \mathbf{3.2}$$

Failure distribution functions often used in the maintenance context and in the reliability analysis work are the exponential, Rayleigh, and Weibull distributions. This section discusses these three distribution. Brief is for the first two distributions ; exponential and Rayleigh.

Exponential distribution

Exponential distribution is one that is most widely used in reliability engineering. The distribution failure density function, the cumulative distribution function, the reliability function, the mean value, and the variance are given by

$$f(t) = \lambda e^{-\lambda t} \qquad \textbf{3.3}$$

for $\lambda>0$, $t>0$

where λ is the scale parameter. In reliability work it is known as the constant failure rate of an item.

t is time

$$F(t) = 1 - e^{-\lambda t} \qquad \textbf{3.4}$$

$$R(t) = e^{-\lambda t} \qquad \textbf{3.5}$$

$$\mu = 1/\lambda \qquad \textbf{3.6}$$

$$\sigma^2 = 1/\lambda^2 \qquad \textbf{3.7}$$

Rayleigh distribution

Raylieigh distribution is also used in reliability engineering. The distribution failure density function, the cumulative distribution function, the reliability function, the mean value, and the variance are given by

$$f(t) = \theta t \exp[-\theta t^2/2] \qquad \textbf{3.8}$$

for $t>0$

where θ is the parameter

t is time.

$$F(t) = 1-\exp[-\theta t^2/2] \quad 3.9$$

$$R(t) = \exp[-\theta t^2/2] \quad 3.10$$

$$\mu = (\pi/2\theta)^{1/2} \quad 3.11$$

$$\sigma^2 = (2/\theta)(1-\pi^2/4) \quad 3.12$$

Weibull distribution

Weibull distributions was developed by Weibull and plays an important role in the statistical analysis especially to describe component failures. The Weibull distribution combines the mathematical tractability of the exponential distribution with the flexibility of the Rayleigh distribution. Exponential and Rayleigh distributions are special cases of the Weibull distribution. Therefore, it is one of the most flexible distributions, and it can be used to represent various types of physical phenomena. There are three parameters of this distribution and the probability density function, f(t), of the distribution is defined by

$$f(t) = (\beta/\eta^{\beta})(t-\gamma)^{\beta-1}\exp[-((t-\gamma)/\eta)^{\beta} \quad 3.13$$

for $t > \gamma$ and $\beta,\eta,\gamma > 0$

where β is the shape parameter

η is the scale parameter, or characteristic life - it is the life at which 63.2 % of the population will have failed.

γ is the location parameter, or failure free time or minimum life.

The Weibull cumulative distribution function, F(t) is obtained by substituting Eq. (3.3) in Eq. (3.1) and evaluate its integral.

$$F(t) = 1 - \exp -[(t-\gamma)/\eta)]^{\beta} \quad 3.14$$

Figure 3.1 shows how the shapes of Weibull probability density function (pdf) change as β changes from β=1/5 to β=1/2, β=1, β=3/2, β=2, β=3, and β=5. Some of the specific characteristics of the Weibull pdf. are the following:

1. For 0<β<1 : f(t) decreases sharply and is convex as t increases beyond the value of γ

2. For β=1 : it becomes the two-parameters exponential distribution as a specific case.

$$F(t) = (1/\eta) \exp -[(t-\gamma)/\eta]$$

3. For β>1 : f(t) assumes wear-out type shapes, with increasing failure rate and for 2.6<β<5.3 it may be approximated as a normal probability density function.

