CHAPTER II

THEORETICAL BACKGROUND

2.1 An overview of Neural Networks

Human brain is highly complex as it consists of billions of the specially built cells called neurons. Each neuron consists of four major components: soma, axon, dendrite and synapse as shown in Figure 2.1.

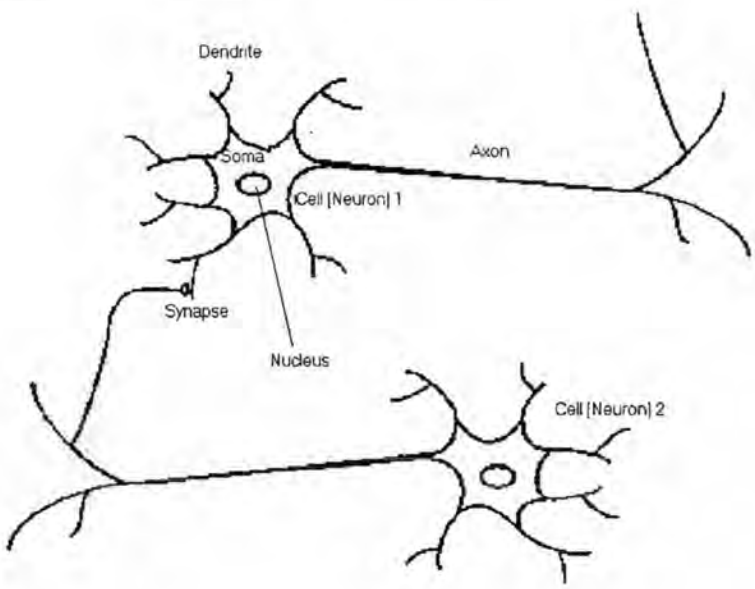

Figure 2.1: Two interconnected biological cells

The input and output signals to the soma of a neuron are transmitted along the axon and dendrite, while the synaptic resistance controls the strength of the signal. A neuron learns to generate a particular signal by adjusting the synaptic resistance.

Neurons are connected to an enormous network called neural network. The synaptic resistance is referred as the weight of a neuron. There are three types of biological learning mechanisms which are.

1. **Supervised learning:** A neuron is forced to generate a target signal associated with a specific input pattern and to reproduce this target signal whenever the specific input pattern occurs.

2. **Unsupervised learning:** There is no target signal generated with a particular input pattern. A neuron competitively adjusts its weight equal to the value of the input pattern.

3. **Reinforcement learning:** It is a mixture of the supervised and unsupervised learning under the environment that the target connot be explicitly.

For artificial neural network, it is notably resemble to the biological learning, as each neuron is designed to have the same components similar to a biological neuron in the human brain as follows:

Soma	corresponds to	neuron node
Axon	corresponds to	output
Dendrite	corresponds to	input
Synapse	corresponds to	weight

2.2 Activation Function

An activation function, denoted by $f(v_k)$, defines the output of a neuron k in terms of the induced local field v_k. Here, we identify the three basic types of the activation functions:

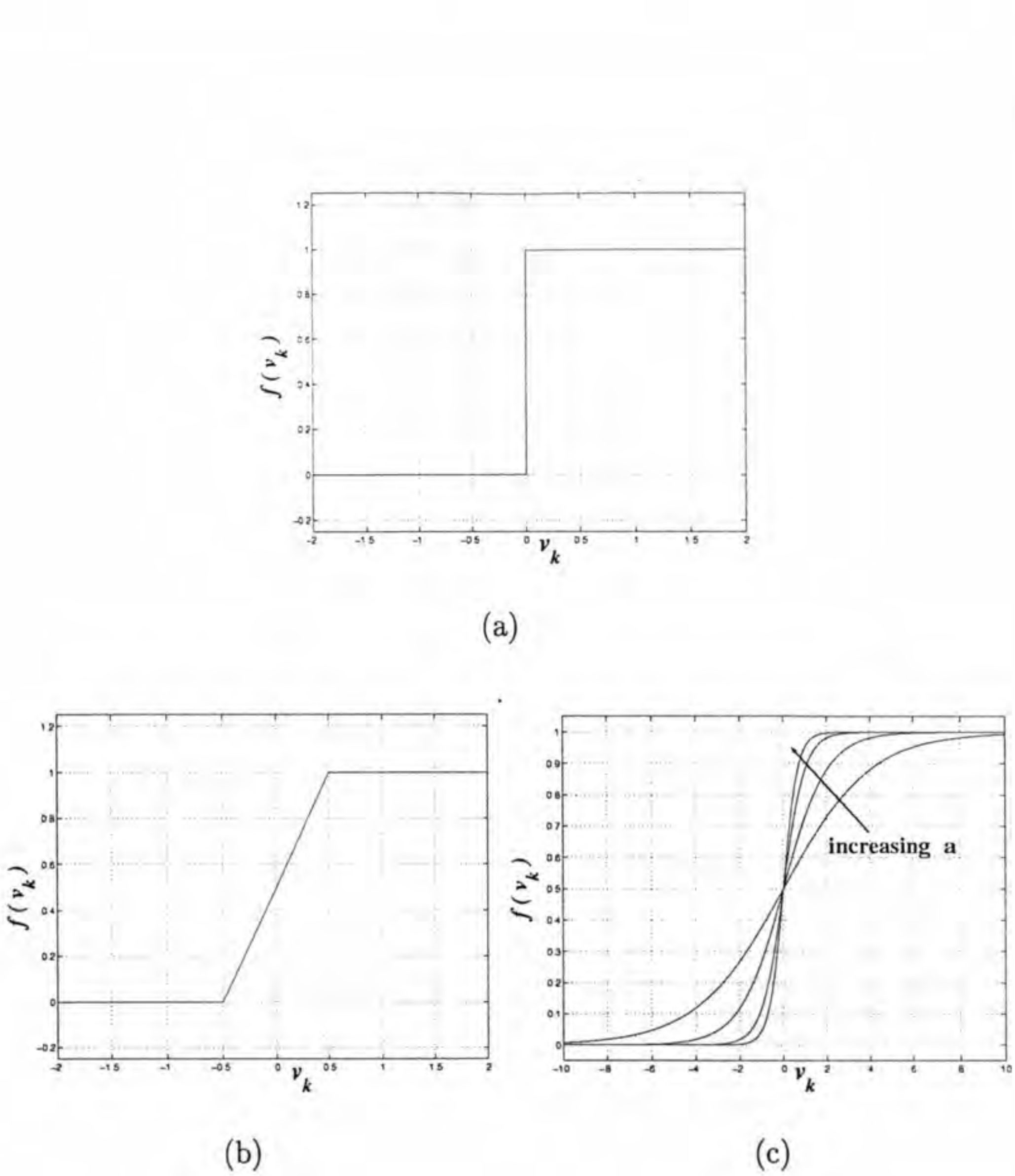

Figure 2.2: Three different types of activation Functions (a)Threshold function, (b)Piecewise-Linear Function and (c)Logistic Activation Function

1. **Threshold function**

   For this type of activation function, described in Fig. 2.2(a), we have

$$f(v_k) = \begin{cases} 1 & if \quad v_k \geq 0 \\ 0 & if \quad v_k < 0 \end{cases} \tag{2.1}$$

   This form of threshold function is commonly referred to as a Heaviside function.

The value of a v_k is computed by

$$v_k = \sum_{j=1}^{m} w_{kj} x_j + b_k \tag{2.2}$$

where m is the dimension of input of the neuron k, w_{kj} is the synaptic weights at link j of neuron k, x_j is the input signals at link j and b_k is the bias of neuron k. Such a neuron is referred as the McCulloch-Pitts model, in recognition of the pioneering work done by McCulloch and Pitts(1943). In this model, the output of a neuron takes on the value of 1 if the induced local field of that neuron is nonnegative, and 0 otherwise.

2. **Piecewise-Linear Function**

For the piecewise-linear function described in Fig. 2.2(b), we have

$$f(v_k) = \begin{cases} 1, & v_k \geq +\frac{1}{2} \\ v_k, & +\frac{1}{2} > v_k > -\frac{1}{2} \\ 0, & v_k \leq +\frac{1}{2} \end{cases} \tag{2.3}$$

where the amplification factor, which is a slope of a linear region, is a constant value. This form of an activation function may be viewed as an approximation to a non-linear amplifier. Therefore, the piecewise-linear function becomes a threshold function if the amplification factor of the linear region is made infinitely large.

3. **Logistic Activation Function**

A logistic activation function, whose graph is s-shaped, is by far the most common form of activation function used in the construction of artificial neural networks. It is defined by

$$f(v_k) = \frac{1}{1 + exp(-av_k)} \tag{2.4}$$

where a is the slope parameter of the logistic activation function. By varying the parameter a, we obtain a logistic activation function of different slopes, as illustrated in Fig. 2.2(c). In fact, the slope at the origin equals $a/4$. When the slope parameter approaches infinity, the logistic activation function simply becomes a threshold function. Also notice that the logistic activation function is differentiable, whereas the threshold function is not.

