# CHAPTER IV

# CONCLUSION

## Summary

In chapter I, we mentioned two classes of two-dimensional electron systems., the metal-oxide-semiconductor (MOS) layers and semiconductor heterojunctions. A MOS inversion layer consists of a metallic layer as an electrode, an oxide layer and a semiconductor layer. When we apply a voltage $V_g$, across the metal and the semiconductor , the valence and conduction bands of the semiconductor are bent. When the bottom of the conduction band is pushed down below $E_F$ near the interface of semiconductor and insulator, electrons are accumulated at the bottom of the conduction band. The electron system may be regarded as 2D since the electrons are confined within the interface region and move relatively freely along the interface. The MOS system is quite convenient in that the concentration n of 2D electrons can be varied by varying the gate voltage, which changes the degree of bending of the conduction band. For the heterojunctions such as semiconductor heterostructures, in which two kinds of semiconductors are put together by molecular beam epitaxy, we have a well defined interface between the two materials with little disorder. However, it is difficult to attach gate electrodes to heterostructures, so that the electron concentration may not be varied by $V_g$. Thus semiconductor heterostructures are characterized by small degree of randomness, whereas MOS inversion layers have the advantage of variable electron concentration.

Then the quantum Hall effect, a direct consequence of Landau quantization, was shortly reviewed. This phenomenon was first found by von Klitzing et al. in 1980.

The main features of the quantum Hall effect are the existence of the Hall plateau and the dissipationless current flow in the region of a Hall plateau. It is now believed that the impurities of a 2D system are responsible for a dissipationless current flow in the region of the Hall plateaus. The impurities broaden each Landau level into an energy band width and it is expected that states near the center of such a band will be delocalized whereas the states in the gap between Landau levels are localized. Thus, whenever the Fermi level is pinned in the mobility gap (localized states), the diagonal component of the resistivity vanishes and the transport properties remain constant explaining the presence of a Hall plateau. In the next section of chapter I we have mentioned the direct measurement of the density of states in the MOS Si(100) structure which was performed by Kukushkin and Timofeev. In this experiment they measured directly the energy distribution of the intensity radiation spectrum I(E) which is proportional to n(E), at a fixed filling of the Landau levels. Their experimental result was shown in Fig. 8. For the last section of chapter I, we have briefly reviewed the SCBA theory of Ando and the LOCA theory of Gerhardts. The density of states of their models are elliptical shape and Gaussian shape respectively. The theoretical results were shown in Fig. 9.

In chapter II, we briefly reviewed the propagator and Feynman path integral. The propagator or probability amplitude of a particle to go from $\vec{x}'$ at time $t'$ to $\vec{x}''$ at time $t''$ , according to Feynman' s ideas can be expressed in the path integral form

$$K(\vec{x}'',\vec{x}';t'',t') = \lim_{N\to\infty} \left(\frac{m}{2\pi i\hbar\varepsilon}\right)^{3N/2} \iint\ldots\int \exp\left[ \frac{i}{\hbar}\left\{\sum_{i=1}^{N} \left( \frac{m}{2\varepsilon}(\vec{x}_i-\vec{x}_{i-1})^2 - \varepsilon V(\vec{x}_i)\right)\right\}\right] d^3x_1 d^3x_2 \ldots d^3x_{N-1}. \quad (4.1)$$

or in a less restrictive notation as

$$K(\vec{x}'',\vec{x}';t'',t') = \int \exp\{\frac{i}{\hbar}S[\vec{x}'',\vec{x}']\}D[\vec{x}(t)]. \qquad (4.2)$$

Then the free particle which can be evaluated exactly using (4.1) has been examined in one dimension. For the system with a quadratic Lagrangian, it was shown that the propagator can be evaluated exactly using van Vleck-Pauli' formula

$$K(\vec{x}'',\vec{x}'';t'',t') = F(t'',t')\exp\{\frac{i}{\hbar}S_{cl}[\vec{x}'',\vec{x}']\}, \qquad (4.3)$$

where

$$F(t'',t') = \det[\frac{i}{2\pi\hbar}\frac{\partial^2 S_{cl}[\vec{x},\vec{x}']}{\partial\vec{x}''\partial\vec{x}'}]^{1/2}. \qquad (4.4)$$

We then examined the problem of an electron confined in two dimensions under the influence of a homogeneous transverse magnetic field and nonlocal harmonic oscillator potential. To apply the van Vleck-Pauli formula, we first transform the actual problem into the problem of an electron moving in the presence of magnetic field, electric field and harmonic force using Stratonovich transformation. Then the classical action of the transformed problem is evaluated exactly using 2x2 matrices introduced by Papadopoulos in the calculation of a harmonically bound charge. The propagator for the original problem is obtained by taking the Gaussian average of the electron propagator in the transformed problem. After getting the exact propagator we then considered the density of states of the electron when the electric field is zero using the standard formula,

$$n(E) = \frac{V}{2\pi\hbar}\int_{-\infty}^{\infty} e^{(i/\hbar)ET} K(0,0;T)dT, \qquad (4.5)$$

we found the density of states of such an electron ,

$$n(E) = (1/2\pi)\alpha\beta mA \sum_{s=0} \sum_{n=0} \delta\frac{[(s+1/2)\hbar\Omega+(n+1/2)\hbar\Omega-E]}{[(s+1/2)\hbar\Omega+(n+1/2)\hbar\Omega-E]} . \quad (4.6)$$

When the nonlocal harmonic oscillator potential goes to zero, (4.6) reduces to a well-known density of states of an electron confined in two dimensions under the influence of a homogeneous transverse magnetic field,

$$n(E) = \frac{\Omega mA}{2\pi\hbar} \sum_{n=0}^{\infty} \delta[E-(n+1/2)\hbar\Omega]. \quad (4.7)$$