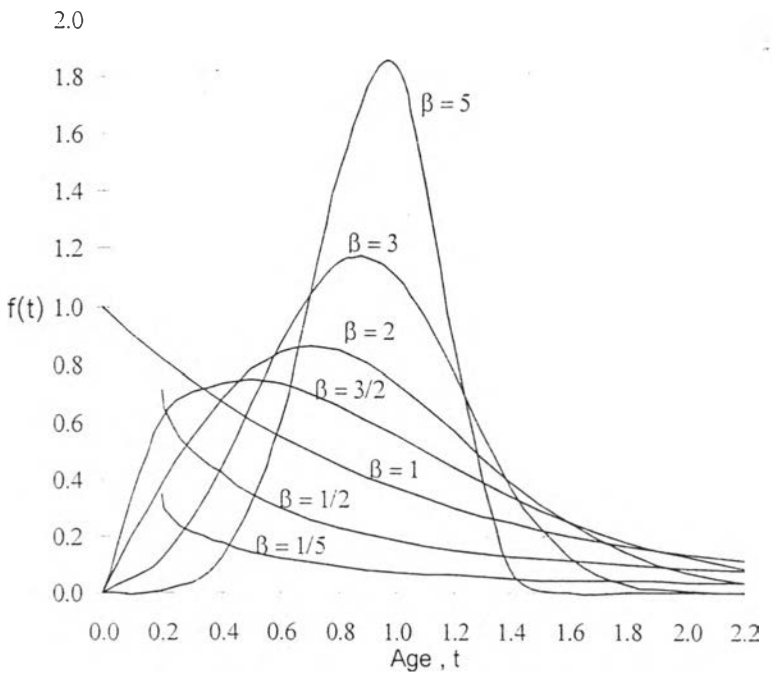


Figure 3.1 Weibull distribution for various values of β, η=1, and γ=0

If η increases while β and γ are constant, the distribution gets stretched out to the right and its height decreases, but maintains its shape and location. On the other hand, if η is decreases while β and γ are constant, the distribution gets stretched out to the left and its height increases. The effects of η on the f(t) are shown in figure 3.2.

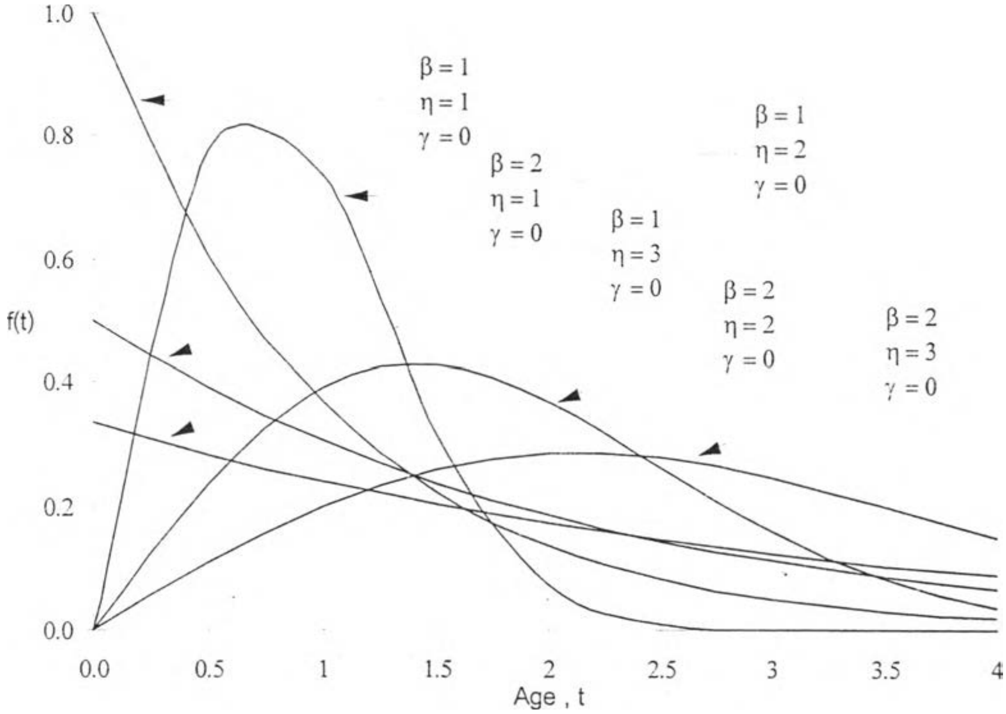


Figure 3.2 The effects of η on the Weibull distribution.

