CHAPTER 2

THEORY

2.1 The distribution of steady state potential in a thin conducting sheet of constant thickness.

Consider a thin sheet of conducting material of which thickness and resistivity are uniform. The boundary of the conducting sheet is held at different electric potential, so that the current flowing in the sheet and a certain potential distribution are set up. Consider a small element of the sheet bounded by lines parallel to x and y axes.

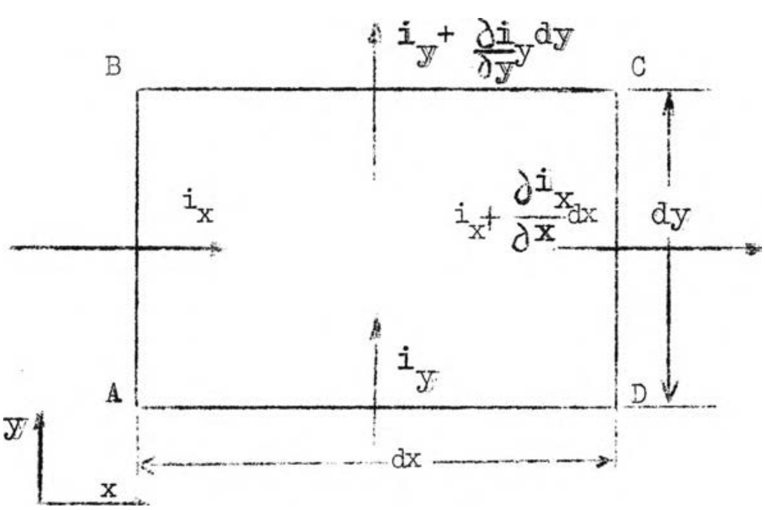

Fig.1 Flow of current through a small element of the conducting sheet

Let i_x, i_y = Component of current density in x and y direction

V = Electrical Potential

ρ = Resistivity

In fig 1. Since

Current flow on face AB $= i_x dy$

" " " $CD = \left(i_x + \frac{\partial i_x}{\partial x} dx\right) dy$

" " " $DA = i_y dx$

" " " $BC = \left(i_y + \frac{\partial i_y}{\partial y} dy\right) dx$

total current entering = total current leaving

$$i_x dy + i_y dx = \left(i_x + \frac{\partial i_x}{\partial x} dx\right) dy + \left(i_y + \frac{\partial i_y}{\partial y} dy\right) dx$$

$$\frac{\partial i_x}{\partial x} dx dy + \frac{\partial i_y}{\partial y} dy dx = 0$$

Divide by dxdy

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} = 0$$

By Ohm's law

$$i_x = -\frac{1}{\rho}\frac{\partial V}{\partial x}, \quad i_y = -\frac{1}{\rho}\frac{\partial V}{\partial y}$$

$$\frac{\partial V}{\partial x^2} + \frac{\partial V^2}{\partial y^2} = 0 \qquad (2.1)$$

2.2 General Torsion Theory

2.2.1 Torsion Function

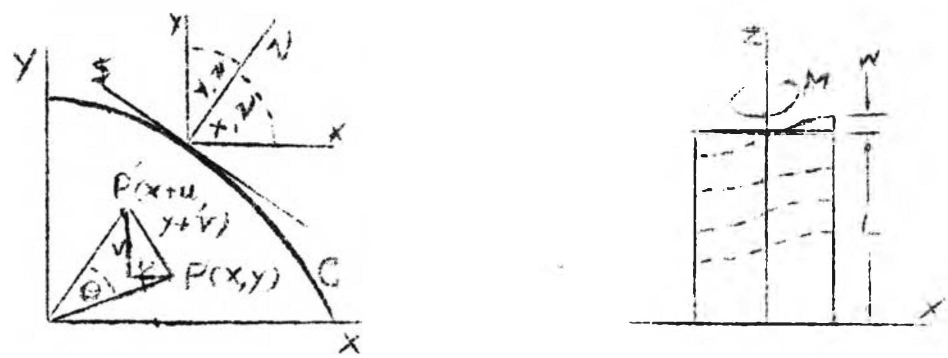

Fig.2

Co-ordinate axes and displacements of shaft

Consider the general case of a long elastic cylindrical shaft, of which the cross-section is simply-connected region, is subjected to no body force and free from external forces on it lateral surface. The generaters of the shaft are parallel to z - axis. One end is fixed in the plane z = 0, while the other end, in the plane z = 1 is twisted through a small angle θ by a couple of magnitude M whose moment is directed along the axis of the shaft.

