CHAPTER I

INTRODUCTION TO TWO-DIMENSIONAL SYSTEM

Introduction and Historical Review

In an ideal two-dimensional system a magnetic field perpendicular to the plane of electrical confinement leads to full quantization of the electron motion (1). The energy spectrum consists of sharp Landau levels separated by the cyclotron energy $\hbar\Omega$. In real system the Landau levels are broadened due to scattering by impurities, phonon or other scattering mechanisms (2). In the simplest approximation the levels are described by a level width Γ. In the case of a high magnetic field, where $\hbar\Omega >> \Gamma$, real gaps appear between the Landau levels. This leads to an oscillatory structure for practically all physical quantities as a function of the magnetic field.

Examples of real two-dimensional electron systems (2DES) are electrons confined to interfaces in GaAs-GaAlAs layered heterostructures and electrons at surfaces in Si metal-oxide semiconductor field effect transistors (MOSFETS). The most fundamental quantity underlying all the physical properties of these systems is the form of the density of states (DOS), n(E). Earliest interest was in electron transport properties, the integer quantum Hall effect (3,4) and the fractional quantum Hall effect (3,5), and in distinguishing localized from delocalized states. However, transport measurements do not readily observe the density of states available to the electrons. Recently several experiments (6-13) measuring single electron behaviour determine the density of states directly. These find that the Landau levels are significantly broadened, due to disorder in the sample, and there is a substantial density of states lying between Landau levels not obtained in existing calculations (2,14,15). An important outstanding

problem (9,14), is to identify the origin of the large density of states between Landau levels in a direct and consistent manner. Indeed recent experiments of specific heat (6-9), capacitance (10,11), magnetization (12,13) and radiative recombination spectra (16), of a two-dimensional system show that Landau levels have large widths and a large density of states between Landau levels (approximately 25% of the density of states of a free two-dimensional electron, n_0).

Begining with the pioneering work of Ando and Uemura (2), there have been several calculations (2,14) of the broadening of the Landau levels due to disorder. Most use a perturbative approach, the self consistent Born approximation. This leads to a narrow density of states centered at each Landau level with n(E) = 0 between Landau levels. A numerical simulation (15) exhibits a large Landau level width and an asymmetric shape presumably due to bound states. Wegner (17) has obtained the density of states for the lowest Landau level centered at $E_0 = (1/2)\hbar\Omega$. Most theories consider electrons interacting with disorder having an interaction of zero range. Typically they use the white noise model in which the variance $W(\vec{r}-\vec{r}\,')=<V(\vec{r})V(\vec{r}\,')> - <V(r)>^2$ of the fluctuating potential $V(\vec{r})$ has zero correlation length, $W(\vec{r}-\vec{r}\,') = W(0)\delta(\vec{r}-\vec{r}\,')$. None of these theories predict a significant density of states lying between Landau levels.

To overcome this defect, Gornik et al. (9) added a constant density of states, xn_0, to the Landau level values with x adjusted to fit experiment ($x \simeq 0.25$). Similarly, Gudmundsson and Gerhardts (14) have added additional disorder, attributed to long range sample inhomogeneity, to further broaden the Landau levels. However, the origin of the additional inhomogeneity and why it must be added separately is not entirely clear nor satisfactory.

We present here a straightforward evaluation of the density of states in the presence of disorder by using the Feynman path integral. The essential point is to keep the correlation length, L, of disorder finite from the outset. The white noise model is then obtained by letting $L \to 0$. The detailed evaluation will be contained in chapter III.

Two-Dimensional Electron System in a Strong Magnetic Field

Two-dimensional electron systems can be realized in several classes of systems. One example is the electrons trapped on the surface of liquid helium below 4.2 K. Since we can not make the concentration of electrons too high in this system, chiefly because the liquid surface can not sustain too many electrons pressed to the surface by an electric field, the electrons form a classical gas with the Boltzmann distribution. There are two classes of system in which we can make degenerate two-dimensional electrons with electrons occupied up to the Fermi energy E_F: metal-oxide-semiconductor (MOS) space charge layers (Fig. 1) and semiconductor heterojunctions (Fig. 2).

A MOS inversion layer consists of a metallic layer as electrode, an oxide layer as an insulator and a semiconductor layer. The most widely used MOS inversion layer is made from p-type silicon with SiO_2 as an insulator. When we apply a voltage V_g, which is called the gate voltage, across the metal and the semiconductor, the valence and conduction bands of the semiconductor are bent as is shown in Fig. 1.

