CHAPTER II

RADIO ASTRONOMY FUNDAMENTALS

This chapter provides the important concept for the measurement the radio signal for the celestial objects. We start with the blackbody radiation connected to the *brightness B* which is the physical quantity of interest. The other related will be all so considered. From the observational point of view ,the physical quantity which we can develop from the electromagnetic energy radiated is the power. It is useful to relate them to the brightness of the distance object and the treatment is in the next part. The last one is about the coordinate and time system which play the important role on the astronomical observation.

Blackbody Radiation

All objects at temperatures above absolute zero radiate energy in the form of electromagnetic waves. In 1859, G. R. Kirchoff showed that a good radiator is also a good absorber. A perfect object which can absorb or reflect energy incident from them is called *blackbody*.

For the blackbody of temperature T, the energy (in form of power) radiated per unit area per unit bandwidth per unit angle, called *brightness B* at the observed frequency ν is

$$B = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1} \tag{2.1}$$

where B = brightness, W m^{-2} Hz^{-1} sr^{-1}

h = Planck's constant (= 6.63 x 10^{-34} J-s)

ν = frequency, Hz

c = velocity of light (= 3 x 10^8 m s^{-1})

k = Boltzmann's constant (= 1.38x10^{-23} J K^{-1})

T = absolute temperature, K

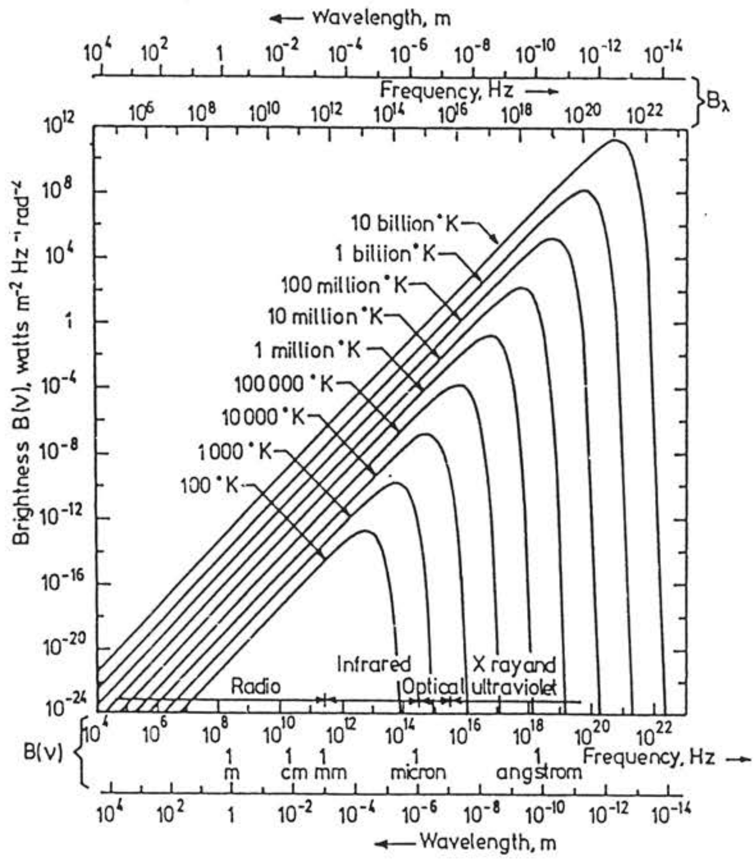

Fig 2.1 Blackbody radiation curve (Rohlfs, 1990).

This law is called *Planck's radiation law* which proposed by Planck in 1901. Referring to Fig 2.1, we can see that the higher temperature has the higher brightness. For a constant temperature, the curve reach the peak at a certain frequency and roll off if the observed frequency is higher than the peak frequency. In logarithmic scale, we can see that the curve of low frequency seem linearly. It is according to the special case when the situation of h << kT occur as in the case of radio astronomy. The approximation can be derived by Taylor's expansion.

$$e^{h\nu/kT} - 1 \cong 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$$

Introducing the result to (2.1)

$$B = \frac{2h\nu^3}{c^2}\frac{kT}{h\nu} = \frac{2kT}{\lambda^2} \tag{2.2}$$

Where λ = wavelength, m.

The approximation is the *Rayleigh-Jeans radiation law* which is useful approximation in the radio part of the spectrum.

Although the Rayleigh-Jeans approximation is a special case of the more general Planck's radiation law, it may be derived directly by consideration of the classical behavior of a blackbody radiator. The Rayleight-Jeans law antedates the Planck's law before the quantum concept was established.

Brightness Distribution and Incident Power

In the observational situation, the electromagnetic energy is falling on the earth surface in any direction. By the aid of the directional antenna, the brightness of the observed is distinguished from the other signal.