Since the deformation is small, the angle θ is assumed to be proportional to the distance of the section from the fixed base. Thus,

$$\theta = \alpha z$$

where α is the twist per unit length.

According to Saint - Venant's Semi Inverse Method, the deformation of the twisted shaft consists of:-

1. The rotation of the cross section of shaft is in such a manner that every diameter seen on the plane of the cross-section remains straight and rotates through the same angle. Thus the displacements along x and y axis correspond to the rotation of cross section are

$$u = -\alpha zy, \qquad v = \alpha zx$$

2. The cross sections are warped and each cross - section is warped in the same way. It means that the warping of the cross-section is independent of z. The displacement along z - axis can be expressed as a function of x and y only. This leads us to assume that the displacement along the z-axis in form of

$$w = \alpha\psi(x,y)$$

where $\varphi(x,y)$ is called "Torsion function or Warping function."

By simple calculation of the stress corresponding to displacements, the strain components in fig.3 will be

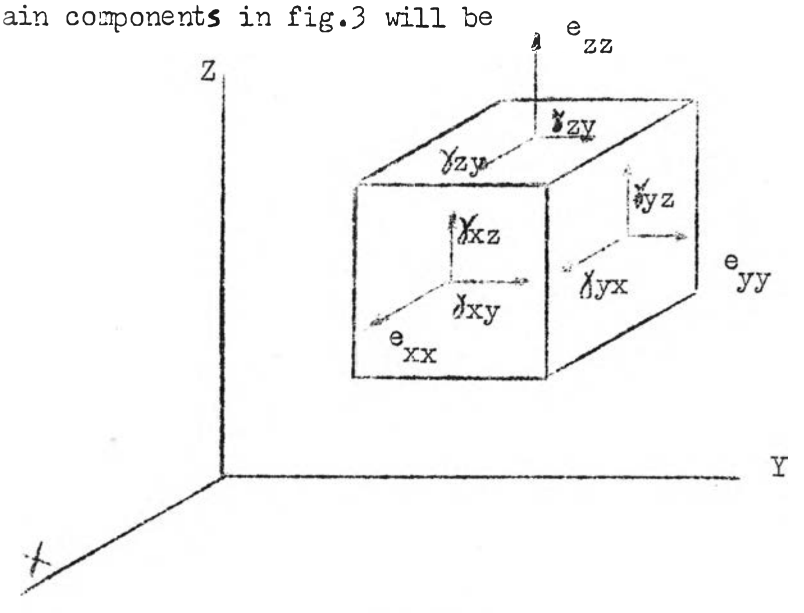

Fig.3

Strain components in an element in three dimensional form.

$$e_{xx} = \frac{\partial u}{\partial x} = 0$$

$$e_{yy} = \frac{\partial v}{\partial y} = 0$$

$$e_{zz} = \frac{\partial w}{\partial z} = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \alpha\left(\frac{\partial \varphi}{\partial x} - y\right)$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \alpha\left(\frac{\partial \varphi}{\partial y} + x\right)$$

According to Hooke's law

$$e_{xx} = \frac{1}{E}\left[\sigma_{xx} - \eta(\sigma_{yy} + \sigma_{zz})\right]$$
$$e_{yy} = \frac{1}{E}\left[\sigma_{yy} - \eta(\sigma_{zz} + \sigma_{xx})\right]$$
$$e_{zz} = \frac{1}{E}\left[\sigma_{zz} - \eta(\sigma_{xx} + \sigma_{yy})\right]$$

$$\gamma_{xy} = \frac{1}{\mu}\tau_{xy}\,, \quad \gamma_{yz} = \frac{1}{\mu}\tau_{yz}\,, \quad \gamma_{xz} = \frac{1}{\mu}\tau_{zx}$$