When the bottom of the conduction band is pushed down below E_F near the interface of p-type Si and SiO_2, electrons are accumulated at the bottom of the conduction band there. The electron system may be regarded as two-dimensional (2D), since the electrons are confined within the interface region and move relatively freely along the interface. This type of MOS system is called an inversion layer, because the

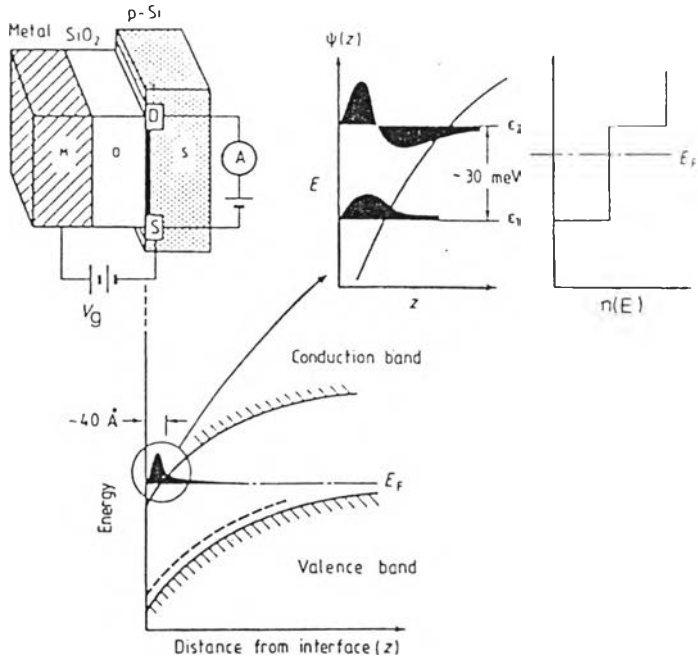

Fig.1 Two-dimensional electron system in the metal-oxide-semiconductor (MOS) inversion layer. The S and D represent source and drain (usually n-type doped region) respectively, V_g gate voltage and E_F Fermi energy. Bending in valence and conduction bands is depicted together with wave function $\psi(z)$ and density of states n(E).

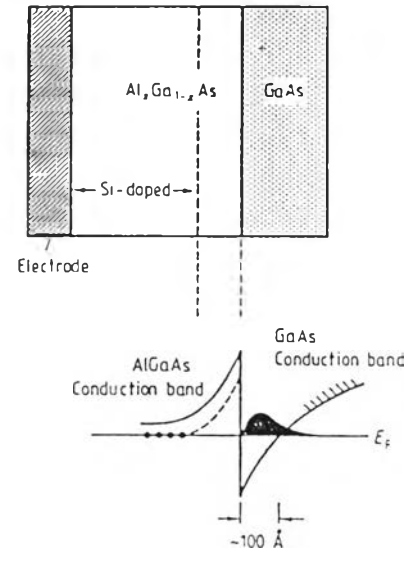

Fig.2 Two-Dimensional electron system in the semiconductor heterostructure. The figure depicts the case of selective doping in AlGaAs.

carrier in this sample is the electron while the bulk semiconductor is p-type. The motion of an electron in this situation is described by the Schrodinger ' s equation,

$$H\Psi(\vec{r}) = E\Psi(\vec{r}), \qquad (1.1)$$

where $$H = \vec{p}^2/2m^* + V(z), \qquad (1.2)$$

and m* is the effective mass of the carrier, z is the direction perpendicular to the interface and V(z) represents the bottom of the conduction band and a Schottky barrier of the oxide layer. If we ignore scatterers or other imperfections in the system, the motion within the xy-plane is free with the wave function given by

$$\Psi(\vec{r}) \propto \exp[\, i(k_x x + k_y y)\,]\, f_n(z), \qquad (1.3)$$

where $f_n(z)$ is the wavefunction in a potential well formed by V(z) with a quantum number n. For the bound states quantized with V(z), $f_n(z)$ has discrete energy levels $\varepsilon_1, \varepsilon_2, \ldots\ldots$. Since the density of states for free electrons in 2D space is a constant, the total density of states comprises a series of step functions (Fig. 3). When $E_F < \varepsilon_1$, the electrons behave as a purely 2D system. The current within the 2D system is driven by a voltage applied across the two electrodes in Fig. 1 by S (source) and D (drain). The MOS system is quite convenient in that the concentration n of 2D electrons can be varied in the same sample in a range as wide as $n \simeq 0 - 10^{13}$ cm^{-2} by varying the gate voltage, which changes the degree of bending of the conduction band. The concentration is proportional to V_g apart from a small threshold.

Similar 2D electrons can be realized in semiconductor heterostructures (Fig.2), in which two kind of semiconductors are put together. The most typical one is the

GaAs-Al $_x$Ga$_{1-x}$ As ($0< x \leq 1$) heterostructure. The lattice constant of GaAs and Al_xGa_{1-x} As are almost the same (the difference being $\cong 0.1$ %), so that when one material is grown on top of another by molecular beam epitaxy, we have a well defined interface between the two materials with little disorder. Unlike MOS system, it is rather difficult to attach gate electrodes to heterostructures, so that the electron concentration may not be varied by V_g. Thus semiconductor heterostructures are characterized by small degree of randomness, whereas MOS inversion layers have the virtue of variable electron concentration.