The Weibull failure rate, the mean value, and variance are given by

$$\lambda(t) = (\beta/\eta^{\beta})\, t^{\beta-1} \qquad \textbf{3.15}$$

$$\mu = \eta\Gamma[(1+\beta)/\beta] \qquad \textbf{3.16}$$

$$\sigma^2 = \eta^2[\Gamma(1+2/\beta)-\Gamma^2(1+1/\beta)] \qquad \textbf{3.17}$$

3.2 Failure data analysis techniques

The failure data analysis techniques are used to estimate distribution parameters. The graphical technique is a very popular one because of some advantages, such as it is easy to understand, easy to visualize the theoretical distribution, and easy to analyze both complete and incomplete data.

3.2.1 Weibull probability paper

The Weibull graph paper is an important material of the Weibull distribution because it is a quick and easy way to determine the Weibull parameters and decide whether results are acceptable or not. Cumulative percentage failure is plotted against time to failure then:

- If a straight line results the parameters can be read directly from the paper.
- If the line is not straight but curved or broken, then certain procedure have to be followed to obtain parameters and draw conclusions.
- If there is no relationship, it suggests a particular type of underlying distribution, the extreme value distribution, or that the data is critically examined.

Weibull probability paper is constructed as following:

$$R(t) = 1\text{-}F(t) = \exp\left[-((t-\gamma)/\eta)^{\beta}\right]$$

Assume $\gamma=0$, that is minimum life or failure free time = 0

$$1\text{-}F(t) = \exp\left[-(t/\eta)^{\beta}\right]$$

$$1/[1\text{-}F(t) = \exp (t/\eta)^{\beta}$$

Taking logarithms of both sides:

$$\ln\left[1/(1\text{-}F(t))\right] = (t/\eta)^{\beta} \qquad \text{(since } \ln e = 1)$$

Taking logarithm again:

$$\ln \ln [1/(1-F(t))] = \beta \ln t - \beta \ln \eta \qquad \textbf{3.18}$$

This is in a straight line form of $y = ax + b$ which:

$y = \ln \ln [1/(1-F(t))]$

$a = \beta$

$x = \ln t$

$b = -\beta \ln \eta$

Therefore, if the Weibull graph paper is used whose y axis is $\ln \ln [1/(1-F(t))]$ and whose x axis is $\ln t$, a straight line will result when cumulative percentage failure is plotted against time to failure.

In this case we assume $\gamma=0$, therefore if there is some minimum life before the first failure, the x axis will be $(t-\gamma)$ and for obtaining a straight line an estimate of γ must be subtracted from each failure time (the method of estimating γ will be shown following).

3.2.2 Cumulative percentage - Median ranks

For a large amount of data the cumulative percentage for each class of the frequency distribution could be used. However, in many cases, the data collected may be not large enough to construct a frequency distribution. So, if a straight percentage is taken to represent cumulative percentage it will overestimate the proportion failing. For example, if there are 10 items are tested and the first failure is given a value of 10%, there will be a bias because

(a) of sampling error
(b) the first failure is further from 0% than the last is from 100% and likewise for the remaining results.

Failure No.		1	2	3	4	5	6	7	8	9	10
Cumulative Percentage	0	10	20	30	40	50	60	70	80	90	100

To correct this, median ranks should be used. The median rank is the 50% point of the assumed Beta distribution of the i th event out of n. Median ranks can be obtained from tables (shown in Appendix A) or from an approximation due to Benard as following.

$$\text{Median rank} = (i - 0.3)/(n + 0.4) \qquad \textbf{3.19}$$

where n = sample size

i = failure order number

So for the sample of 10 items the cumulative percentage are:

Failure No.	1	2	3	4	5	6	7	8	9	10	
	0	10	20	30	40	50	60	70	80	90	100
Cumulative Percentage	6.7	16.2	25.9	35.5	45.1	54.8	64.5	74.1	83.8	93.3	

3.2.3 Procedure for plotting on Weibull probability paper

There are 10 steps of this procedure described in Chartwell 6572 as:

1. Draw up three column:

   Failure Number   Time to Failure   Median Rank

2. Put failure times in order, lowest to highest, in column 2.

3. Find the column for the sample size in the Median rank tables or calculate from the Benard equation and then complete the column 3.