E = modulus of elasticity,

μ = shear modulus

η = Poisson's ratio

The stress components will be

$$\tau_{yz} = \mu\alpha\left(\frac{\partial\varphi}{\partial y} + x\right) \qquad (2.2)$$

$$\tau_{zx} = \mu\alpha\left(\frac{\partial\varphi}{\partial x} - y\right) \qquad (2.3)$$

$$\tau_{xy} = \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$$

By substituting this value in the differential equations[1] of equilibrium

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} + X = 0$$
$$\frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{yz}}{\partial z} + Y = 0 \qquad (2.4)$$
$$\frac{\partial\sigma_{zz}}{\partial z} + \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + Z = 0$$

---

1 Timoshenko, S.P. and Goodier, J.N. Theory of Elasticity. P236

and the boundary condition

$$\bar{X} = \sigma_{xx} \cos(\nu, x) + \tau_{xy} \cos(\nu, y) + \tau_{xz} \cos(\nu, z)$$

$$\bar{Y} = \sigma_{yy} \cos(\nu, y) + \tau_{yz} \cos(\nu, z) + \tau_{xy} \cos(\nu, x) \qquad (2.5)$$

$$\bar{Z} = \sigma_{zz} \cos(\nu, z) + \tau_{xz} \cos(\nu, x) + \tau_{yz} \cos(\nu, y)$$

where $\cos(\nu, x)$, $\cos(\nu, y)$, $\cos(\nu, z)$ are direction cosines of the external normal to the surface of the body at the point under consideration.

The differential equations of equilibrium will be satisfied if the torsion function $\varphi(x,y)$ satisfies the Laplace's equation,

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad \text{in } R \qquad (2.6)$$

where R is the region of the cross-section of cylinder.

To satisfy the boundary conditions on the lateral surface of the cylinder.

$$\left(\frac{\partial \varphi}{\partial x} - y\right) \cos(x, \nu) + \left(\frac{\partial \varphi}{\partial y} + x\right) \cos(y, \nu) = 0 \qquad \text{on } C$$

C is the boundary of the cross-section of the cylinder.

But $\dfrac{\partial \varphi}{\partial x} \cos(x, \nu) + \dfrac{\partial \varphi}{\partial y} \cos(y, \nu) = \dfrac{d\varphi}{d\nu}$

So the boundary condition can be written in the form of:-

$$\frac{d\varphi}{d\nu} = y \cos(x, \nu) - x \cos(y, \nu) \qquad \text{on } C \qquad (2.7)$$

The problem of solving Laplace's equation in the region R whose normal derivative is prescribed on the boundary of the region, such as the problem of solving eq.(2.6) and boundary condition (2.7) is known as "Neumann problem."

Since $$M = \iint_R (x\tau_{zy} - y\tau_{zx})\, dxdy \qquad (2.8)$$

By substitute of the eq.(2.2) and eq.(2.3)

$$M = \mu\alpha\iint_R \left(x^2 + y^2 + x\frac{\partial\varphi}{\partial y} - y\frac{\partial\varphi}{\partial x}\right) dxdy \qquad (2.9)$$

Eq. (2.6) is identical to the eq.(2.1). If the boundary potential of the conducting sheet is prescribed in form of

$$\frac{dV}{d\nu} = y\cos(x,\nu) - x\cos(y,\nu) \qquad \text{on } C$$

which is identical to the eq.(2.7). Then torsion function φ and potential V are analogous. It is not convenient to prescribe this type of boundary condition on the boundary. Another function which has simpler boundary condition should be introduced.