The quantum Hall effect is a direct consequence of the Landau quantization, the quantum states in a magnetic field. We first consider the quantum motion of electrons in magnetic fields. Like a classical Larmor motion, the quantum state of an electron in a magnetic field corresponds to a rotation called cyclotron motion. An easy way to represent this state is to express the coordinates of an electron as

$$x = X + \xi \; ; \; y = Y + \eta \; , \qquad (1.4)$$

where (X,Y) is the coordinate of the center of the cyclotron motion, while (ξ,η) is the relative coordinate around the center. The Hamiltonian of a free electron system in a magnetic field is given by

$$H_O = (1/2m^*)[\, \vec{p} + (e/c)\vec{A}\,]^2 \qquad (1.5)$$

where $\vec{p}$ is the momentum of an electron and $\vec{A}$ is the vector potential with $\vec{\nabla} x \vec{A} = \vec{B}$. One can verify that (ξ,η) rotates with an angular frequency equal to the cyclotron frequency

$$\Omega = eB/m^*c. \qquad (1.6)$$

Note that ξ and η are subjected to an uncertainty of order ℓ because of the commutation relation

$$[\xi,\eta] = -i\ell^2, \qquad (1.7)$$

where ℓ is the cyclotron radius given by

$$\ell = (c\hbar/eB)^{1/2}. \qquad (1.8)$$

The Hamiltonian is in turn written in terms of (ξ,η) as

$$H_o = (\hbar\Omega/2\ell^2)[\,\xi^2 + \eta^2\,], \qquad (1.9)$$

so that the energies of the Hamiltonian are quantized into discrete Landau levels,

$$E_n = [n + 1/2\,]\hbar\Omega \quad (n = 0,1,2...). \qquad (1.10)$$

The key aspect of this quantization is obviously the completely discrete energy spectrum, which is hard to concieve in a normal bulk system. If we apply a magnetic field to a 3D system, for instance, an electron can move freely along the direction of the magnetic field with the corresponding orbit being a helix. The density of states of the 3D quantum system, shown in Fig.3, comprises a series of continous band arising from the motion along the magnetic field, unlike the 2D case. In this sense the Landau quantization is perfect in 2D.

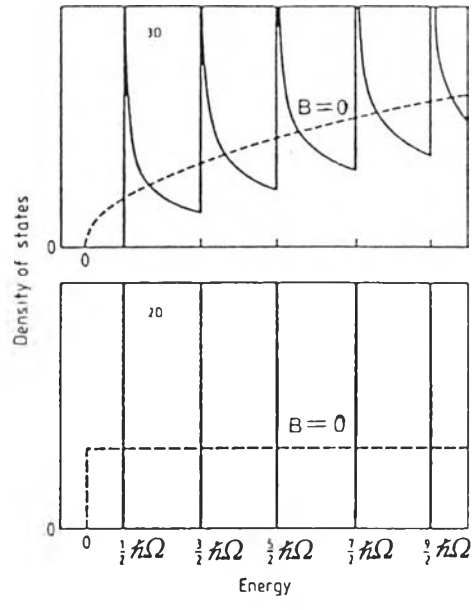

Fig.3 Density of states for two and three-dimensional electron system in magnetic field.

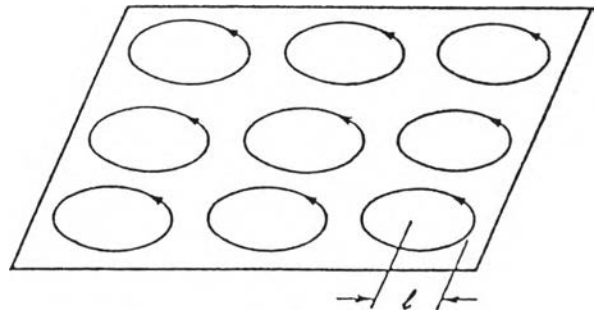

Fig.4 The degeneracy (total number of states) in a Landau level in a two- dimensional system is roughly equal to the number of circles of radius ℓ (cyclotron radius) covering the system.

For every Landau level in a 2D system, the degeneracy (number of states belonging to the level) is given by (Fig.4),

$$N_L = L^2/(2\pi \ell^2), \qquad (1.11)$$

where L is the linear dimension of the system. This degeneracy comes from the fact that all cyclotron motions with different center coordinates have the same energy. Note that X and Y, too, have an uncertainty of order ℓ because of the commutation relation,

$$[X,Y] = i\ell^2. \qquad (1.12)$$

By using the degeneracy concept, we can specify the concentration n of electrons by the Landau level filling factor, which is defined as

$$\upsilon = 2\pi\ell^2 n\,. \qquad (1.13)$$

This dimensionless quantity indicates the filling Landau level, e.g. $\upsilon = 3$ means that the lowest three Landau levels are just filled.

So far, we have mentioned the free system. In real, say, semiconductor systems, there always exists randomness arising from impurities and the roughness of the semiconductor interfaces. In the presence of randomness, the Hamiltonian is given by

$$H = H_O + V(\vec{r}), \qquad (1.14)$$

where $V(\vec{r})$ is the random potential. Now the system is dominated by the dynamics of the center coordinate (X,Y) of the cyclotron motion. The degeneracy of the states with different (X,Y) in a Landau level is now lifted. This implies that the density of states is no longer a series of sharp lines, each Landau level becomes a band with width Γ. We call this band a Landau sub-band, the nature of these sub-bands depends quite strongly on the mode of spatial variation of the random potential. Ando and Uemura (2) assumed that $V(\vec{r})$ varies rapidly within a length scale of cyclotron radius ℓ. Such a potential may be constructed as

