APPENDIX A

Sample of Calculations

A.1 Determination of fitting equation of boundary potential.

The fitting equation of boundary potential which was recorded during the experiment is determined by means of method of least square. The boundary condition is in form of:-

$$V = S_1 (x^2+y^2) + c_1$$

In case of the square used in this experiment, one variable is equal to 10, the equation is reduced to

$$V = S_1 x^2 + c_1$$

Let $X = x^2$ for square and $X = x^2+y^2$ for the other,

$$V = S_1 X + c_1$$

The coefficient S_1 and the constant C_1 can be determined by means of the method of least square as follow:-

$$mS_1 + \left(\sum_{i=1}^{m} X_i\right) c_1 = \sum_{i=1}^{m} V_i$$

$$\left(\sum_{i=1}^{m} X_i\right) S_1 + \left(\sum_{i=1}^{m} X_i^2\right) c_1 = \sum_{i=1}^{m} X_i V_i \qquad (B.1)$$

m = number of datas.

The following table shows a part of the calculating table.[1]

---

1 Full datas of x, y and V are in table 1.

x	y	V	X	X^2	X·V
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
10.0	5.2	2.80	27.04	731.16	75.71
10.0	5.4	3.10	29.16	850.31	87.77
10.0	5.6	3.22	31.36	1.983.45	100.98
10.0	5.8	3.43	33.64	1,131.65	115.39
10.0	6.0	3.63	36.00	1,296.00	130.68
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
sum		324.374	3,234.00	190,137.43	18,891.82

Substituting into eq.(A-1)

$$98\ S_1 + 3234\ c_1 = 324.87$$

$$3234\ S_1 + 190{,}137.43\ c_1 = 18{,}891.82$$

$$S_1 = .09796$$

$$c_1 = .08250$$

Thus the fitting equation of boundary potential is

$$V = .09796\ x^2 + .0825$$

A.2 Transformation of potential to conjugate function.

On the boundary, the values of conjugate function are

$$\psi = \frac{1}{2}(x^2 + y^2)$$

For shaft of square cross section, the boundary potential is given by eq. (2.27)

$$V = 0.1 (x^2 + y^2) - 10$$

So the relation between ψ and V are

$$\psi = 5 (V + 10) \qquad (A.2)$$

also from eq. (2- 31) and eq. (2-34)

For rectangular $\psi = 5 (V + 2.5)$ (A.3)

For I - section $\psi = \frac{10}{3} (V + 0.6)$ (A.4)

For example, the potential 2.4 V on square shaft will represent the conjugate function,

$$\psi = 5 (2.4 + 10.0) = 62.0$$

but on I - section, it will represent

$$\psi = \frac{10}{3} (2.4 + 0.6) = 10.0$$

A.3 Construction of the shear stress line.

The conjugate function at any point can be transform to shearing stress function by eq. (2.15)

$$\phi = \psi - \frac{1}{2}(x^2 + y^2) = \psi - \frac{1}{2}r^2$$

The shear stress lines can be constructed by means of the equipotential line obtained from the experiment as follow:-

Let $\psi_0, \psi_1, \psi_2, - - - - -, \psi_n$ be the values of conjugate function corresponding to equipotential line.

According to the experiment, the increment of potential between the equipotential lines $\Delta\psi$ are constant.

Choose $\psi_0 = 0$, thus $\psi_1 = \Delta\psi$, $\psi_2 = 2\Delta\psi$, $\psi_3 = 3\Delta\psi, - - - -, \psi_n = n\Delta\psi$

Take the origin as center of circles whose radius are $r_1, r_2, \ldots, r_n$

Let $r_0 = 0$ at origin, $r_1 = \sqrt{2\Delta\psi}$, $r_2 = \sqrt{4\Delta\psi}, - - -, r_n = \sqrt{2n\Delta\psi}$

Consider the value of shear stress function at the intersection of the circles and the equipotential line.

	ψ_0	ψ_1	ψ_2	- - - - - - - - - -	ψ_n
r_1	$-\Delta\psi$	0	$\Delta\psi$	- - - - - - - - - -	$n\Delta\psi$
r_2	$-2\Delta\psi$	$-\Delta\psi$	0	- - - - - - - - - -	$(n-1)\Delta\psi$
⋮	⋮	⋮	⋮		⋮
r_m	$-m\Delta\psi$	$-(m-1)\Delta\psi$	$-(m-2)\Delta\psi$	- - - - - - - - - -	$(n-m)\Delta\psi$

We may conclude that as n = 1, 2, 3, 4,.......

the intersection point of ψ_n and r_n will form shear stress line = 0

" " " ψ_n " r_{n-1} " " " " $= \Delta\psi$

— — — — — — — — — — — — — — — — — — —

— — — — — — — — — — — — — — — — — — —

— — — — — — — — — — — — — — — — — — —

" " " ψ_n " r_{n-m} " " " " $= m\Delta\psi$

A-4. Determination of shear stress components.