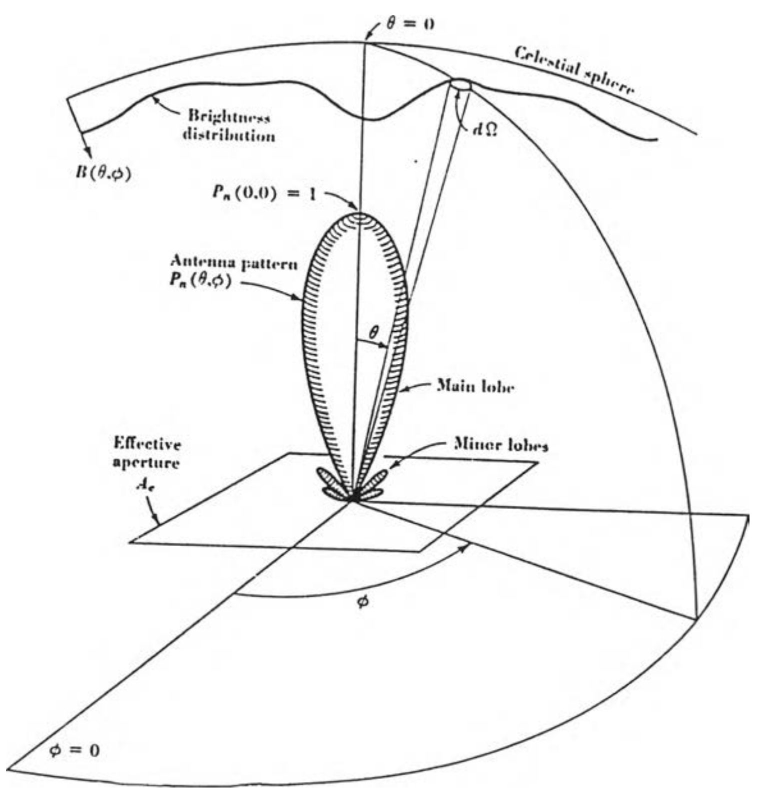

Fig 2.2 Antenna power pattern and the brightness distribution of the sky (Kraus, 1986)

The response of the antenna to the radiation is called the *power pattern* P_n. It is the normalized quantity which is the function of angle θ and ϕ, as suggested in Fig 2.2.

It is reasonable to imagine that the antenna collecting the energy from the sky incident on an arbitrary area. The energy collecting area is called the *effective aperture* A_e of the antenna.

The P_n and A_e are characteristic of antenna, regardless the physical appearance of the antenna. They will be discussed in detail by the next chapter.

Hence, the infinitesimal power dW from a solid angle dΩ of the sky incident on the antenna with the effective aperture Ae and the power pattern $P_n(\theta,\phi)$ may be expressed by

$$dW = B(\theta,\phi) \cdot P(\theta,\phi) \cdot A_e d\Omega d\nu$$

where dW = infinitesimal power, W

$B(\theta,\phi)$ = brightness of the sky from direction (θ,ϕ), W m^{-2} sr^{-1} Hz^{-1}

$P_n(\theta,\phi)$ = normalized power pattern of the antenna

A_e = effective aperture of the antenna, m^2

dΩ = infinitesimal solid angle of the sky, sr^{-1}.

dν = infinitesimal element of the detected bandwidth, Hz

By integration over all direction, we obtain the amounts of total power received by antenna at its terminal as follow

$$W = A_e \int_{\nu_o - \frac{\Delta\nu}{2}}^{\nu_o + \frac{\Delta\nu}{2}} \iint_{\Omega} B(\theta,\phi) \cdot P_n(\theta,\phi) \cdot d\Omega d\nu \qquad (2.3)$$

The ν_o is the operating frequency and $\Delta\nu$ is the bandwidth of the antenna. When the signal is feed through the receiver, the bandwidth of the signal is limited by predetection section of the receiver. Hence, we may considered $\Delta\nu$ as the bandwidth of the receiver.

A the previous situation, the power per unit bandwidth is more pertinent than the power contain in an arbitrary bandwidth, W. This power per unit bandwidth is often called the *spectral power w* which can be expressed in the following form

$$dw = B(\theta,\phi) \cdot P(\theta,\phi) \cdot A_e d\Omega$$

where dw = infinitesimal spectral power, W/Hz.