$$V(\vec{r}) = V_O \Sigma_i \delta(\vec{r} - \vec{r}_i), \qquad (1.15)$$

where $\vec{r}_i$ is the position of the ith scatterer. In this case, the motion of the electron may be regarded as a quantum hopping of the center of the cyclotron motion with the hopping distance $\simeq \ell$ for each jump. The potential equation (1.15), is characterized by the dimensionless concentration of scatterers,

$$c_i = 2\pi\ell^2 n_i, \qquad (1.16)$$

where n_i is the original concentration of the scatterers and c_i represents the average number of scatterers within a circle of radius ℓ. The electronic structure of the system depends strongly on c_i. When $c_i >> 1$(dense scatterers), the description of the system by the self-consistent Born approximation (Ando and Uemura 1974) becomes applicable (Fig.5). In this case, the density of states for each Landau sub-band is semi-elliptic with a width given by

$$\Gamma = 2c_i (V_O/2\pi\ell^2). \qquad (1.17)$$

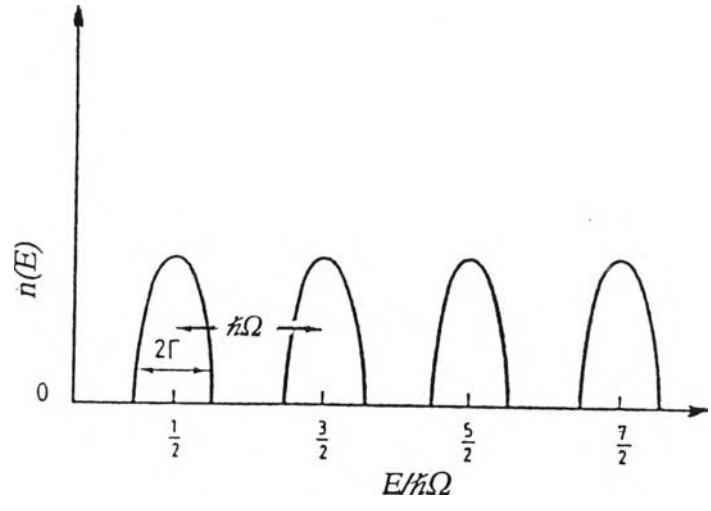

Fig.5 Density of states (semi-elliptic) for the disordered two-dimensional electron system in a strong magnetic field in the self-consistent Born approximation (SCBA).

The Quantum Hall Effect

In 1980 von Klitzing, Dora and Pepper (18) found quite a remarkable phenomenon for 2D electrons in Si MOS inversion layers. Fig.6 shows a typical result. In this figure we show the diagonal resistance R_{xx} and the Hall resistance R_{xy} as a function of the gate voltage V_g ($\propto$ electron concentration) in a Si (100) MOS inversion layer in a magnetic field B = 19T.

It is remarkable that the resistance R_{xx} vanishes in regions of finite width of V_g between adjacent Landau sub-bands. If we turn to the R_{xy}, the Hall resistance takes a constant value over the region in which R_{xx} =0. This phenomenon has already been observed by Kawaji and Wakabayashi (19). The region where R_{xx} = 0 and R_{xy} = constant is called a plateau. What von Klitzing et al. found is that the value of R_{xy} in

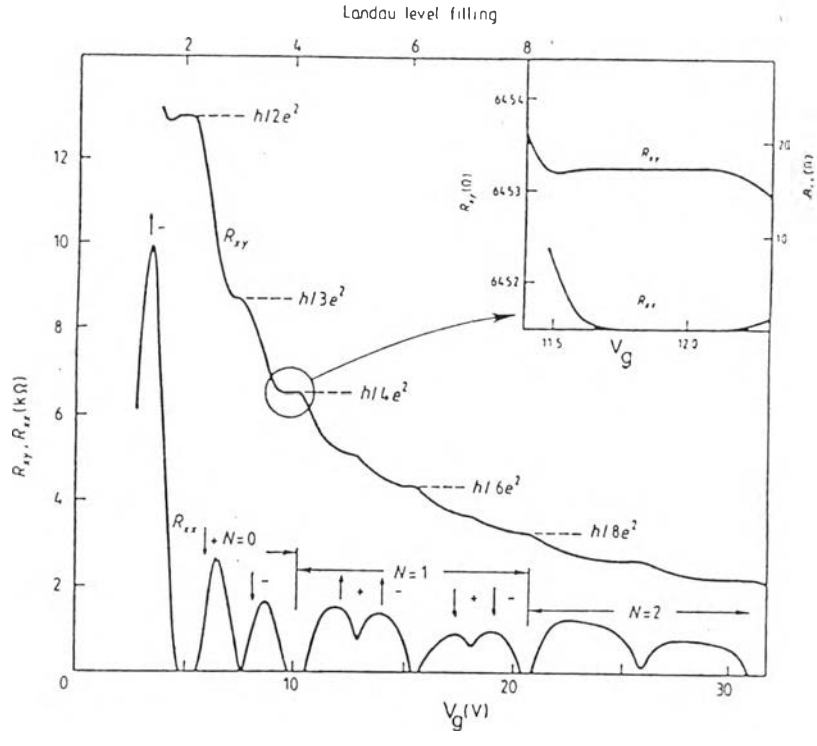

Fig.6 Experimental result for the quantum Hall effect in a Si(100) MOS inversion layer in a magnetic field of B = 19T at T = 1.5K. The diagonal resistance R_{xx} and Hall resistance R_{xy} are shown as a function of gate voltage V_g, which is proportional to the electron concentration. The oscillation in R_{xx} is labelled by the Landau index N, spin (↑↓) and valley (±). The upper scale indicates the Landau filling factor $2\pi\ell^2 n$. The inset shows a detail of the plateau between N = 0 and N = 1 Landau levels.