2.2.2 Conugate function

Since the torsion function $\varphi(x,y)$ is harmonic in the region of cross section, it is possible to construct an analytic function $\varphi + i\psi$ of complex variable $x + iy$ where $\psi(x,y)$ is the conjugate hormonic function, related to $\varphi(x,y)$ by the Cauchy - Riemann equations

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}, \qquad \frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

The function $\psi(x,y)$ is called "Conjugate function."

Eleminate ψ by differentiating the first with respect to y, the second with respect to x and substracting, then

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad \text{in R} \qquad (2.10)$$

The boundary condition of can be determined by eq.(2.7) by using Cauchy-Riemann equation and the notation that

$$\cos(x,\nu) = \frac{dy}{ds} = \frac{dx}{d\nu}, \cos(y,\nu) = -\frac{dx}{ds} = \frac{dy}{d\nu}$$

$$\frac{d\varphi}{d\nu} = y\frac{dy}{ds} + x\frac{dx}{ds}$$

but
$$\frac{d\varphi}{d\nu} = \frac{\partial \varphi}{\partial x}\cdot\frac{dx}{d\nu} + \frac{\partial \varphi}{\partial y}\cdot\frac{dy}{d\nu}$$

$$= \frac{\partial \psi}{\partial y}\cdot\frac{dy}{ds} + \frac{\partial \psi}{\partial x}\cdot\frac{dx}{ds} = \frac{d\psi}{ds}$$

$$\therefore \quad \frac{d\psi}{ds} = y\frac{dy}{ds} + x\frac{dx}{ds}$$

The boundary condition is then defined as

$$\psi = \tfrac{1}{2}(x^2+y^2) + \text{Constant} \qquad \text{on C} \qquad (2.11)$$

From eq.(2.2), (2.3) and (2.9), the stress components and twisting moment are expressed in term of conjugate function.

$$\tau_{yz} = -\mu\alpha\left(\frac{\partial \psi}{\partial x} - x\right) \qquad (2.12)$$

$$\tau_{zx} = \mu\alpha\left(\frac{\partial \psi}{\partial y} - y\right) \qquad (2.13)$$

$$M = \mu\alpha \iint_R \left(x^2 + y^2 - x\frac{\partial\psi}{\partial x} - y\frac{\partial\psi}{\partial y}\right) dxdy \tag{2.14}$$

If the boundary potential of the conducting sheet is prescribed in form of

$$V = \tfrac{1}{2}\,(x^2 + y^2) + \text{Constant} \qquad \text{on } C.$$

which is identical to the eq.(2.11). The conjugate function ψ and potential V are analogous.

The boundary potential in this case is simple. It is more convenient to make the analogy between the potential V and the conjugate function ψ .

2.2.3 Shearing Stress Function

The torsion problem may also be formulated in term of a function $\phi(x,y)$ which is defined as

$$\phi(x,y) = \psi(x,y) - \tfrac{1}{2}\,(x^2 + y^2) \tag{2.15}$$

by differentiation.

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial x} - x \quad , \quad \frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial y} - y \tag{2.16}$$

then the stress components will be

$$\tau_{zx} = \mu\alpha\frac{\partial\phi}{\partial y} \tag{2.17}$$

$$\tau_{zy} = -\mu\alpha\frac{\partial\phi}{\partial x} \tag{2.18}$$

Differentiate eq.(2.16), the first with respect to x and the second with respect to y and add

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2 \qquad (2.19)$$

Thus the function $\phi(x,y)$ satisfies Poisson's equation. The function $\phi(x,y)$ is known as "Shearing stress function."

From eq. (2.5), Since $\bar{X} = \bar{Y} = \bar{Z} = 0$ and $\cos(\nu, z) = 0$ the boundary condition reduces to

$$\tau_{xz}\cos(\nu,x) + \tau_{yz}\cos(\nu,y) = 0$$

Substituting eq.(2.11) and (2.12) into the boundary condition.

$$\frac{\partial \phi}{\partial y}\cdot\frac{dy}{ds} + \frac{\partial \phi}{\partial x}\cdot\frac{dx}{ds} = \frac{d\phi}{ds} = 0$$

$$\phi = \text{constant} \qquad (2.20)$$

along the boundary of cross section.