The shear stress components can be calculated from both ψ and ϕ since the experiment is done on ψ, so we will calculate from ψ.

Consider on the axis y = constant, the value of V is function of x only. Assuming the fitting equation is the fourth degree polynomial function.

$$V = a + bx + cx^2 + dx^3 + ex^4$$

According to the method of least square, the co-efficient of fitting equation can be determined from the following equation.

$$ma + \left(\sum_{i=1}^{m} x_i\right) b + \left(\sum_{i=1}^{m} x_i^2\right) c + \left(\sum_{i=1}^{m} x_i^3\right) d + \left(\sum_{i=1}^{m} x_i^4\right) e = \sum_{i=1}^{m} \psi_i$$

$$\left(\sum_{i=1}^{m} x_i\right) a + \left(\sum_{i=1}^{m} x_i^2\right) b + \left(\sum_{i=1}^{m} x_i^3\right) c + \left(\sum_{i=1}^{m} x_i^4\right) d + \left(\sum_{i=1}^{m} x_i^5\right) e = \sum_{i=1}^{m} x_i \psi_i$$

$$\left(\sum_{i=1}^{m} x_i^2\right) a + \left(\sum_{i=1}^{m} x_i^3\right) b + \left(\sum_{i=1}^{m} x_i^4\right) c + \left(\sum_{i=1}^{m} x_i^5\right) d + \left(\sum_{i=1}^{m} x_i^6\right) e = \sum_{i=1}^{m} x_i^2 \psi_i$$

$$\left(\sum_{i=1}^{m} x_i^3\right) a + \left(\sum_{i=1}^{m} x_i^4\right) b + \left(\sum_{i=1}^{m} x_i^5\right) c + \left(\sum_{i=1}^{m} x_i^6\right) d + \left(\sum_{i=1}^{m} x_i^7\right) e = \sum_{i=1}^{m} x_i^3 \psi_i$$

$$\left(\sum_{i=1}^{m} x_i^4\right) a + \left(\sum_{i=1}^{m} x_i^5\right) b + \left(\sum_{i=1}^{m} x_i^6\right) c + \left(\sum_{i=1}^{m} x_i^7\right) d + \left(\sum_{i=1}^{m} x_i^8\right) e = \sum_{i=1}^{m} x_i^4 \psi_i$$

For example, when they are substituted by th· datas in table 7

$$18a + 107.57b + 735.77c + 5530.37d + 44253.6e = 23.365$$

$$107.57a + 735.77b + 5530.37c + 44253.6d + 369139\,e = 121.527$$

$$735.77a + 5530.7b + 44253.6c + 369139d + 3.16779\times10^{6}e = 720.032$$

$$5530.37a + 44253.6b + 369{,}139c + 3.16779\times10^{6}d + 27.7376\times10^{6}e = 4{,}733.13$$

$$44253.6a + 369{,}139b + 3.16779\times10^{6}c + 27.7376\times10^{6}d + 246.507\times10^{6}e = 33633.8$$

Solve the equation, we get

a = 1.686, b = .0184, c = -.00190, d = -.000138, e = -.000154

Thus $V = 1.686 + .0184x - .0019x^2 - .000138x^3 - .000154x^4$

$$\frac{\partial V}{\partial x} = .0184 - .0038x - .000414x^2 - .000616x^3$$

Consider eq. (A -2)

$$\psi = 5\,(V + 10)$$

$$\frac{\partial \psi}{\partial x} = 5\,\frac{\partial V}{\partial x}$$

$$\frac{\tau_{yz}}{\mu\alpha} = -\frac{\partial \psi}{\partial x} + x$$

for example at x = 8.0

$$V_f = 1.686 + .0184\times8 - .0019(8)^2 - .000138(8)^3 - .000154(8)^4 = 1.010$$

$$\psi_f = 5\,(1.010 + 10) = 55.05$$

$$\frac{\partial V_f}{\partial x} = .0184 - .0038(8) - .00414(8)^2 - .000616(8)^3 = -.3539$$

$$\frac{\partial \psi_f}{\partial x} = -.3539 \times 5 = -1.7695$$

$$\frac{\tau_{yz}}{\mu\alpha} = 1.7695 + 8.00 = 9.7695 \approx 9.77$$

Since the method of solving the fitting equation is rather laboriously if there are several sets of datas, so a computer program is provided in Appendix (B)

If a single point in the region is considered, it is much easy to approximate from it's neighborhood points by the following method. For example, to evaluate $\frac{\tau_{yz}}{\mu\alpha}$ at x = 8.0, y = 0.