The activation functions defined in Eqs. (2.1), Eqs. (2.3) and Eqs. (2.4) range from 0 to +1. It is sometimes desirable to have the activation function ranging from -1 to +1, in which case the activation function assumes an antisymmetric form with respect to the origin; that is, the activation function is an odd function of the induced local field. Specifically, the threshold function of Eqs. (2.1) is now defined as

$$f(v_k) = \begin{cases} 1, & if \quad v_k > 0 \\ 0, & if \quad v_k = 0 \\ -1, & if \quad v_k < 0 \end{cases} \tag{2.5}$$

which is commonly referred to as a signum function. For the corresponding form of a logistic activation function, we may use a hyperbolic tangent function, defined by

$$f(v_k) = tanh(v_k) \tag{2.6}$$

2.3 Radial Basis Function (RBF) Neural Networks

2.3.1 Radial Basis Function Networks

Radial basis functions are simply a class of functions. In principle, they could be employed in any sort of model (linear or nonlinear) and any sort of network (single-layer or multi-layer). However, since Broomhead and Lowe's 1988 seminal paper, radial basis function networks (RBF networks) have traditionally been associated with radial functions in a single-layer network such as shown in the Figure 2.3.

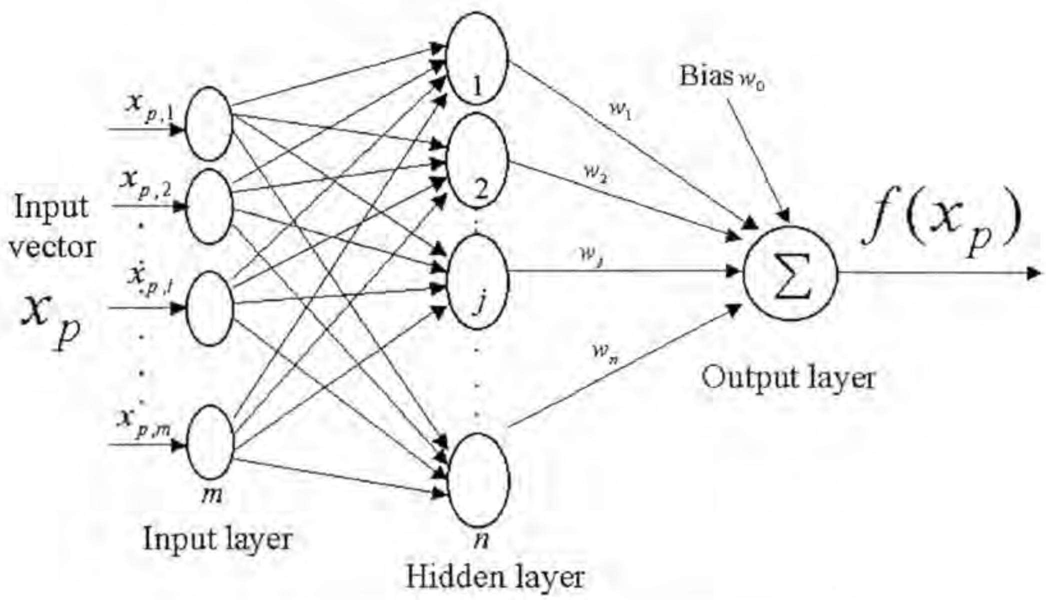

Figure 2.3: The structure of RBF Neural Network

From the Fig. 2.3, let $\mathbf{X} = \{\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_p}, \ldots, \mathbf{x_z}\}$ be the set of input vectors. An input vector $\mathbf{x_p}$ in a m dimensional space is defined as $\mathbf{x_p} = [x_{p,1}, x_{p,2}, \ldots, x_{p,i}, \ldots, x_{p,m}]^T$. Each $x_{p,i}$ is fed forward to n hidden neurons whose outputs are linearly combined with weights w_j into the network output $f(\mathbf{x_p})$.

A RBF network is nonlinear if the basis functions can move or change size or if there is more than one hidden layer. Normally, we use a single-layer networks with functions which are fixed in position and size. There are few basis functions involved with the RBF. Gaussian distribution function is one of the basis functions which is typically used in the RBF.

2.3.2 Gaussian Function

A Gaussian distribution function is defined by

$$f_k(\mathbf{x_p}) = exp(-\frac{1}{\sigma_k^2}\left\|\mathbf{x_p} - \mu_\mathbf{k}\right\|^2) \tag{2.7}$$

where i is an index of an input data vector, k is an index of a hidden unit, $\mathbf{x_p}$ is an input vector i, $\mu_\mathbf{k}$ is the center vector of RBF hidden unit k, σ_k is the width vector of RBF hidden unit k, $f_k(\mathbf{x_p})$ is an output of neuron k, and $\left\|\mathbf{x_p} - \mu_\mathbf{k}\right\|^2$ is the Euclidean distance between $\mathbf{x_p}$ and $\mu_\mathbf{k}$.