Recall eq. (2.8)

$$M = \iint_R (x\,\tau_{zy} - y\,\tau_{zx})\,dxdy$$

$$= -\mu\alpha\iint_R \left(x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y}\right)dxdy$$

$$= -\mu\alpha\iint_R \left[\frac{\partial (x\phi)}{\partial x} + \frac{\partial (y\phi)}{\partial y}\right]dxdy + 2\mu\alpha\iint_R \phi\,dxdy$$

By Green's Theorem

$$M = -\mu\alpha\int_C \phi\left[x\cos(x,\nu) + y\cos(y,\nu)\right]ds + 2\mu\alpha\iint_R \phi\,dxdy$$

Choose $\phi = 0$ on C.

$$M = 2\mu\alpha\iint_R \phi\,dxdy \qquad (2.21)$$

The twisting moment expressed in term of shearing stress function is simpler than expressed in term of conjugate function or torsion function.

It is advisable to determine the torsional stiffness in terms of ϕ Although the shear stress function, ϕ, is not analogous to the potential V in the electrical analogy, it can, however, be determined from the conjugate function by mean of eq. (2.15). Then the twisting moment can be approximated as shown in section A - 5 appendix A.

Consider a family of curves, in the plane of the cross - section obtained by setting ϕ = constant, these curves are called "Lines of shearing stress" or "Shear stress lines."

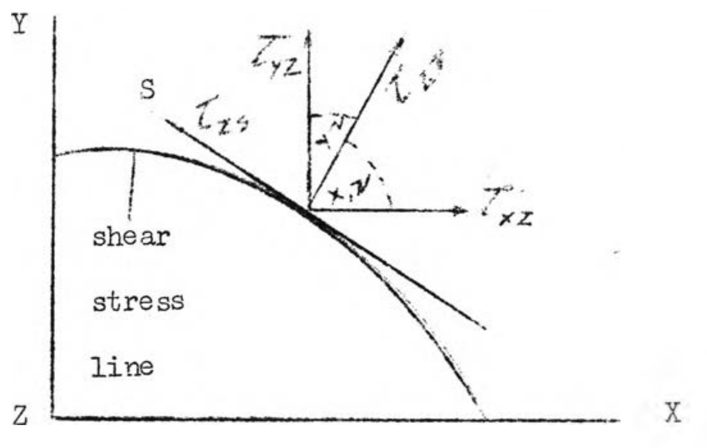

Fig.4 Shear stress components on the shear stress line

$$\frac{\partial \phi}{\partial s} = 0$$

$$\frac{\partial \phi}{\partial y} \cdot \frac{dy}{ds} + \frac{\partial \phi}{\partial x} \cdot \frac{dx}{ds} = 0$$

$$\tau_{xz} \frac{dy}{ds} - \tau_{yz} \frac{dx}{ds} = 0$$

$$\tau_{xz} \cos (x, \nu) + \tau_{yz} \cos (y, \nu) = 0 \qquad (2.22)$$

The eq.(2.22) indicates that the components of shear stress normal to the shear stress lines ($\tau_{z\nu}$ in fig.4) are alway zero, thus, the shear stress at any point in the twisted shaft is in the direction tangential to the shear stress line.(τ_{zs} in fig.4).

The magnitude of the resultance shear stress τ is.

$$\tau = \tau_{zs} = \tau_{yz} \cos (x, \nu) - \tau_{xz} \cos (y, \nu)$$

$$= -\mu\alpha\left(\frac{\partial \phi}{\partial x}\frac{dx}{d\nu} + \frac{\partial \phi}{\partial y}\cdot\frac{dy}{d\nu}\right)$$

$$= -\mu\alpha \frac{d\phi}{d\nu} \tag{2.23}$$

Thus, the magnitude of resultant shearing stress at any point is indicated by the closeness of shear stress lines at that point and the maximum shear stress acts at the point where the shear stress lines are closest together.