It is observed that y = 0

$$x = 7.38, \quad V = 1.2$$

$$x = 8.00, \quad V = 1.0$$

$$x = 8.56, \quad V = 0.8$$

A second degree polynomial function of x is assigned to be the value of V.

$$V = a + bx + cx^2$$

$$\text{at } x = 7.38, \quad a + 7.38b + 54.4644c = 1.2$$

$$\text{at } x = 8.00, \quad a + 8.00b + 64c = 1.0$$

$$\text{at } x = 8.56, \quad a + 8.56b + 73.2736c = 0.8$$

$$c = -.02929$$

$$b = .1279$$

$$a = 1.85$$

$$V_8 = 1.85 + .1279x - .02929x^2$$

$$= 1.85 + .1279 \times 8 - .02929(8)^2 = .9986$$

$$\frac{\partial \psi_s}{\partial x} = .1279 - .05858x$$

$$= .1279 - .05858 \times 8 = -.34074$$

$$\frac{\partial \psi_s}{\partial x} = -.34074 \times 5 = -1.7037$$

$$\frac{\tau}{\mu\alpha} = -(-1.7039) + 8 = 9.7039 \approx 9.70$$

which is less than 1% different from the former value

A-5 Determination of the torsional stiffness.

From eq. (2 - 17)

$$M = 2\mu\alpha \iint_R \phi \, dxdy$$

Since $\iint_R \phi \, dxdy = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum_{k=1}^{n} \phi_k \cdot \Delta x_k \cdot \Delta y_k$

The integral part can be approximated by dividing the area of specimen into many equal sqaures. The value of ϕ at the center of each square is determined by interpolation. Since the specimen represents a quarter of cross-section.

$$\iint_R \phi \, dxdy \approx 4 \sum_{k=1}^{m} \phi_k \cdot \Delta x_k \cdot \Delta y_k$$

$$\frac{M}{\mu\alpha} \approx 8 \sum_{k=1}^{m} \phi_k \cdot \Delta x_k \cdot \Delta y_k \qquad (A - 5)$$

In case of equal squares.

$$A_k = \Delta x_k \, \Delta y_k \qquad = \text{constant}$$

$$\frac{M}{\mu\alpha} = 8A \sum_{k=1}^{m} \phi_k$$

For example, the square specimen is divided into 25 equal squares, each square is equal to 4 units as shown in fig. 20. The value of ϕ of each square are in table 18, we get

$$\sum_{k=1}^{21} \phi_k = 709.6$$

$$\frac{M}{\mu\alpha} = 8 \times 4 \times 709.6 = 22,707.2$$

A - 6 Determine K and K_1

From eq. (2 - 27) and (2 - 28)

$$K = \frac{\tau_{max}}{\mu\alpha a}$$

$$K_1 = \frac{M}{\mu\alpha a^e b}$$

for example, square specimen. $\frac{M}{\mu\alpha} = 22,707.2$, $\frac{\tau_{max}}{\mu\alpha} = 13.47$

$$K = \frac{13.47}{20} = .6735 \approx .674$$

$$K_1 = \frac{22,707.2}{20^4} = .1419$$

APPENDIX B

COMPUTER PROGRAM AND FLOW CHART

B.1 COMPUTER PROGRAM FOR DETERMINING FITTING EQUATION.

```
C        DETERMINE EMPIRICAL FØRMULA IN PØLYNØMIAL FØRM BY
C        LEAST SQUARE METHØD
C
         DIMENSIØN P (200), PHI (200), A(7,7), B(7), C(7), X(7), E(7,7)
C
C        READ DATA
C        ND = NØ.ØF PRØBLEMS PUNCH IN A CARD
C        N  = NØ.ØF EQUATIØNS, M = NØ.ØF DATAS PUNCH IN A CARD
C        P  = VALUE ØF VARIABLES, PHI = VALUE ØF FUNCTIØNS PUNCH 4 SETS EACH CARD
C
         READ (2,5) ND
    5    FØRMAT (I2)
         DØ 500 NN = 1, ND
         READ (2,10) N,M
   10    FØRMAT (I2,I4)
         DØ 20 I = 1,M,4
         READ (2,15) P(I), PHI(I), P(I+1), PHI(I+1), P(I+2), PHI(I+2),P(I+3),PHI(
        1I+3)
         WRITE (3,15) P(I),PHI(I),P(I+1),PHI(I+1),P(I+2),PHI(I+2),P(I+3),PHI(
        1I+3)
   15    FØRMAT (8F9.4)
   20    CØNTINUE
         IF(N.LE.0.ØR.N.GT.7.ØR.M.GT.200) GØ TØ 480
         IF(M.LT.N) GØ TØ 480
```

```
C
C         CLEAR ARRAY
C
          DØ 25 I = 1,7
          C(I) = 0.0
          D = 0.0
          X(I) = 0.0
          B(I) = 0.0
          DØ 25 J = 1,7
          A(I,J) = 0.0
          E(I,J) = 0.0
   25     CØNTINUE
C
C         CALCULATE CØNSTANT AND CØEFFICIENTS
C
          DØ 35 I = 1,N
          DØ 30 J = 1,N
          NA = I + J - 2
          DØ 30 K = 1,M
          A(I,J,) = A(I,J) + P(K)**NA
          E(I,J) = A(I,J)
   30     CØNTINUE
          DØ 35 L = 1,N
          C(I) = C(I) + PHI(L)*P(L)**(I-1)
   35     CØNTINUE
```