Also, the spectral power from the radiated energy of the entire sky received at the antenna's terminal can be written as

$$w = A_e \iint_{\Omega} B(\theta,\phi) \cdot P_n(\theta,\phi) \cdot d\Omega \qquad (2.4)$$

The relation (2.3) and (2.4) is valid for the antenna which is able to collect the electromagnetic energy at any polarization. If the radiation is of an incoherent, unpolarized nature, only fraction of the incident power will be received since only antenna is responsive to only certain polarization component (Kraus, 1986 ; Brown and Lovell, 1958). For the linearly polarized antenna, the fraction is 1/2. Hence the (2.3) and (2.4) can be respectively written for such antenna as

$$W = \frac{1}{2} A_e \int_{\nu_o - \frac{\Delta\nu}{2}}^{\nu_o + \frac{\Delta\nu}{2}} \iint_{\Omega} B(\theta,\phi) \cdot P_n(\theta,\phi) \cdot d\Omega d\nu \tag{2.5}$$

$$w = \frac{1}{2} A_e \iint_{\Omega} B(\theta,\phi) \cdot P_n(\theta,\phi) \cdot d\Omega \tag{2.6}$$

As (2.5) and (2.6), we can determine the received output power for arbitrary bandwidth $\Delta\nu$, if we know the spectral power, by the following relation.

$$W = \int_{\nu_o - \frac{\Delta\nu}{2}}^{\nu_o + \frac{\Delta\nu}{2}} w \cdot d\nu \tag{2.7}$$

Temperature and Noise

In the previous section we considered antenna as the electromagnetic energy's collector from the brightness distribution around them. It has already been discussed the mechanical of antenna as the radiometer for determining the temperature of distant region of space.

In 1928, H. Nyquist was proposed that the noise power per unit bandwidth available at the terminal of a resister R and temperature T is given by

$$w = kT \tag{2.8}$$

where w = spectral power or power per unit bandwidth, w/Hz

k = Boltzmann's constant (=1.38 x 10^{-23} J/K)

T = absolute temperature, K.

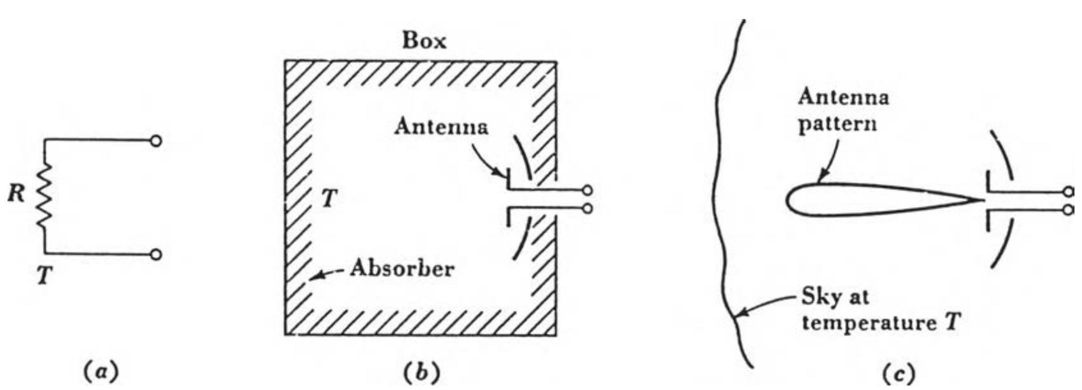

Fig 2.3 Thermal noise at the terminals of the resister (a), the antenna in blackbody enclosure (b), and antenna in the sky (c) (Kraus, 1986).

The thermal motion of electrons in the resister produce a current which form the random input to the terminal. The mean value of this current will be zero, but the rms. value of the current i, $\langle i^2 \rangle$ represent the power in the resister as expressed by

(2.8). The spectral power of the resister is independent of the value of the resister (Kraus, 1986; Rohlfs, 1990).

As Fig 2.3b, if the resister is replaced by a lossless matched antenna enclosing by a blackbody enclosure with ambient temperature T, the spectral power measured at the antenna terminals remain the same value as resister of temperature T since the intrinsic impedance of the antenna.

Now the enclosure has been removed and replaced by the sky with temperature T as shown in Fig 2.3c . The antenna sees the sky through the power pattern. Since the antenna embedded at temperature T, the brightness $B(\theta,\phi)$ are the same in all directions. By Rayleigh-Jeans approximation as indicated by (2.2), we have

$$w = \frac{kT}{\lambda^2} A_e \iint_{\Omega} P_n(\theta,\phi) d\Omega$$

If we defined the integration of the antenna's power pattern on the right side of the equation as the *beam solid angle* Ω_A of the antenna (discussed in the next chapter), which is

$$\Omega_A = \iint_{\Omega} P_n d\Omega$$

we have

$$w = \frac{kT}{\lambda^2} A_e \Omega_A$$

The product of effective aperture A_e and the beam solid angle Ω_A of the antenna equals to the square of the operating wavelength, say $A_e\Omega_A = \lambda^2$ (derived in the next chapter), yields

$$w = kT$$

It can be referred from this result that the antenna behave as the resister R of temperature T. However, the spectral power is the same but the antenna see any portion of the sky unequally since the power pattern is the function of directions.