2.3 Method of analogy and supply boundary condition.

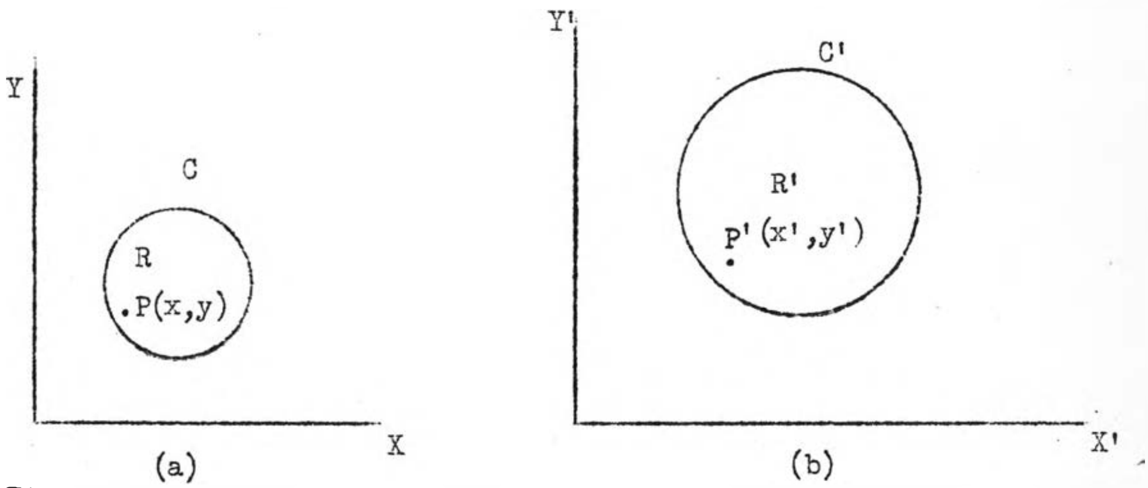

Fig.5 Co-ordinate axes and the region of the cross-section of the shaft and of the conducting sheet under consideration

Figure 5 (a) and 5 (b) are geometric similarity. Figure 5(b) represents the conducting sheet and figure 5(a) represents the cross - section of shaft. The conducting sheet is the enlarged scale model of the cross - section of shaft by the factor of n. Point P (x,y) in figure 5(a) correspond to point P'(x',y') in figure 5(b) by

$$x' = nx \;,\; y' = ny$$

The potential distributed in a thin conducting sheet as described in section (2.1) is the analogous system of torsion problem in term of conjugate function if the boundary potential is prescribed in form of

$$V = \tfrac{1}{2}\,(x^2 + y^2) + \text{constant}$$

This equation does not specify the unit of potential. A coefficient "S_1" is introduced so that when the value of expression $(x^2 + y^2)$ increases 1 unit, the potential V increases S_1 Volts. The boundary potential suppling to conducting sheet in figure 5(b) is then expressed in term of co-ordinate (x', y') as

$$V = S_1(x'^2 + y'^2) + C_1 \quad \text{Volt} \qquad (2.24)$$

The value of ψ on boundary C in figure 5(a) is prescribed by eq. (2.11)

$$\psi = \tfrac{1}{2}\,(x^2 + y^2) + \text{constant}$$

In this experiment, the constant is alway chosen to be zero. So the value of ψ prescribed on boundary C in figure 5(a) is governed by equation

$$\psi = \tfrac{1}{2}\,(x^2 + y^2) \qquad (2.25)$$

One unit of ψ is represented by S_2 volts and because the constant of ψ and V can be chosen arbitrarily. The potential V on point $P'(x',y')$ in R' represent the conjugate function ψ on the corresponding point P (x,y) in R by relation

$$V = S_2\psi + c_2 \tag{2.26}$$

The coefficient S_1 and constant c_1 in eq.(2.24) can be determined from the maximum and minimum value of potential and the co - ordinates. Coefficient S_2 and constant c_2 in eq.(2.26) can be determined from two pairs of corresponding points on boundary calculated from eq.(2.24) and (2.25)

The dimension of eachlength measured in this experiment is expressed in term of unit only. The unit of conducting sheet may differ from the unit of the cross-section of the shaft.