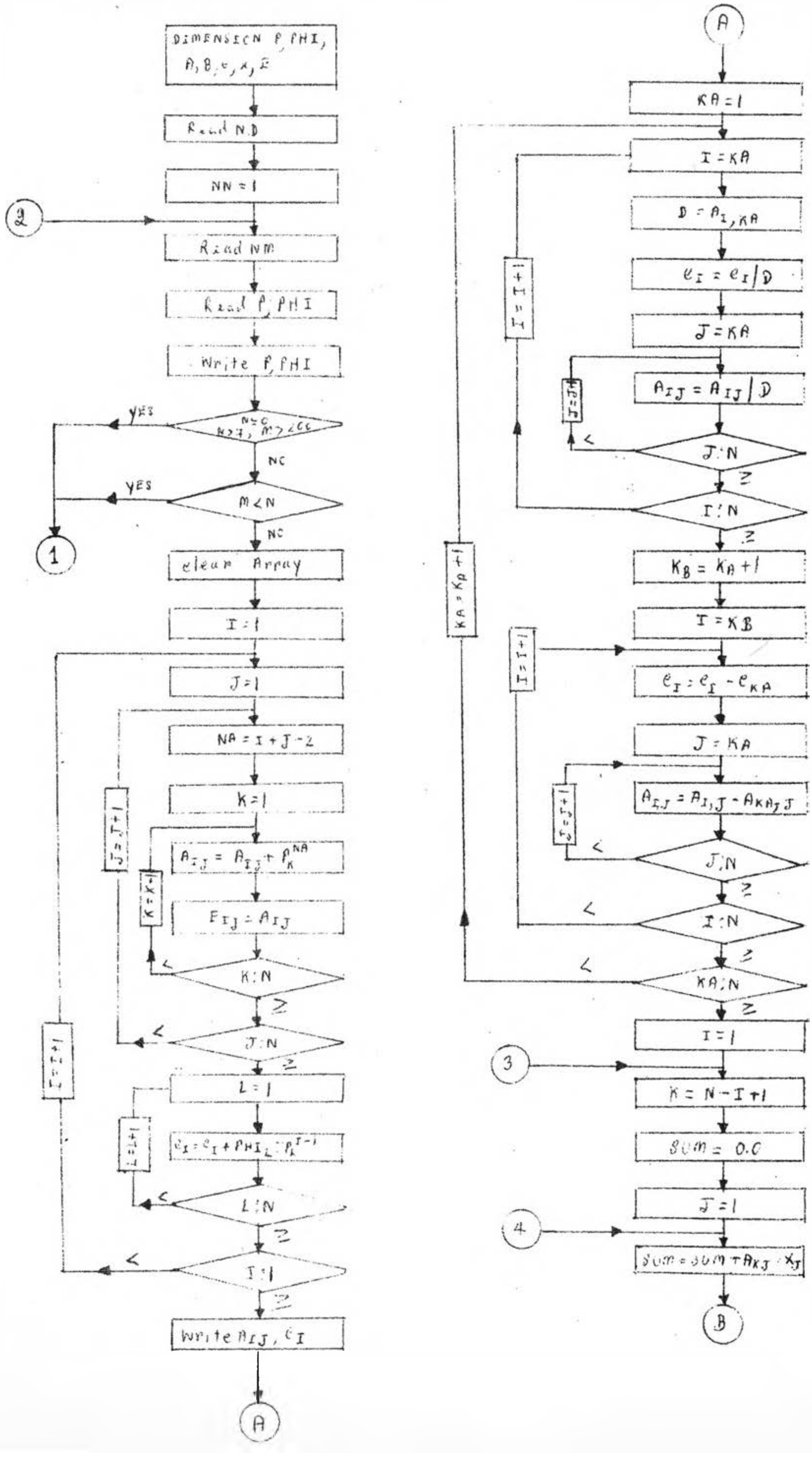
```
      WRITE (3,40)
   40 FØRMAT (6HOGIVEN/)
      DØ 60 I = 1,N
      WRITE (3,50) (A(I,J), J = 1,N), C(I)
   50 FØRMAT (1Ho,8 (E 13.6, 2X) )
   60 CØNTINUE
C
C     SØLVE SIMULTANEØUS EQUATIØN BY METHØD ØF DIVISIØN BY LEADING
C     CØEFFICIENT
C
C     PERFØRM DIVISIØN
C
      DØ 180 KA = 1,N
      DØ 110 I = KA, N
      D = A (I,KA)
      C(I) = C(I)/D
      DØ 110 J = KA,N
      A(I,J)= A(I,J)/D
  110 CØNTINUE
C
C     PERFØRM SUBSTRACTIØN
C
      KB = KA+1
      DØ 160 I = KB, N
      C(I) = C(I) - C(KA)
```