The spectral power measured at the antenna terminal can be determined by (2.6). It determines the temperature of the distance regions in space couple to the system through the radiation resistant of the antenna (Kraus, 1986). The temperature of the antenna resistant is called the *antenna temperature* T_A. It is important to note that it is not the temperature of the antenna structure (Kraus, 1986).

This property is important for astronomical purpose since it is possible to determine the temperature from the remote object in the sky. If the brightness distribution of the sky, $B(\theta,\phi)$, is considered as the radiation of black body with temperature $T_s(\theta,\phi)$, the spectral power can be expressed as

$$w = \frac{1}{2} A_e \iint_{\Omega} \frac{2kT_s(\theta,\phi)}{\lambda^2} P_n(\theta,\phi) d\Omega = kT_A$$

$$T_A = \frac{A_e}{\lambda^2} \iint_{\Omega} T_s(\theta,\phi) P_n(\theta,\phi) d\Omega$$

Since $A_e\Omega_A = \lambda^2$, we have

$$T_A = \frac{1}{\Omega_A} \iint T_s(\theta,\phi) P_n(\theta,\phi) d\Omega \quad (2.9)$$

We can refer this relation that the antenna temperature is the integration of the temperature of the source in all direction weighting by the antenna pattern. It is impossible to determine the exact temperature of a certain object if the main lobe of the antenna is insufficiently sharp. It is suffered from the temperature received by the minor lobe. As the result, the pin-point beam antenna is the major requirement of the modern radio telescope.

The source temperature, $T_s(\theta,\phi)$ is the *equivalent temperature or brightness temperature* which may be equal to the thermal temperature of the source if the radio noise power is due to thermal emission. However, if the noise can be generated by a non-thermal mechanism, such as plasma or synchrotron oscillation which the equivalent temperature is greater than the thermal temperature of the source (Kraus, 1986).

Flux Density

In the previous sections we discussed the measured power from the distributed source around the antenna. The individual source will be treated in this section.

Discrete radio source is classified as the *point source* when the source subtends the infinitesimal sky portion compared to the solid angle of the antenna's main beam. The largest sources may be classified as the *localized* or *extended source*. In common practice, it may be regard the sources of less diameter than 1^o as the localized source and the sources with diameter of more than 1^o as the extended source (Kraus, 1986).

For any discrete source, the integral of the brightness over the source yield the total source *flux density S*. Thus,

$$S = \iint_{source} B(\theta,\phi)d\Omega \tag{2.10}$$

where S = flux density of source, W m^{-2} Hz^{-1}

This unit is inconveniently large in practice. The flux density are normally expressed in the smaller unit of Jansky (Jy) which identical with the *flux unit* found in the older literature and is defined as $1x10^{-26}$ W m^{-2} Hz^{-1} (Rohlfs, 1990, Christiansen and Hogbom, 1985). The name Jansky is after the pioneer radio astronomer Carl G. Jansky.

When the source is observed with and antenna of power pattern $P_n(\theta,\phi)$, the *observed flux density S_0* is

$$S_o = \iint_{source} B(\theta,\phi)P_n(\theta,\phi)d\Omega \qquad (2.11)$$

The observed flux density is usually less than the actual flux density from (2.10). If the observed source is the point source and situated at the main beam direction the $P_n(\theta,\phi) \cong 1$, the observed flux density approach to the true value. From this reason, the flux density is appropriate quantity to express the intensity of the point source. For the sources of the larger extent, the flux density refer to some portion of the source. Hence, for the largest sources, the brightness $B(\theta,\phi)$ or the brightness temperature $T_s(\theta,\phi)$ of the source should be applied.

Following from the previous reason, when the antenna is aligned with the localized or extended source and $P_n(\theta,\phi)$ in minor lobe is considerably small, the ***observed*** or ***apparent brightness B_o*** is then

$$B_o = \frac{\iint B(\theta,\phi)P_n(\theta,\phi)d\Omega}{\iint P_n(\theta,\phi)d\Omega} = \frac{S_o}{\Omega_A} \qquad (2.12)$$

The variation of the flux density S with frequency is called the *flux density spectrum*. The integration $S_o(\theta,\phi)$ over a operating bandwidth $\Delta\nu$ yield the *total flux density S'* in this band of frequency $\Delta\nu$, thus

$$S' = \int_{\nu_o - \frac{\Delta\nu}{2}}^{\nu_o + \frac{\Delta\nu}{2}} S d\nu \qquad (2.13)$$

It follows that the total (observed) power W at the receiver of bandwidth $\Delta\nu$ is given by

$$W = \frac{1}{2} A_e \int_{\nu_o - \frac{\Delta\nu}{2}}^{\nu_o + \frac{\Delta\nu}{2}} \iint_{\Omega} B(\theta,\phi) P_n(\theta,\phi) d\Omega d\nu$$

$$= \frac{1}{2} A_e S'_o \qquad (2.14)$$

where S'_o = observed flux density spectrum from the source, if the minor lobe of the antenna considerably small.