any plateau quantized into a universal quantity with an astonishing accuracy. For a plateau between Nth and (N+1)th level, the Hall resistance is given by

$$R_{xy} = h/Ne^2 \text{ (N integer)}, \qquad (1.18)$$

where the right-hand side only contains physical constants. This is the quantum Hall effect. The experimental accuracy for the above formula is of the order 1 PPM (= 10^{-6}) which is subsequently reduced to as small as 0.1 PPM (20). In other words, a plateau is flat within this accuracy, and the value of a plateau itself is given by (1.18) within this accuracy for any sample of Si MOS inversion layer, or, in later experiments, for any sample of GaAs-AlGaAs heterostructures.

Explanation of the Quantum Hall Effect (21)

The main features of quantum Hall effect which require a physical understanding are the existence of the Hall plateau and the dissipationless current flow in the region of a Hall plateau (R_{xx}=0).

A simple explanation is as follows. As mentioned above the electrons subjected to a high magnetic field perpendicular to the plane of the system occupy discrete energy levels called Landau levels. Each Landau level is highly degenerate. The number of electrons per unit area in a Landau level (ignoring spin for simplicity) is eB/h . In a situation when i Landau levels are completely full, the carrier density n is given by

$$n = ieB/h . \qquad (1.19)$$

The Hall resistivity is related to the Lorentz force acting on the electrons drifting in the applied magnetic and electric fields. It is given by

$$R_H = B/ne . \qquad (1.20)$$

Eq. (1.19) and (1.20) give the quantum Hall condition in Eq. (1.18). The quantum Hall effect is observed when the thermal energy kT is very small compared to the magnetic energy $\hbar\Omega$ so that the excitation of electrons from the last full Landau level i to the next higher Landau level can be neglected. To satisfy this condition one needs low temperatures and high magnetic fields to observe the quantum Hall effect. In GaAs heterojunctions this condition can be satisfied at relatively lower magnetic field or higher temperature because of the smaller effective mass of carriers and hence a larger magnetic energy as compared to the Si-MOS.

The above explanation does not hold in reality however. The probability that i Landau levels will be found completely full in a real physical system is zero because the carrier density is specified. As soon as an extra electron is added to the 2D system with i Landau levels completely full, the extra electron fills the (i+1)th Landau level and the Fermi energy jumps discontinously. What is required then is a reservior of electrons which would keep i Landau levels completely full by transfering electrons to and from the 2D system when the electron density is varied over a limited range.

The above explanation ignores any impurities present in the system. The presence of impurities can explain the existence of a dissipationless current flow (vanishing of linear resistance) in the region of Hall plateaus. The impurities broaden each Landau level into an energy band of finite width. It is expected that states near the

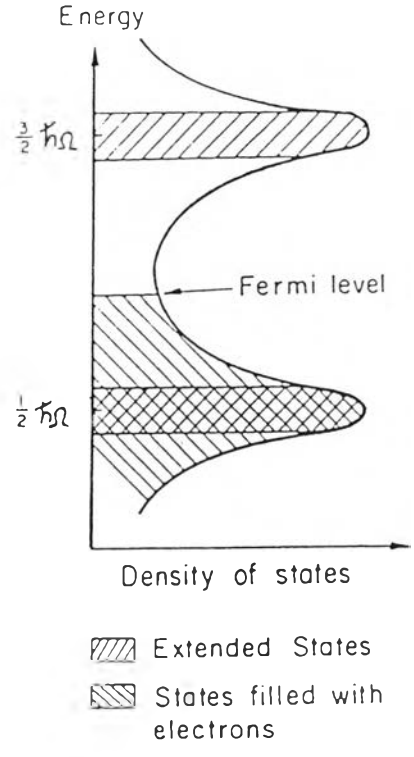

Fig.7 A schematic diagram showing Landau levels broadened into bands by impurities. Only the states near the center of the bands are extended states. The Fermi level is shown to lie in the mobility gap between two Landau levels.

center of such a band will be delocalized (extended in space) whereas a strong enough impurity potential will cause the appearance of localized states in the gap between the Landau levels (Fig. 7). Only the delocalized states take part in transport processes and can carry current for example. In the presence of localized states the Fermi level remains pinned in the gap between two Landau levels for finite range of n (or B). The impurity levels act as the reservoir which keeps i Landau levels completely full by transfering electrons to and from the 2D system of the extended states (current carrying states) when the electron density is varied over a limited range. Thus the density of delocalized states at the Fermi level $n(E_F)$ remains zero for this range of n (or B). The

scattering of electrons (which is responsible for electrical resistance) is proportional to $n(E_F)$. Hence, whenever the Fermi level is pinned in a gap (also called a mobility gap because of the absence of current carrying extended states near the Fermi level), the diagonal component of the resistivity vanishes and the transport properties remain constant explaining the presence of a Hall plateau.