There are three types of specimen used in this experiment as shown in fig.9. The square conducting sheet of 10 units x 10 units represents the square cross- section of shaft of 10 units x 10 units.A unit ofthe sheet in this case is equal to 25 m.m. ,but a unit of cross section of shaft equal to 25/n m.m.. This conducting sheet then represents all size of the cross-sections of the shafts that has geometric similarity. The values of the conjugate function at any corresponding points remain the same for all sizes of the cross-sections.

In this thesis, the potential supply is 15 V. In the case of a square, as in figure 8(a) the maximum and minimum boundary potential is 10 V and 0 V respectively. Maximum potential is at the point (10,10) and minimum potential is at the points (0,10) and (10,0)

From eq.(2.24)

$$V = S_1 \cdot (x^2+y^2) + c_1$$

$$10 = 200\, S_1 + c_1$$

$$0 = 100\, S_1 + c_1$$

$$S_1 = 0.1, \quad c_1 = 10$$

The boundary potential is then governed by the equation.

$$V = 0.1\ (x^2+y^2) - 10 \qquad \text{Volt.} \qquad (2.27)$$

Since either the x or the y co-ordinate are always equal to 10 units, the boundary potential (2.27) is reduced to

$$V = 0.1\, x^2 \text{ or } 0.1\, y^2 \qquad \text{Volt.} \qquad (2.28)$$

From eq. (2.25)

at point (10,10) $\qquad \psi = 100.0$

at point (0,10) $\qquad \psi = 50.0$

The value of S_2 and c_2 can be determined from eq.(2.26)

$$10 = 100\, S_2 + c_2$$

$$0 = 50\, S_2 + c_2$$

$$S_2 = 0.2, \quad c_2 = 10.0$$

The potential V relates to the conjugate function ψ by the equation.

$$V = 0.2\psi + 10.0 \tag{2.29}$$

$$\frac{\partial V}{\partial x} = 0.2\frac{\partial \psi}{\partial x} \quad , \quad \frac{\partial V}{\partial y} = 0.2\frac{\partial \psi}{\partial y} \tag{2.30}$$

By the same method, for the rectangular conducting sheet of 5x10 unit, the boundary potential is governed by

$$V = 0.1\ (x^2 + y^2) - 2.5 \qquad \text{Volt} \tag{2.31}$$

The relation between potential and conjugate function are

$$V = 0.2\psi - 2.5 \tag{2.32}$$

$$\frac{\partial V}{\partial x} = 0.2\frac{\partial \psi}{\partial x} \quad , \quad \frac{\partial V}{\partial y} = 0.2\frac{\partial \psi}{\partial y} \tag{2.33}$$

In the case of the I cross-section in fig.9, the maximum boundary potential is 10.2 V. The boundary potential is governed by

$$V = 0.15\ (x^2 + y^2) - 0.6 \tag{2.34}$$

The relation between potential and conjugate function

$$V = 0.3\psi - 0.6 \tag{2.35}$$

$$\frac{\partial V}{\partial x} = 0.3\frac{\partial \psi}{\partial x} \quad , \quad \frac{\partial V}{\partial y} = 0.3\frac{\partial \psi}{\partial y} \tag{2.36}$$

2.4 Analytical equations

For rectangular shaft, the torsion function, conjugate function, shear stress components and twisting moment have been solved as followed:-[1]

---

[1] Sokolnikoff, I.S. Mathematical Theory of Elasticity. P.130-133.