If the radiating source of temperature $T_s(\theta,\phi)$ subtends a solid angle Ω_s, the observed flux density can be calculated by aid of the Raleigh-Jean approximation. Thus,

$$S_o = \frac{2k}{\lambda^2} \iint_{\Omega_s} T_s(\theta,\phi) d\Omega \qquad (2.15)$$

From above equation, If the source have uniform temperature distribution T_s $(\theta,\phi) = T_s$, we have

$$S_o = \frac{2kT_s}{\lambda^2}\Omega_s. \tag{2.16}$$

The Minimum Detectable Temperature and Flux Density

The temperature which we measured at the receiver is not only the antenna temperature T_A as discussed in the previous section, but also the temperature from another noise source in the system such as the transmit line or the receiver itself.

The transmission line have an certain impedance act as the resister in the Nyquist's theorem. Hence, it add the noise to the signal received from antenna. By the way the antenna temperature is higher than the actual value.

In the receiver, the component emitted the thermal noise. This noise is amplified along with any external signal and cannot in principle be distinguished from it, if there are no any calibration measure (Rohlfs, 1990).

The *sensitivity* or *minimum detectable temperature* can be defined as the value of rms. noise temperature. The system noise can be reduced to desire extent by increasing integration time (after detection). Increasing predetection bandwidth, or by taking average more than one observation (Kraus, 1986). The increasing integration time and averaging process give a change of cancellation for the noise fluctuation. The increasing bandwidth increase the noise and also the more cancellation is accomplished. The noise level is the minimum detectable level. Since the *root mean* *square (rms.)* value of the signal, the noise level may be represented by rms. level. If the receiver is of the total-power type (see the next chapter), the sensitivity may be written as (Kraus, 1986; Brown and Lovell, 1958; Rohlfs, 1990)

$$\Delta T_{min} = \Delta T_{rms} = \frac{T_{sys}}{\sqrt{\Delta\nu \cdot t \cdot n}} . \qquad (2.17)$$

where ΔT_{min} = sensitivity or minimum detectable temperature, K

ΔT_{rms} = rms. system noise temperature K

T_{sys} = system noise temperature = $T_A + T_R$,K

T_A = antenna temperature

T_R = noise temperature for the receiver

$\Delta\nu$ = predetection bandwidth, Hz

t = integration time, s

n = number of record average

According to the Rayleigh-Jeans approximation, the *minimum detectable brightness* ΔB_{min} can be determined by

$$\Delta B_{min} = \frac{2k}{\lambda^2} \frac{T_{sys}}{\sqrt{\Delta\nu \cdot t \cdot n}} . \qquad (2.18)$$

Since the spectral power measured at the antenna terminal have the property as the thermal noise as discussed in the previous section, we have

$$w = kT_A = \frac{1}{2}A_e \iint_{\Omega} B(\theta,\phi)P_n(\theta,\phi)d\Omega = \frac{1}{2}A_e S_o$$

$$S_o = \frac{2kT_A}{A_e} \tag{2.19}$$

Substitute the minimum detectable temperature in (2.17) to the antenna temperature T_A in (2.19), we have the *minimum detectable flux density* ΔS_{min} which is

$$\Delta S_{min} = \frac{2k}{A_e}\frac{T_{sys}}{\sqrt{\Delta\nu \cdot t \cdot n}} . \tag{2.20}$$

Astronomical Coordinate and Time

It is essential to understand the astronomical coordinate system and the measure of time when we are interested in study the celestial object.

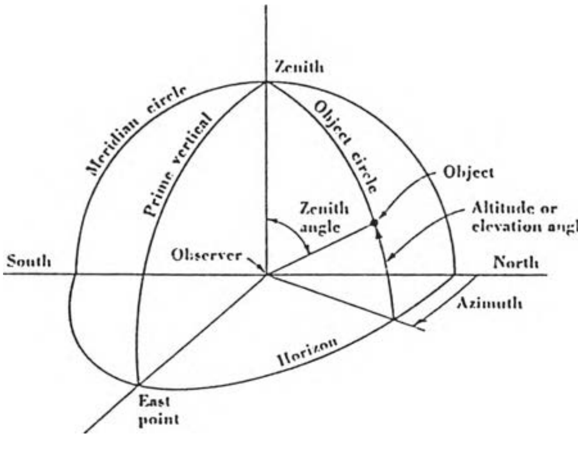

Fig 2.4 Horizontal coordinate system (Kraus, 1986).