Direct Measurement of the Density of States of a Two-Dimensional Electron Gas

In summary, from the explanation of the quantum Hall effect, the region between the Landau levels has a finite density of states but these states cannot take part in the transport because they are localized, non-current-carrying states. Recently, the expectation of non zero density of states between Landau levels was confirmed by experiments of specific heat (6-9), capacitance (10,11) and magnetization (12,13) of a two-dimensional electron gas. However, most of these experiments deal with the density of states dn/dE_F measured on the Fermi surface, also called the thermodynamic density of states and it is different from n(E). Recently, Kukushkin and Timofeev(16) proposed to determine the density of 2D electron states in a transverse magnetic field by a direct method based on measurements of the luminescence spectra connected with radiative recombination of 2D electrons with nonequilibrium photoexcited holes in the Si(100) MOS structure. In this method they measured the energy distribution of the intensity of radiation spectra I(E) ($I(E) \propto n(E)$) at a fixed filling of the Landau levels by electrons. The experimental result is shown in Fig.8.

From Fig.8, it is shown that the density of states between Landau levels is not vanishingly small and the width Γ depends on both the filling factor υ and the Landau index N. The evaluation of the density of states of our model and the comparison with the direct measurement of Kukushkin and Timofeev will be contained in chapter III.

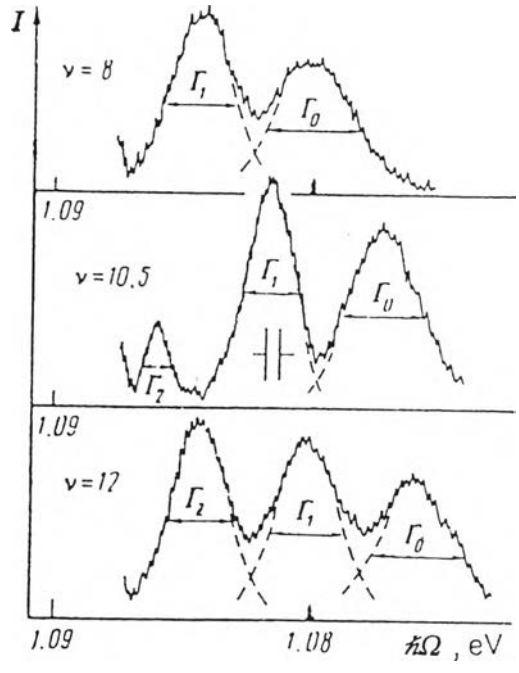

Fig.8 2D-electron radiative recombination spectra obtained at B = 7T, T=1.6K at different fillings of the Landau levels (υ).

Theoretical Models of the Density of States of a Two-Dimensional Electron Gas (3)

As we have mentioned in the introduction of this chapter, there have been several calculations of the broadening of the Landau levels due to disorder. Among them the so called self-consistent-Born approximation (SCBA) is known to be most established and the simplest one free from various difficulties of divergence caused by the singular nature of the density of states. In the SCBA, effects of scattering from each scatterer are taken into account in the lowest Born approximation, while those of the level broadening are considered in a self-consistent way within the frame work of the Green's function formalism. So that in this section we first briefly review the theory

and summarise the characteristic results. According to Ando et al.'s ideas(2), the Hamiltonian is given by

$$H = H_0 + H_1, \tag{1.21}$$

with $$H_0 = (1/2m)[\vec{p}^2 + e\vec{A}/c]^2, \tag{1.22}$$

$$H_1 = \sum_i \sum_\mu v^\mu(\vec{r} - \vec{r}_i, z_i), \tag{1.23}$$

where $v^\mu(\vec{r} - \vec{r}_i, z_i)$ is the effective two-dimensional potential of the μth kind of scatterer located at $(\vec{r}_i, z_i)$. By choosing the symmetric gauge $\vec{A} = (-By/2, Bx/2)$, an eigen function of H_0 is given by

$$\Psi_{NX}(\vec{r}) = (1/\sqrt{L})\exp\left[\frac{ixy}{2\ell^2} - \frac{ixY}{\ell^2}\right]\chi_N(x-X), \tag{1.24}$$

with $$\chi_N(x) = (2^N N!\sqrt{\pi}\ell)^{-1/2}\exp\left[-\frac{x^2}{2\ell^2}\right]H_N(x/\ell), \tag{1.25}$$

where $H_N(y)$ is Hermite' s polynomial and L^2 is the area of the system. The Hamiltonian is rewritten as

$$H_0 = \sum_{NX} E_N a^+_{NX} a_{NX}, \tag{1.26}$$

$$H_1 = \sum_i \sum_\mu \sum_{NX} \sum_{N'X'} (NX| v^\mu(\vec{r} - \vec{r}_i, z_i) |N'X') a^+_{NX} a_{N'X'}, \tag{1.27}$$

where a^+_{NX} and a_{NX} are the creation and destruction operators, respectively. The Green's function can be written as

$$G_N(E)\delta_{NN'}\delta_{XX'} = \langle 0 | a_{NX}(E-H)^{-1} a^+_{NX} | 0 \rangle, \tag{1.28}$$

where I0) represents the vaccuum state and <.....> means an average over all configurations of scatterers. The Green's function is diagonal with respect to the Landau level index N and center coordinate X and independent of X. The density of states is given by

$$n(E) = -(1/\pi)\sum_{NX} \mathrm{Im}\, G_N(E + i0)$$

$$= -(1/\pi)\frac{1}{2\pi\ell^2}\sum_{N} \mathrm{Im}\, G_N(E+i0). \qquad (1.29)$$