4. Plot Median Rank against Time to Failure on the appropriate Weibull probability paper.

5. If the data appears to be a straight line draw the line, paying more attention to the higher cumulative percentage points than lower cumulative percentage points.

6. For a straight line, write in a value of Minimum life, γ equals to 0 at the top of the paper.

7. Draw a perpendicular from the line through the Estimation point at the top left hand side of the paper.

8. Where the perpendicular line crosses the β scale, read off the value of β and write it at the top of the paper.

9. Project across from "η Estimator" on the vertical axis to the line and then down and read the value off the horizontal axis. Fill in this value of η, the characteristic Life, at the top of the paper.

10. Read the values off the P_{μ} scale where the perpendicular line crosses it. Locate this value on the vertical scale and project across to the line and down. Read off the value on the horizontal scale which is the Mean Life, μ. As a check that the line has been drawn correctly, calculate the mean time to failure from the original data and compare it with the Mean Life read off the graph.

3.2.4 Procedure for dealing with a curved plot

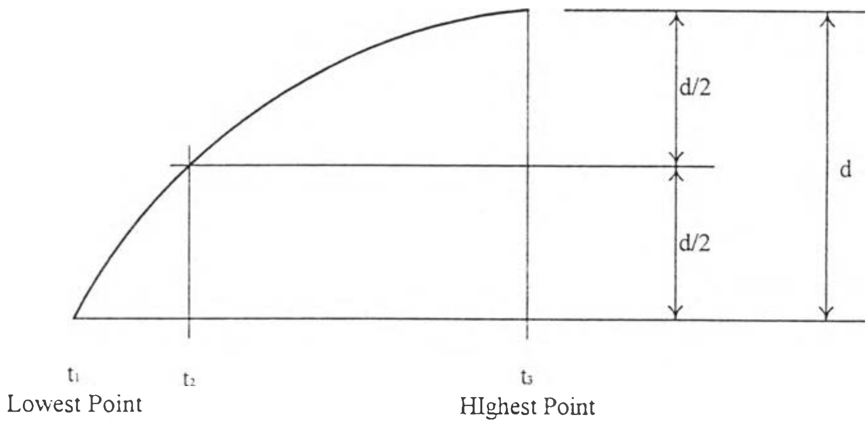
The procedure consists of 9 steps as shown following.

1. Plot the results on Weibull probability paper as described above. If a curve results, first check that there is only one failure mode.

2. If the plot is convex when viewed from the top of the paper conclude that there is some minimum life before failures start to occur.

3. If the plot is concave when views from the top of the paper conclude that the items had already started deteriorating before they were put on test.

4. Estimate a value of γ. This is best done by using the formula below.



The distance d is the linear vertical distance, it is not read off the vertical axis. t_2 is read off the plot then γ is found from:

$$\gamma = t2 - \frac{(t3 - t2)(t2 - t1)}{(t3 - t2) - (t2 - t1)} \qquad \textbf{3.20}$$