We should start with the *horizontal coordinate system* which is used to describe the position of object at the observer's site. As the perspective of observer on the Earth, the sky appears the hemisphere as shown by Fig 2.4 .

The point overhead the observer is called *zenith*. It may also be imagined another hemisphere which complete the sphere. The point of another site of the zenith is called *nadir*. The great circle which passed through the zenith and nadir and oriented in direction north and south is called the *observer's meridian*.

In this system, we can identify and object by the *azimuth, Az* or horizontal angle from the North clockwise to the circle which pass the poles (i.e., zenith and nadir) through the object of interest, and the *altitude* Alt, or elevation angle from the observer's horizon to the object. The altitude may be replaced by the *zenith distance Z*, which is the angle from the zenith to the object of interest, if it is more convenient. The identification is shown in Fig 2-4.

It is clearly to see the simplicity of this system, but the system depend on the position of the observer in the Earth's surface and also the celestial objects are moving from east to west when the time is elapsed.

The other *equatorial coordinate* in this situation, is introduced. Although, we know that the Earth have the motion relatively to the other object in the universe, it remain useful to apply the geocentric concept to measure the position of celestial object. As the revelation of the observer on the Earth, the sphere of the sky rotated around an axis which pass through the fixed poles in the sky rather than the Earth have rotation in the space. The sphere of the sky contain all celestial objects is called the *celestial sphere*. The poles of the celestial sphere moving around coincide with the north and south poles of the Earth as seen in Fig 2.5 .

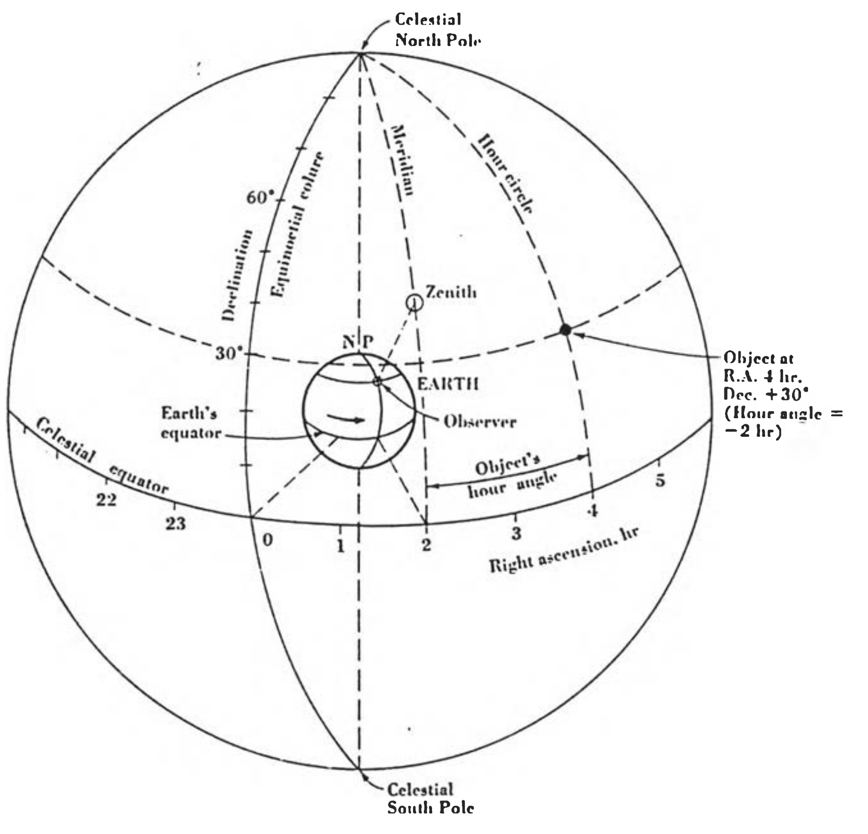

Fig. 2.5 Equatorial coordinate system (Kraus, 1986).

The poles are *north celestial pole (NCP)* and *south celestial pole (SCP)* respectively. The great circle which is the extension of the Earth's equator is also called the *celestial equator*. All of the celestial object except the sun and the other members of the solar system are seem to occupy fixed position in the celestial sphere since the immensely distance from the Earth.

The coordinate of a celestial object are given by the *right ascension* α and the *declination* δ. The great circle which pass through the object and the celestial poles is called the *hour circle*. The declination is the angle from the celestial equator to the object of interest measured along the hour circle. The right ascension is the angle between and arbiter point at the celestial equator to the object's hour angle. The arbitrary point of reference is the *vernal equinox* γ which is the point where the orbital path of the sun on the celestial sphere intersect with the celestial equator and the sun will ascend to the northern hemisphere after situate at that point.