The Green' s function is calculated perturbationally, using iteratively

$$(E-H)^{-1} = (E-H_O)^{-1} + (E-H_O)^{-1}H_1\,(E-H)^{-1}, \qquad (1.30)$$

and is usually written in terms of the self-energy $\Sigma_N(E)$ as

$$G_N(E) = G^{0}_N(E) + G^{0}_N(E)\Sigma_N(E)\, G_N(E), \qquad (1.31)$$

with $G^{0}_N(E) = (E-E_N)^{-1}$. In sufficiently a strong magnetic field one can neglect couplings between different Landau levels . When E E_N, one gets

$$\Sigma_N(E) = (1/4)\,\Gamma^{2}_N\, G_N(E), \qquad (1.32)$$

$$\text{with} \quad \Gamma_N = 4(2\pi\ell^2)\sum_{\mu}\int dz\, N^{\mu}_i(z)\sum_{m}[\,(Nm\,|\,v\,(z)\,|\,Nm)\,]^2, \qquad (1.33)$$

and $N^{\mu}_i(z)$ is the density of scatterers of the μth kind. The density of states becomes

$$n(E) = \frac{1}{2\pi\ell^2}\sum_{N}\left[\,1 - \left(\frac{E-E_N}{\Gamma_N}\right)^2\,\right]^{1/2}. \qquad (1.34)$$

The density of states for each Landau level has a semielliptic form with the width Γ_N.

The nature of level broadening Γ_N depends strongly on the range of scattering potentials. In the case of short range scatterers, one can replace $v^{M}(\vec{r},z)$ by $V_O\delta(\vec{r})$ and get a constant width $\Gamma_N=\Gamma$ as in Eq. (1.17), the level broadening is essentially independent of the Landau level as shown in Fig. 5. In the case of long range scatterers, on the other hand, one gets

$$\begin{aligned} \Gamma_N &\simeq 4 \sum_{M} \int dz\, N_i^{M}(z) \int d\vec{r}\, [v^{M}(\vec{r},z)]^2 \\ &= 4 \langle (V(\vec{r}) - \langle V(\vec{r}) \rangle)^2 \rangle, \end{aligned} \tag{1.35}$$

where $V(\vec{r})$ is the local potential energy due to scatterers and <....> means the average over their configurations. The level width is independent of the Landau level and is expressed by the fluctuation of the local potential energy.

In order to see explicitly the range dependence of the level width, take for instance scatterers with the Gaussian potential $v^{M}(\vec{r}, z) = \frac{V^{M}(z)}{\pi L^2} \exp[-(\vec{r}/L)^2]$, which approaches to a δ-potential as the range L goes to zero. After a little manipulation, for a ground Landau level N =0, one has

$$\Gamma_O^2 = \frac{\Gamma^2}{1+(L/\ell)^2}. \tag{1.36}$$

The absolute value of the level width depends not only on the functional form of the potential, i. e. its range, but also on the multiplicative factor which appears in the potential. The level width decreases as the range increases, because the model potential which becomes weaker with larger L has been assumed.

However, the SCBA is not sufficient in the energy region close to the spectral edges, which is clear if we consider the unphysical sharp cutoff of the density of states (see Fig. 5).

Gerhardts (22) used the method of cumulant expansion in calculating the density of states. The Green' s function is written as

$$G_N(E) = \int_0^\infty dt \exp[-i(Et/\hbar)\ (0\ I\ a_{NX} K(t)\ a^+_{NX}\ I\ 0) \qquad (1.37)$$

with $K(t) = < \exp [\ -(i/\hbar)Ht\ ] >$. (1.38)

In the interaction representation one gets

$$K(t) = \overset{0}{K}(t) < T \exp [\ -(i/\hbar) \int_0^t d\tau\ H_1(\tau)\ ] >, \qquad (1.39)$$

with T being a time-ordering operator,

$$H_1(t) = \exp [\ (i/\hbar)H_0 t\ ]\ H_1 \exp [\ -(i/\hbar)H_0 t\ ], \qquad (1.40)$$

and $\overset{0}{K}(t) = \exp [\ -(i/\hbar)H_0 t\ ]$. (1.41)

A cumulant expansion of the time-ordered exponential in Eq. (1.39) gives

$$< T\exp [\ -(i/\hbar) \int_0^t d\tau\ H_1(\tau)\ ] > = \exp [\ \sum_{v=1}^{\infty} (1/v!)\ C_v\ ]. \qquad (1.42)$$