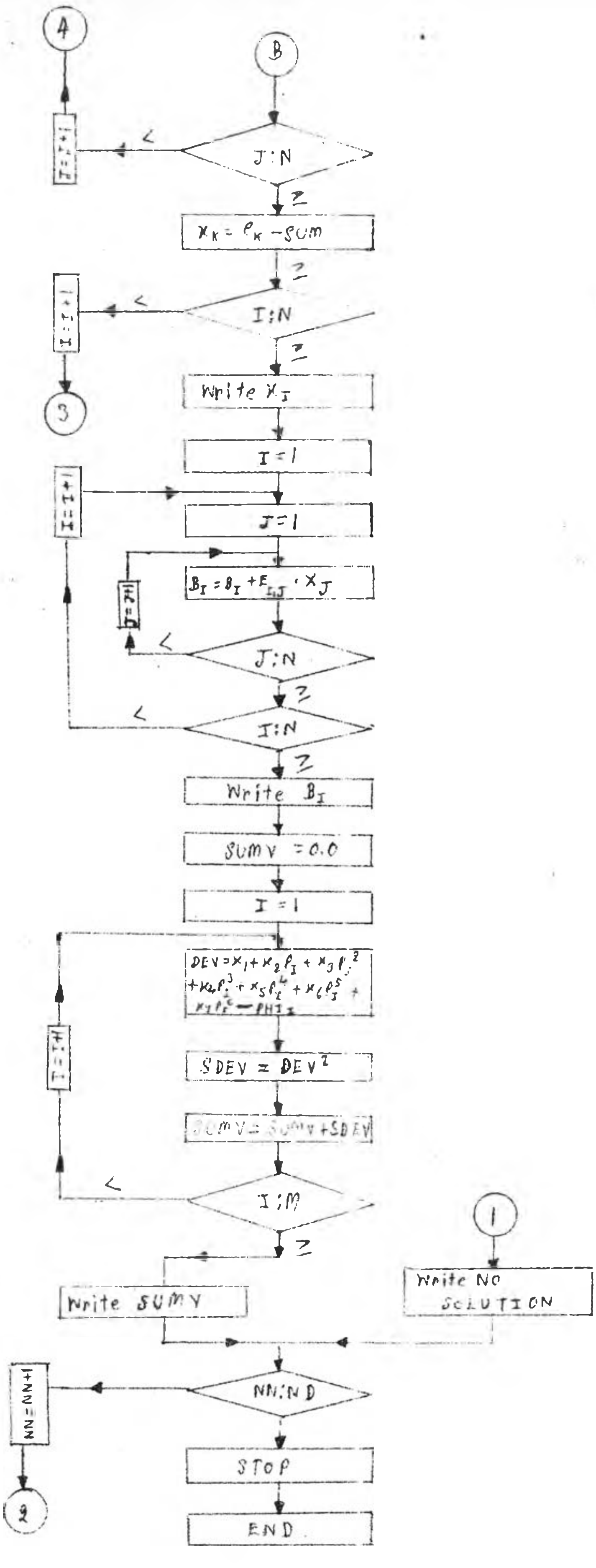
```
          DØ 160 J = KA, N
          A(I,J) = A(I,J) - A(KA,J)
    160   CØNTINUE
    180   CØNTINUE
C
C         CØMPUTE RØØTS ØF THE SIMULTANEØUS EQUATIØNS
C
          DØ 210 I = 1,N
          K = N - I+1
          SUM = 0.0
          DØ 205 J = 1, N
          SUM = SUM + A(K,J)*X(J)
    205   CØNTINUE
          X(K) = C(K) - SUM
    210   CØNTINUE
          WRITE (3,220)
    220   FØRMAT (27HØRØØTS ØF THE EQUATIØNS ARE /)
          DØ 240 I = 1,N
          WRITE (3,230) I, X(I)
    230   FORMAT (3HOX(, I2,2H) =, E 13.6 )
    240   CØNTINUE
C
C         CHECK RØØTS ØF EQUATIØNS
C
```

```
      DØ 260 I = 1,N
      DØ 260 J = 1,N
      B(I) = B(I) +E(I,J)*X(J)
  260 CØNTINUE
      WRITE (3,270)
  270 FØRMAT (32H0 CHECK RØØTS FRØM LEFT HAND SIDE/)
      DØ 290 I = 1,N
      WRITE (3,280) I, B(I)
  280 FORMAT (3H0B (I2,2H) = , E15.8)
  290 CØNTINUE
C
C     CØMPUTE SUM ØF SQUARE ØF DEVIATIØN
C
      SUMV = 0.0
      DØ 310 I = 1,M
      DEV=X(1)+X(2)*P(I)+X(3)*P(I)**2+X(4)*P(I)**3+X(5)*P(I)**4+X(6)*P(I
     1)**5+X(7)*P(I)**6-PHI(I)
      SDEV = DEV**2
      SUMV = SUMV + SDEV
  310 CØNTINUE
      WRITE(3,320)SUMV
  320 FØRMAT (30H0 SUM ØF SQUARE ØF DEVIATIØN =, E15.8)
      WRITE (3,300)
  300 FORMAT (121H0************---------------------------------*
     1******--------------------------------------------*
```

```
         2///)
         GØ TØ 500
480      WRITE (3,490)
490      FØRMAT (12HONØ SØLUTIØN)
         WRITE(3,300)
500      CØNTINUE
         STØP
         END
```