The right ascension is expressed in unit of hour, minute and second of time $(0^h \leq \alpha < 24^h)$ and the declination is expressed in degree. If the object is north of the celestial equator, the declination is positive and will be negative if south $(-90^\circ \leq \delta \leq +90^\circ)$. For example, the object, as shown in Fig 2.5 situates $\alpha = 4^h 0^m 0^s$ and $\delta = +30^\circ$.

It is useful to relate both system together. Now, we have to know that the position of a distance object depends on the position of the observer and the observation time in the horizontal coordinate system, but it is rather fixed in equatorial coordinate. At first, it should be considered some aspect of time.

The measurement of time is related to the rotation of the Earth, i.e. we define one day $(24^h 0^m 0^s)$ as the period of the Earth's rotation. there are two points of reference use to identify the completeness of the rotation. One is an arbitrary fixed point in the space and the other one is the Sun. The vernal equinox was selected to the reference point in the space.

The period of the vernal equinox situated to the same point in horizontal system of a certain observation site is defined as the *sidereal day* which coincide with the period of the Earth compare to the vernal equinox and have the same value for the period of rotation of celestial sphere. The sidereal day is not common for human whose activities depend on the daylight, so the Sun is more rational to apply than the vernal equinox.

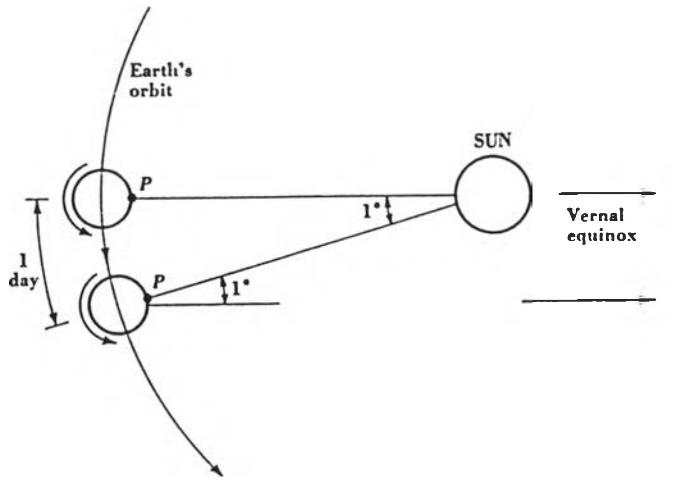

Fig. 2.6 Sidereal and solar day in comparison (Kraus, 1986) .

The solar day is define as same as the sidereal day but the reference point is replaced by the Sun. Since the irregularity of the period depend on the position of the Earth on its ellipse orbit around the Sun, the *mean solar day* is commonly applied. In the Fig 2.6, it can be shown the relation of the sidereal day and solar day. It is obvious that the solar day is longer than the sidereal day. From the observation, the *ratio of the sidereal to mean solar time* ν is

$$\nu = 1.002737903 \quad (2.21)$$

[1]

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[1] US Naval. American ephemeris and nautical Almanac. WS: Government printing office, 1966 cite in Kraus, J. D. Radio astronomy. 2nd. NY: McGraw-Hill, 1986.

since the time of a day depends on the longitude of the observer. The common tine should be applied for communication easily. The common time is the mean solar time at Greenwich (0° longitude) is called the *universal time*, *UT*. It is easily to convert the standard time of the observer to the universal time which is given by

$$UT = ST - TZ \tag{2.22}$$

where UT = universal time

ST = standard time of the observer

TZ = time zone of the observer. *

Where the long time interval are involved, it is convenient to reckon as the *Julian date*. The Julian date reckon time entirely in days instead of days, months or years. The number of days reckon from noon ($12^{h}0^{m}0^{s}$ UT) of 1^{st} January 4713 BC. Now, we have already to relate the previous astronomical coordinate system together. Since the celestial sphere is infinitely large, it may be consider that the sphere of the sky in horizontal coordinate coincide with the celestial sphere in equatorial coordinate. By a little trigonometry, as shown in Fig 2.7, it can be proved that the NCP is the point of $Az = 0^{\circ}$ and $Alt = \theta$ which is the latitude of the observer (if we consider the Earth is perfect sphere) . The object at the zenith distant Z situated at the meridian have the declination δ as given by

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* For Thailand, time zone equal to +7.