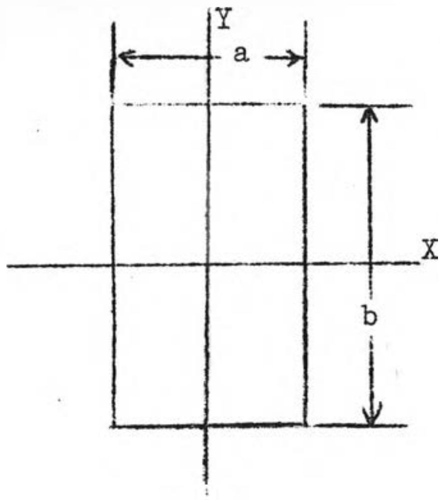

Fig. 6 Dimensions and co-ordinate axes of the rectangular shaft under consideration.

$$\psi(x,y) = xy - \frac{8a^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\,\frac{\sinh k_n y}{\cosh(k_n b/2)}\,.\sin k_n x \qquad (2.37)$$

$$\psi(x,y) = \frac{a^2}{4} + \tfrac{1}{2}(y^2-x^2) - \frac{8a^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\,.\,\frac{\cosh k_n y}{\cosh(k_n b/2)}\,.\cos k_n x \qquad (2.38)$$

Since $\phi(x,y) = \psi(x,y) - \tfrac{1}{2}(x^2+y^2)$

$$\phi(x,y) = \frac{a^2}{4} - x^2 - \frac{8a^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\,.\frac{\cosh k_n y}{\cosh(k_n b/2)}\,.\cos k_n x \qquad (2.39)$$

$$\tau_{zy} = \mu\alpha\left[2x - \frac{8a}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\,.\frac{\cosh k_n y}{\cosh(k_n b/2)}\,\sin k_n x\right] \qquad (2.40)$$

$$\tau_{zx} = -\frac{8a\mu\alpha}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\,.\,\frac{\sinh k_n y}{\cosh(k_n b/2)}\,.\cos k_n x \qquad (2.41)$$

$$M = \mu\alpha\left[\frac{ba^3}{3} - \frac{64a^4}{\pi^5}\sum_{n=0}^{\infty}\frac{\tanh(k_n b/2)}{(2n+1)^5}\right] \qquad (2.42)$$

where $k_n = \frac{(2n+1)\pi}{a}$

a = length of the side parallel to x - axis

b = " " " to y - axis

$b \geq a$

It is more convenient to compare the maximum shear stress and torsional stiffness of rectangular shafts by factors K and K_1. The factors K and K_1 are defined as

$$K = \frac{\tau_{max}}{\mu \alpha a} \tag{2.43}$$

$$K_1 = \frac{M}{\mu \alpha a^3 b} \tag{2.44}$$

For I cross-section, no analytical solutions are established, but the approximate solutions are as followed,

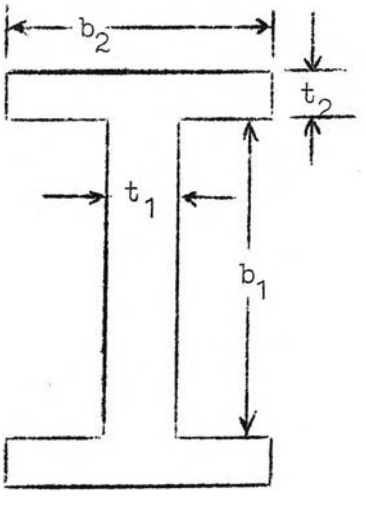

Fig. 7 Dimension of the I cross-section under consideration

$$M = \frac{\mu \alpha}{3}(b_1 t_1^3 + 2\ b_2 t_2^3) \tag{2.45}$$

The corresponding dimensions are shown in fig. 7.

The shear stress of the boundary points far from the corners of cross section is approximated by the equation of the narrow rectangle.

---

[1] Timoshenko, S.P. and Goodier, J.N. Theory of Elasticity P.307-309 and P.321-324.

$$\tau = \mu \alpha t \tag{2.46}$$

where t = thickness at that point

The stress concentration at the reentrant corner is approximated by the approximate equation of the stress concentration at the reentrant corner of the angle.

$$\tau = \mu \alpha t \left(1 + \frac{t}{4r} \right) \tag{2.47}$$

where r = radius of fillet

The values obtained from the abovs equations will be compared to the values obtained experimentally.