B.2 Flow chart for solving fitting equation





A
B
J:N
<
≥
$x_K = P_K - SUM$
I=I+1
I:N
3
Write X_I
I=1
J=1
$B_I = B_I + F_{I,J} \cdot X_J$
J=J+1
J:N
Write B_I
SUMV = 0.0
I = 1
$DEV = x_1 + x_2 P_I + x_3 P_I^2 + x_4 P_I^3 + x_5 P_I^4 + x_6 P_I^5 + x_7 P_I^6 - PHI_I$
$SDEV = DEV^2$
SUMV = SUMV + SDEV
I:M
1
Write SUMV
Write NO SOLUTION
NN:ND
NN=NN+1
2
STOP
END

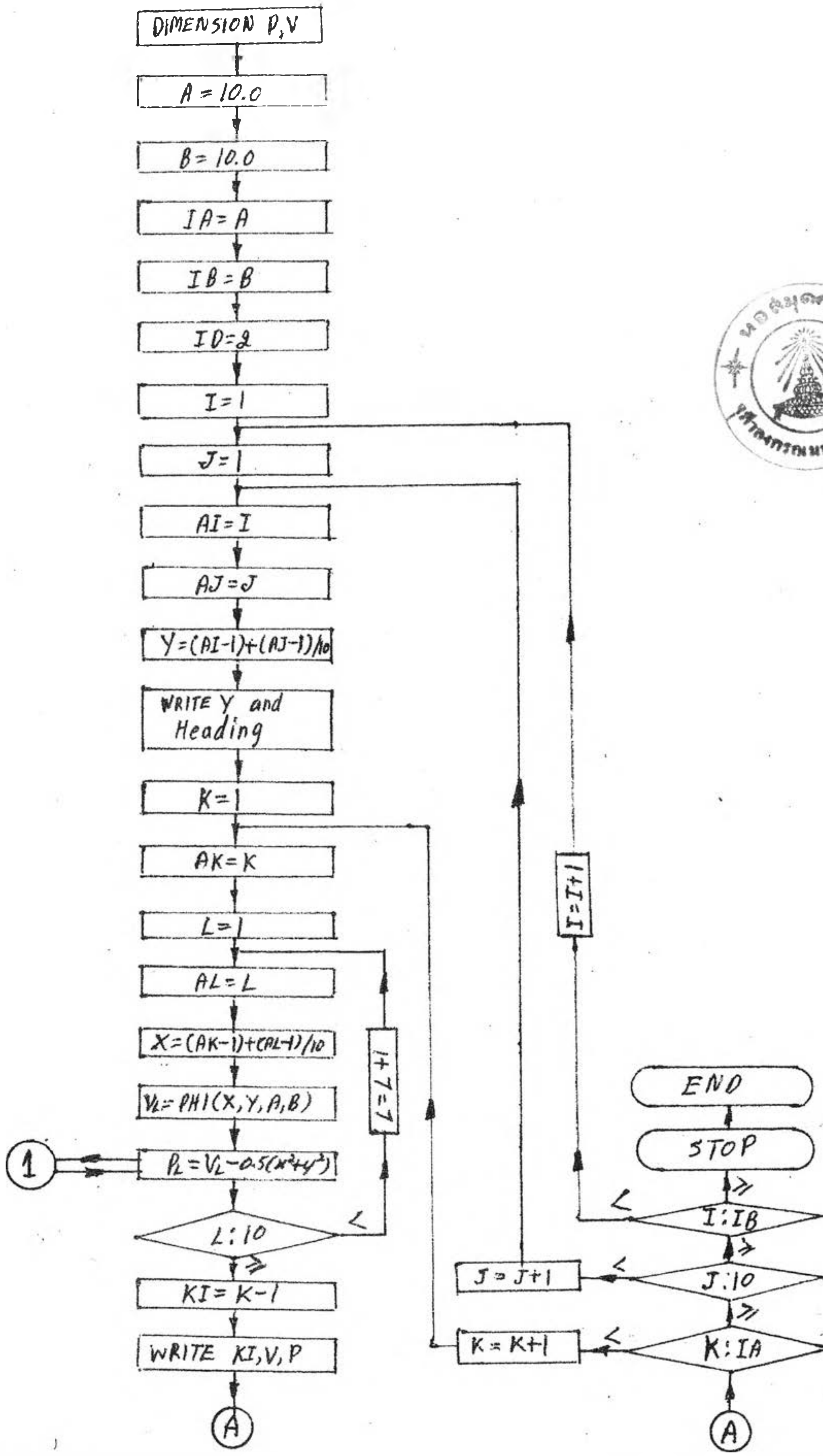
B.3 COMPUTER PROGRAM FOR SOLVING ANALYTICAL EQUATION OF CONJUGATE FUNCTION AND SHEAR STRESS FUNCTION OF THE SQUARE SHAFT.