The first-order term,

$$C_1 = -(it/\hbar) < H_1 >, \qquad (1.43)$$

is trivial and can be neglected. The second-order term is nontrivial and is given by

$$C_2 = (-i/\hbar)^2 T \int_0^t d\tau \int_0^t d\tau' [< H_1(\tau) H_1(\tau') > - < H_1 >^2]. \qquad (1.44)$$

If the the lowest nontrivial term is retained and in a sufficiently strong magnetic field, one gets

$$K_N(t) = (0 \mid a_{NX} K(t) \overset{+}{a}_{NX} \mid 0).$$

$$= \exp [- (i/\hbar) E_N t] \exp [- \frac{\Gamma_N^2 t^2}{8\hbar^2}]. \qquad (1.45)$$

Thus, the density of states takes on a Gaussian form

$$n(E) = \frac{1}{2\pi\ell^2} \sum_N [(\pi/2)\Gamma_N^2]^{-1/2} \exp [-2(\frac{E-E_N}{\Gamma_N})^2], \quad (1.46)$$

which is shown by the dotted line in Fig. 9. This approximation, called the lowest-order cumulant approximation(LOCA), can take into account higher order effects partially and does not cause the unphysical sharp cutoff of the density of states. It gives therefore, a theoretical basis, together with the calculation of Ando, for using the Gaussian form of the density of states in qualitative line shape analysis.

We see that the simple theories of Ando et al.(2) and Gerhardts (22) predict for the density of states broadened Landau levels of an elliptical or a Gaussian shape, depending on the approximation in which the interaction of the electrons with randomly distributed scatterers is taken into account. For a sufficiently strong magnetic field B, the Landau levels are energetically welled separated and the density of states is expected to be zero or exponentially small in the gap between the Landau levels. A number of

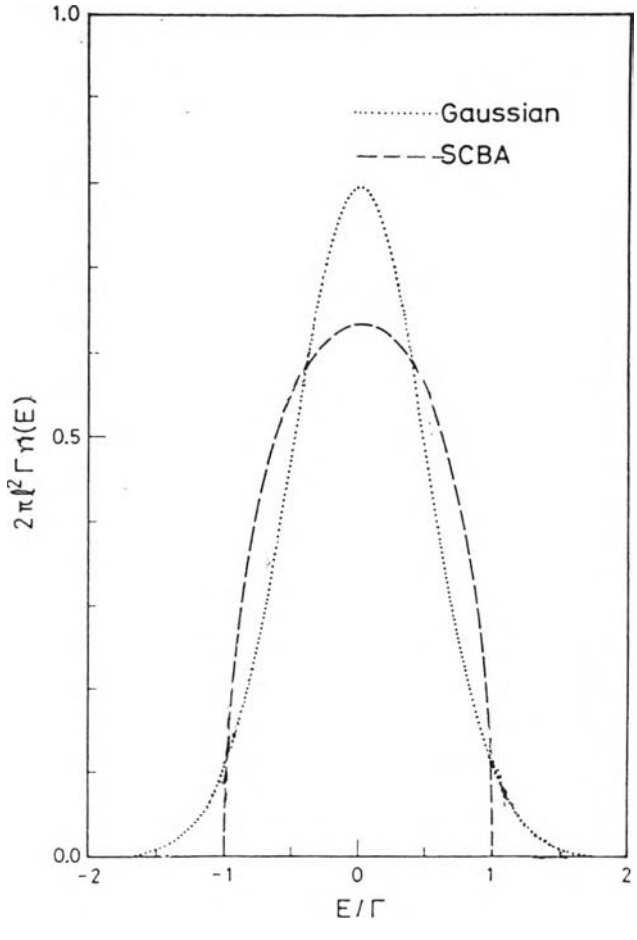

Fig. 9 The density-of-states profile of the ground Landau level calculated by using the self-consistent-Born approximation (dashed line) and the lowest order cumulant approximation (dotted line).

recent experiments produce, however, strong evidence for an unexpectedly large density of states in the Landau gap for the 2DEG in both GaAs-(GaAl)As heterostructures and the MOS-inversion layer . For the evaluation of thermally activated transport in the quantum Hall regime, Gornik et al. (9) and Gudmundsson and Gerhardts (14) proposed the following model of the density of states,

$$n(E) = (1-x)\,[\,(1/\pi\ell^2)\sum_n G(E, E_n; \Gamma)] + xn_o, \qquad (1.47)$$

where only a fraction 1-x of the total states is described by a Gaussian-shaped Landau DOS. Here $E_n = \hbar\Omega(n+1/2)$ is the Landau-level energy, Γ the level broadening, and

$$G(z, z';\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\,[\,-(1/2)\,(\frac{z-z'}{\Gamma})^2\,], \qquad (1.48)$$

a normalized Gaussian distribution. A constant background is represented by a fraction x of the zero-field DOS, n_o. Measurements of equilibrium properties such as magnetization and specific heat seem to support this picture. However, the physical origin of the background DOS remains unclear.

The next chapter we will devote to review the Feynman path integral theory including some exact solutions of solvable -2D problems.