From the Fig. 2.4, $x_{p,1}$ represents the first element of input vector $\mathbf{x_p}$, and $x_{p,2}$ represents the second element of input vector $\mathbf{x_p}$. $f(\mathbf{x_p})$ represents the output obtained from the Gaussian distribution function, which is a symmetric belled shape that has the same width in all dimensions. Therefore, all the input vectors, which have the same distance from the center, will result in the same output $f(\mathbf{x_p})$.

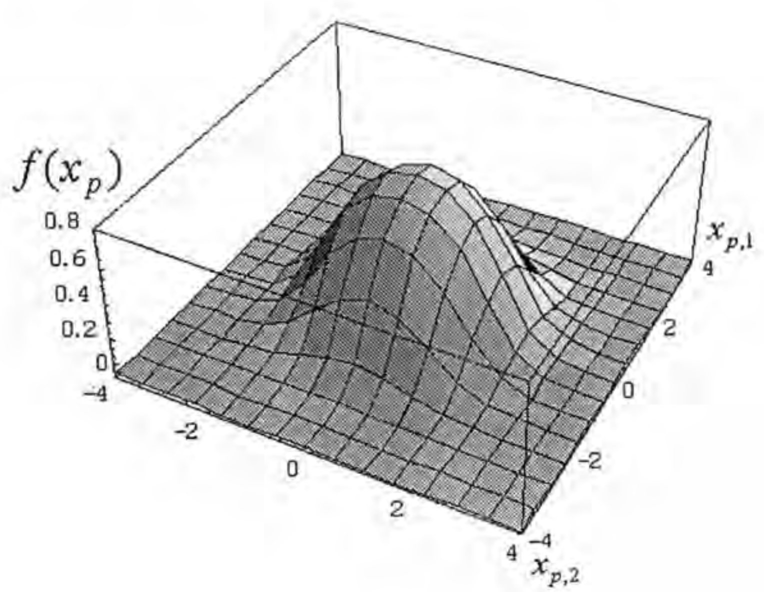

Figure 2.4: Illustration of a Gaussian RBF function

There are some other basis functions which are also generally used in the RBF. The most general formula for any radial basis function (RBF) is multivariate Gaussian function.

2.3.3 Multivariate Gaussian Function

A multivariate Gaussian function is defined by

$$f_k(\mathbf{x_p}) = exp(-\frac{1}{2}\left[\mathbf{x_p} - \mu_\mathbf{k}\right]^T \Sigma^{-1} \left[\mathbf{x_p} - \mu_\mathbf{k}\right]) \tag{2.8}$$

where Σ refers to the covariance matrix of the input data, $\mu_\mathbf{k}$ is the center vector. This multivariate Gaussian function is illustrated in Figure 2.5.

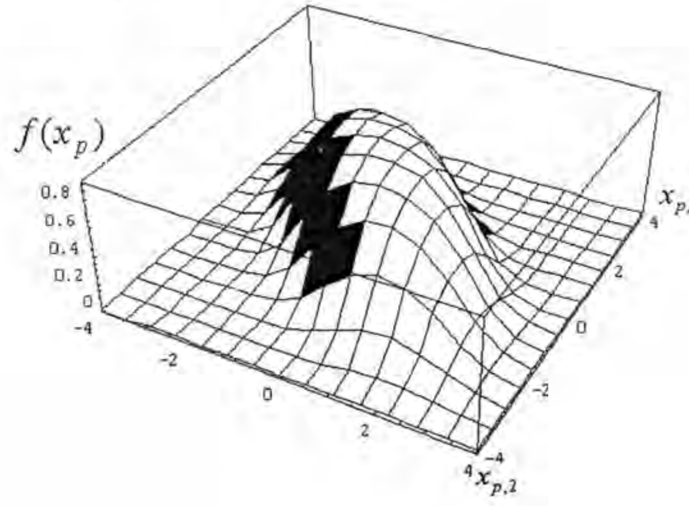

Figure 2.5: Illustration of a multivariate Gaussian function

From the Fig. 2.5, $x_{p,1}$ represents the first element of input vector $\mathbf{x_p}$, and $x_{p,2}$ represents the second element of input vector $\mathbf{x_p}$. $f(\mathbf{x_p})$ represents the output obtained from the multivariate Gaussian function, which is an elliptic shape that possibly has different widths in the different dimensions. Therefore, some input vectors, which have the same distance from the center, may not result in the same output $f(\mathbf{x_p})$.