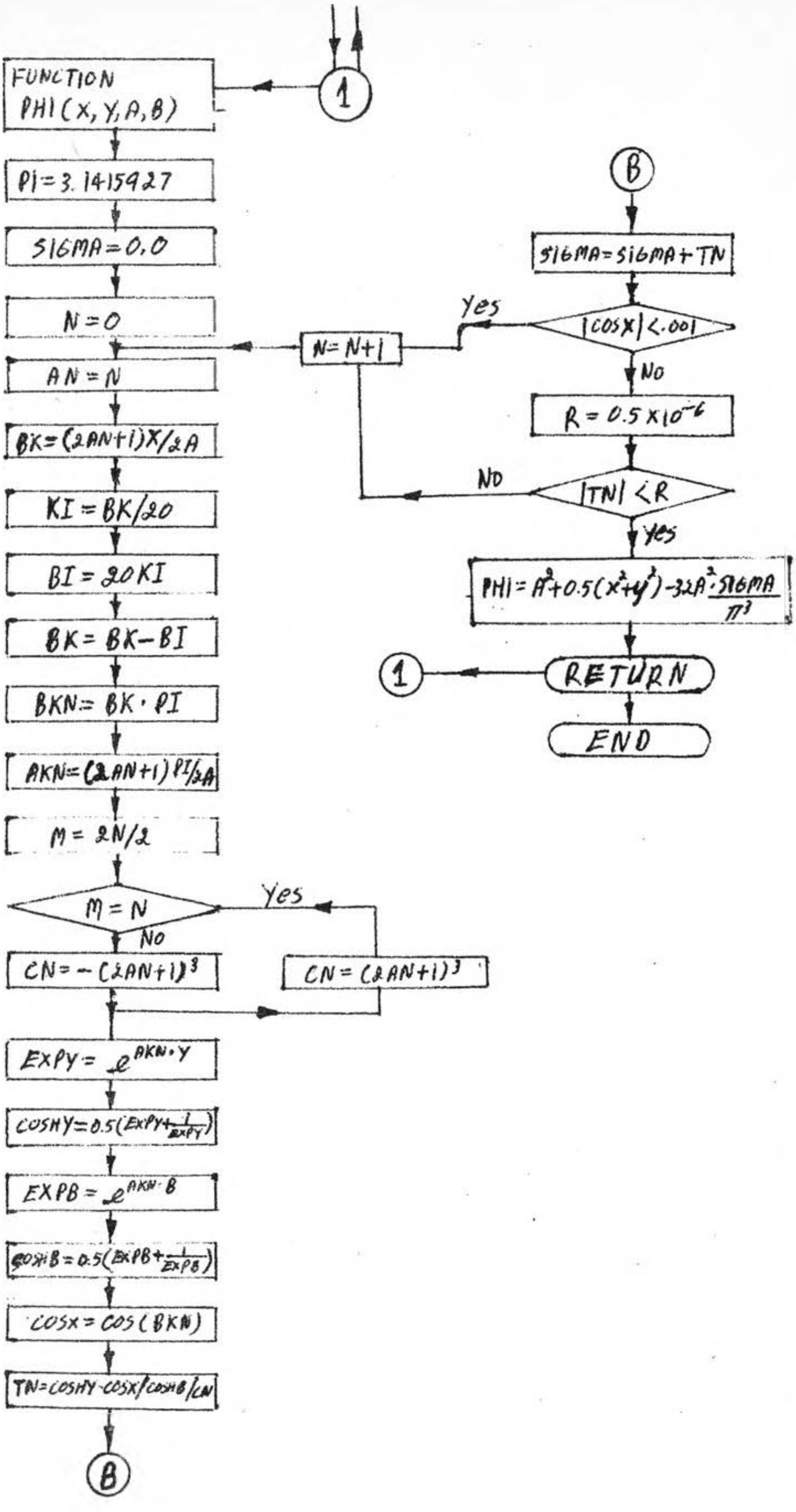
```
      DIMENSIØN P(10),V(10)
      A = 10.0
      B = 10.0
      IA = A
      IB = B
      ID = 2
      DØ 300 I = 1,IB
      DØ 300 J = 1,10,ID
      AI = I
      AJ = J
      Y = (AI-1.) + (AJ - 1.)/10
      WRITE (3,10) Y
10    FØRMAT (3HOY=,F4.1, 24X,21HCØNJUGATE FUNCTIØN,
     135X,17H STRESS FUNCTIØN/)
      WRITE(3,20)
20    FØRMAT(1Hb, 14X, 3Hb.0, 7X, 3Hb.2, 7X,3Hb.4, 7X,3Hb.6,7X,3Hb.8,
     112X,3Hb.0, 7X,3Hb.2, 7X,3Hb.4, 7X,3Hb.6, 7X,3Hb.8/)
      DØ 300 K = 1,IA
      AK = K
      DØ 200 L = 1,10, ID
      AL = L
      X = (AK - 1.) + (AL - 1.)/10.
```

```
      V(L) = PHI(X,Y,A,B)
200   P(L) = V(L) - 0.5* (X**2+Y**2)
230   KI = K - 1
      WRITE (3,240) KI,(V(M), M=1,10,ID), (P(N), N = 1,10,ID)
240   FØRMAT (3Hb.X =, I1,7X,5F10.4, 5X, 5F10.4/)
300   CØNTINUE
      STØP
      END
      FUNCTIØN PHI(X,Y,A,B)
      PI = 3.1415927
      SIGMA = 0.0
      N = 0
100   AN = N
      BK = (2*AN + 1.)*X/(2.*A)
      KI = BK/20
      BI = 20*KI
      BK = BK - BI
      BKN = BK*PI
      AKN = (2.*AN+1.) *PI/(2.*A)
        M = N/2*2
      IF (M.EQ.N) GØ TØ 105