In chapter III, we evaluated the density of states of an electron in a two-dimensional system under the influence of a transverse magnetic field and disorder potential due to impurities. In our model we considered an electron in a system of very dense, random and weak scatterers and we assumed the scattering potential to be a Gaussian potential with finite correlation length L. To obtain the electron density of states we first evaluated the average propagator of the electron using the path integral method. We found the average propagator could be taken into the form

$$K(\vec{r}',\vec{r};T) = \int D[\vec{r}(t)]\, e^{(i/\hbar)S[\vec{r}(t)]}, \quad (4.8)$$

where
$$S[\vec{r}(t)] = \frac{m}{2}\int_0^T dt\, (x^2+y^2+\Omega(xy-yx)) + \frac{i\rho}{2\hbar}\int_0^T\int_0^T dt d\sigma W(\vec{r}(t)-\vec{r}(\sigma)), \quad (4.9)$$

and
$$W(\vec{r}(t)-\vec{r}(\sigma)) = \frac{u^2}{\pi L^2}\exp\{-\frac{[\vec{r}(t)-\vec{r}(\sigma)]^2}{L^2}\}. \quad (4.10)$$

After expressing the average propagator K in terms of a zero-order propagator $K_o$ and then performing the first cumulant approximation, we obtained the average propagator,

$$K_1(0,0;T) = K_o(0,0;T)\exp[-\frac{u^2\rho\pi T}{2(2\pi\hbar L)^2}\int_0^T dt[B(T,y)]^{-1}], \quad (4.11)$$

where
$$K(0,0;T) = (\frac{m}{2\pi i\hbar T})(\frac{\Omega T}{2\sin(\Omega T/2)}), \quad (4.12)$$

and
$$B(T,y) = 1+\frac{4i\hbar\sin[(\Omega/2)(T-y)][\sin(\Omega y/2)]}{m\Omega L^2\sin(\Omega T/2)}. \quad (4.13)$$

We then defined $E_L = \hbar^2/2mL^2$, $\xi_L = u^2\rho/\pi L^2$ and $x = \hbar\Omega/E_L$, finally obtaining the density of states of the electron as

$$n(E) = (S/\pi\hbar)\int_{-\infty}^{\infty} dTK_o(0,0;T)\exp[\, iET/\hbar$$

$$-\frac{\xi_L xT\sin(\Omega T/2)}{2i\hbar\Omega\sqrt{a^2(x,T)-1}}\tan^{-1}[\frac{\sqrt{a(x,T)-1}}{\sqrt{a(x,T)+1}}\tan(\Omega T/4)], \quad (4.14)$$

where
$$a(x,T) = (x/4i)\sin(\Omega T/2) - \cos(\Omega T/2). \quad (4.15)$$

The density of states in (4.14) could not be evaluated analytically and we made a large-T approximation to investigate the density of states at low energies and obtained an analytic form of (4.14) as

$$n(\nu) = \frac{n_o x}{\sqrt{2\pi\Gamma'^2}}\sum_{n=0}^{\infty}\exp[-\frac{(\nu-(n+1/2)x)^2}{2\Gamma'^2}], \quad (4.16)$$

where $\xi'_L = \xi_L/E_L^2$, $\nu = E/E_L$ and $\Gamma'^2 = \Gamma^2/E_L^2$. The density of states from (4.16) for $\xi'_L = 1$, $1 \leq x \leq 4$ and $x = 5$, $1 \leq \xi'_L \leq 5$ were shown in Fig. 13 and Fig.14 respectively.

In the last section of chapter III we have shown the comparisons of our numerical results of (4.51) with the magnetooptic experiments of Kukushkin and Timofeev. We found that by choosing the correlation length L=97Å and the fluctuation parameter $\xi_L$ =6.8 $meV^2$, the numerical and experimental results are in good agreement.

Discussion and Conclusion

From the comparison with experiments in Fig. 14, 15, we see that the present model of the DOS can describe experiments quite accurately. The present model is quite simple but has some essential features. Firstly, the DOS is evaluated nonperturbatively. Thus the DOS n(E) for large values of ($E - E_n$ ) can be obtained. The DOS can therefore be finite for all E and does not vanish between LL' s. Secondly, n(E) is evaluated for arbitrary correlation length L of disorder. This corresponds to electron-impurity ion potentials $v(\vec{r} - \vec{R}_i)$ having a finite range. The importance of using a finite-range potential has recently been stressed by Ando and Murayama (39).

However, to obtain the Gaussian DOS of (4.16), we made approximations. Particularly, we took the long-time limit of the electron propagator which means n(E) is valid for the lowest-lying Landau levels only. This approximation probably masks difficulties that appear with the DOS for higher LL' s. These questions are carefully discussed by Broderix et al. (40).

The method presented in this thesis may be improved in two different directions. One direction is to improve upon the action $S_o$ in the same spirit as done by

Sa-yakanit (31) on his evaluation of path-integral theory of a model disordered system. Another direction is to go beyond the first cumulant. This means that, the average propagator will contain a second cumulant $(1/2)[ <(S - S_0)^2>_0 - < S - S_0>_0^2 ]$. However, from this research, we have shown that the first cumulant approximation is sufficient to obtain an appropriate DOS which compares well with experiments. Furthermore, from our DOS expression, combined with a model of impurity screening, Esfarjani et al. (41) obtained a self-consistent model of LL broadening due to impurities. This leads to an explanation of the quantum Hall oscillation. Finally, we mention that the method developed here may be extended to the electrical conductivity or the electron mobility calculation. The consideration of the electrical conductivity is of particular interest for it may give us a clue to understanding the nature of localization of electrons in disordered systems.