5. Add another column to the table results:

Failure No. Time to Failure Median Rank t-γ

6. Subtract γ from each value of t.

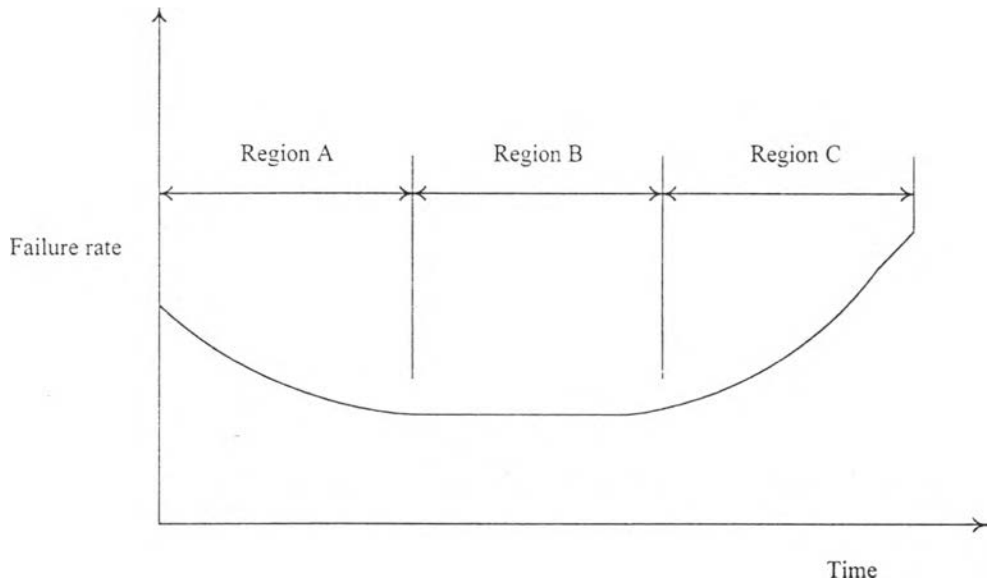
7. Plot Median Rank against t-γ. If the plot still looks curved, adjust the value of γ until it becomes straight.

8. When reading off values of Characteristics Life, Minimum Life, Mean Life, etc. add the value of γ back on.

9. For concave plots γ will have to be added on to t to give a straight line. The procedure for estimating γ will be exactly the same.

3.3 Bath tub curve concept

The effectiveness and economy of preventive maintenance can be maximized by taking account of the time-to-failure distribution of the maintained parts and the failure frequency (failure rate) trend of the system. In general, a device or a part goes through three separate and distinct stages. The first is called a running-in stage (burn-in of infant mortality phase), during which the device is characterized by a decreasing failure rate. The second is called the operating stage (also called the useful life phase), during which we experience a constant failure rate. The third is called the wear-out stage (deterioration) during which the failure rate is increased rapidly. When the failure rate is increasing, it is an indication that the device has aged or become worn. A Weibull distribution can represent the "Bath tub curve" as shown in Figure 3.3.



Region A : a running-in stage

Region B : operating stage

Region C : deterioration (i.e. wear-out)

Figure 3.3 Bath tub curve.

Typically, if a part has a decreasing failure rate (Region A), any replacement will increase the probability of failure. If the failure rate is constant (Region B), replacement will make no difference to the failure probability. **If the part has an increasing failure rate (Region C), then preventive replacement will reduce the probability of equipment failure** in the future and just where these preventive replacements should occur will be influenced by the relative costs. Moreover, if the part has a failure free life (Weibull $\gamma>0$), then replacement before this time will ensure that failures do not occur.

Many of the reasons for the decreasing failure rate region failures are listed in Table 3.1. Most producers of manufactured engineering equipment provide a "running-in period" for their products.

No.	Reason
1	Inadequate quality control
2	Inadequate manufacturing methods
3	Substandard materials and workmanship
4	Wrong start-up and installation
5	Difficulties because of assembly
6	Inadequate debugging
7	Inadequate processes and human error
8	Inadequate handling methods and wrong packaging

Table 3.1 Reasons for "running-in region" failures.

In the operating region of Figure 3.3, the failure rate is constant. In additional, the failures occur unpredictably and therefore, are known as random failures. Some of the reasons for these failures are given in the Table 3.2.

No.	Reason
1	Causes which cannot be explained
2	Human errors, abuse, and natural failures
3	Unavoidable failures: these cannot be avoided by even the most effective preventive maintenance practices
4	Undetectable defects
5	Low safety factors
6	Higher random stress than expected

Table 3.2 Reasons for "operating region" failures

In the wear-out region of Figure 3.3, the failure rate starts increasing. This increase indicates the end of the useful life period of the product in question. Some causes for the failures during the "wear-out period" are listed in Table 3.3. Failures occurring in this period can be reduced with properly designed replacement and preventive maintenance policies and procedures.