      CN = - (2.*AN+1.)**3
      GØ TØ 106
105   CN = (2.*AN+1.)**3
106   EXPY = EXP (AKN*Y)
```

```
      CØSHY = 0.5*(EXPY +1./EXPY)
      EXPB = EXP(AKN*B)
      CØSHB= 0.5*(EXPB +1./EXPB)
      CØSX = COS (BKN)
      TN = CØSHY*CØSX/CØSHB/CN
110   SIGMA = SIGMA + TN
      IF (ABS (CØSX).LT.0.001) GØ TØ 25
      R= 0.5E-6
      IF(ABS(GØSX).LT.0.001) GØ TØ 120
25    N=N+1
      GØ TØ 100
120   PHI=A**2+0.5*(Y**2-X**2)-32.*A**2*SIGMA/PI**3
      RETURN
      END
```

B.4 Flow chart for solving analytical equation of the square shaft





FUNCTION PHI(X, Y, A, B)
PI = 3.1415927
SIGMA = 0.0
N = 0
AN = N
BK = (2AN+1)X/2A
KI = BK/20
BI = 20KI
BK = BK−BI
BKN = BK · PI
AKN = (2AN+1) PI/2A
M = 2N/2
M = N
Yes
No
CN = −(2AN+1)³
CN = (2AN+1)³
EXPY = e^AKN·Y
COSHY = 0.5(EXPY + 1/EXPY)
EXPB = e^AKN·B
COSHB = 0.5(EXPB + 1/EXPB)
COSX = COS(BKN)
TN = COSHY·COSX/COSHB/CN
B
B
SIGMA = SIGMA + TN
|COSX| < .001
Yes
No
N = N+1
R = 0.5 × 10⁻⁶
|TN| < R
No
Yes
PHI = A² + 0.5(X² + Y²) − 32A²·SIGMA/π³
1
RETURN
1
END

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VITA

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