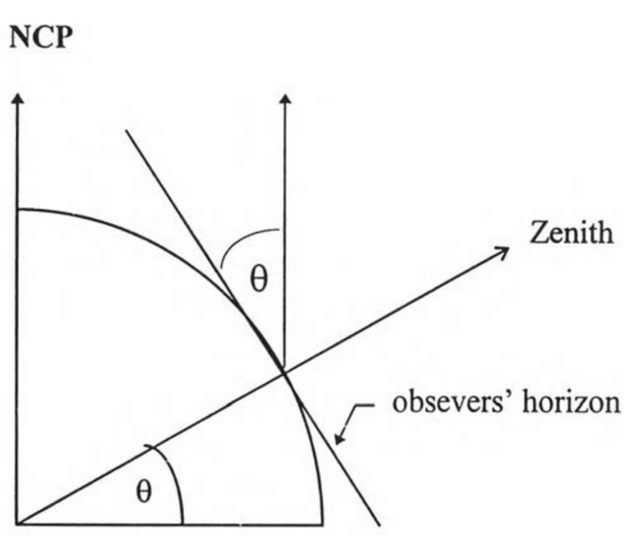

Fig 2.7 Position of NCP compare to the observer horizon.

$$\delta = \theta + Z \tag{2.22}$$

where δ = declination of the object, degree

θ = latitude of the object, degree

Z = zenith distance of the object, degree

Since the celestial sphere rotated around the poles westwards, the celestial object rises from the east and set to the west. We can see that hour circle of an object move as the same. The angle from the observer's meridian measured westwards to the vernal equinox along the celestial equator is called the *local sidereal time, LST*. If the angle is measure to the hour circle of an object, it is called *hour angle HA*. thus, the hour angle of the object with right ascension α can be written as

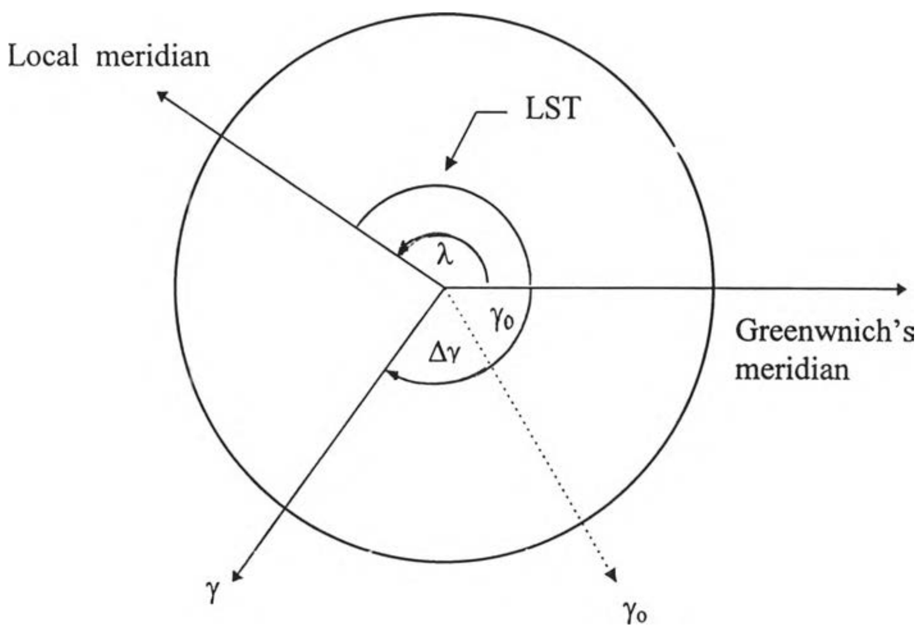

Fig 2.8 Local sidereal time of the observer at longitude λ

$$HA = LST - \alpha \quad (2.23)$$

The local sidereal time, LST is related to the universal time and longitude λ of the observer. From the Fig 2.8, the local sidereal time at UT is given by

$$LST = \lambda + \gamma_o + \Delta\gamma$$

where LST = local sidereal time

λ = longitude of the observers.

γ_o = sidereal time on 0^h UT at Greenwich

$\Delta\gamma$ = deviation of vernal equinox at UT

Since the celestial sphere rotates with the period equal to sidereal day. Hence, the deviation angle $\Delta\gamma$ when the time elapsed as mean solar time, UT, can be given by

$$\Delta\gamma = \nu * UT \tag{2.24}$$

where ν = ratio between sidereal time to solar time (=1.0027379093).

UT = universal time.

Substitute (2.24) to the previous equation, we have

$$LST = \lambda + \gamma_o + \nu * UT \tag{2.25}$$

The sidereal time on 0^h UT at Greenwich γ_o can be found in American ephemeris and Nautical Almanac or by calculate directly from

$$\gamma_o = \gamma_e + 24^h * \nu * (J - J_o) \tag{2.26}$$

where γ_e = Greenwich's sidereal time at 0^h UT of an instant of time (called the *epoch*).

J = Julian date for the observation.

J_o = Julian date of epoch.