No.	Reason
1	Inadequate maintenance
2	Wear due to friction
3	Wear due to aging
4	Wrong overhual practices
5	Corrosion and creep
6	Desighed-in life of the product is short

Table 3.3 Reasons for "wear-out region" failures

3.4 Maintenance costs

The maintenance cost can be divided into 2 major classes:

1. Direct maintenance cost comprising the main costs of
    a. Manpower or labor costs
    b. Material cost
    c. Cost of tools for repairing
    d. Overhead cost

2. Indirect maintenance cost comprising of

a. Equipment costs, such as
   - Accelerated wear because of poor maintenance
   - Excessive spare parts inventory
   - Unnecessary equipment redundancy
   - Disproportionate energy consumption

b. Production related cost, such as
   - Rework because of badly aligned equipment
   - Excessive scrap and material losses
   - Idle work due to breakdown
   - Late shipment because of unplanned down time

c. Product cost due to
   - Quality and reliability issues
   - Lost sales because of long down time periods
   - Warranty claims from dissatisfied customers

3.5 Maintenance system

Typically, maintenance system is subjected to *preventive* and *corrective* maintenance.

3.5.1 Preventive maintenance

Preventive maintenance seeks to retain the system in an operational or available stage by preventing failures from occurring. This can be by servicing, such as tuning or adjusting, lubrication, inspection and corrective action and cleaning. Preventive maintenance also involves the replacement of the marginal parts or components, even though they are still functioning, because they can be a cause of machine failures or downtimes. Preventive maintenance is measured by the time taken to perform the specified maintenance tasks and their specified frequency.

Preventive maintenance can be classified into 3 types as:

1. Opportunity maintenance

Opportunity maintenance involves action taken to prevent future machine failure when the machine is not operating for any reason. The opportunity is taken to conduct preventive activity during repair for previous failure, shift change over, meal breaks, slack time, works holiday shut down etc. characteristics of this type of maintenance are:

- It provides an overall downtime reduction compared to the operate to failure policy.
- It provides better utilization of maintenance personnel.
- It is planned but unscheduled.
- It can only occur if spares and personnel are available when the opportunity arises.
- It enables some control of the spares inventory.

2. Planned preventive maintenance

Planned preventive maintenance involves action taken at appropriate, planned times that reduce the probability of unpredictable failures. Its characteristics are:

- It only appropriate if:
  - the cost of prevention is less than or equal to the total cost of failure.
  - the failure is age related.
- a well functioning machine may be disturbed and maintenance related failures may be introduced.
- it enables the best utilization of spares and personnel.
- disruption to production is minimized.

- it may result in the replacement of part life components, which may be expensive if the components can not be used.

3. Predictive maintenance

Predictive maintenance involves action taken to prevent future machine failure, initiated as the result of regular checks on the condition of the machine or the quality of the product. The characteristics of this type of maintenance are:

- it needs a parameter that indicates the health of a machine or provides warning of impending failure.
- monitoring may or may not require machine shut down.
- parameter analysis may be conducted on-line or off-line.
- it is less wasteful of spares life than planned preventive maintenance.
- it may be possible to plan preventive action.
- statistical process control is a powerful predictive tool in many cases.

3.5.2 Corrective maintenance

Corrective maintenance is undertaken only when it is necessary because of a malfunction or failure. Corrective maintenance includes all action to return a system from a failed to an operating or available state. Generally, the corrective action consists of replacing parts and components which have failed or making general repairs such as splicing a broken lead. Corrective maintenance can be quantified as the mean time to repair (MTTR).

The time to repair, however, includes several activities, usually divided into three groups:

1. Preparation time: finding the person for the job, travel, obtaining tools and test equipment, etc.

2. Active maintenance time: actually doing the job.

3. Delay time: waiting for spares, etc.