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APPENDIX

APPENDIX A

Evaluation of $S_{cl}[\vec{r}'',\vec{r}']$ and $< e^{i\vec{k}.(\vec{r}(t)-\vec{r}(\sigma))} >_o$

In order to evaluate the average $< e^{i\vec{k}.(\vec{r}(t)-\vec{r}(\sigma))} >_o$, it is necessary to establish a characteristic functional (31),

$$< \exp\{ (i/\hbar)\int_0^T dt\, \vec{f}(t).\vec{r}(t) \} >_o,$$

where $\vec{f}(t)$ is any arbitrary function of time, and the average $< .... >_o$ is defined as

$$< \exp\{ (i/\hbar)\int_0^T dt\, \vec{f}(t).\vec{r}(t) \}>_o = \frac{\int D[\vec{r}(t)]\exp\{(i\hbar)[S_o+\int_0^T dt\vec{f}(t).\vec{r}(t)]}{\int D[\vec{r}(t)]\exp\{(i/\hbar)S_o\}}. \quad (A.1)$$

Equation (A.1) suggests that if the action S_o is quadratic, then so is the action $S_o' = S_o + \int^T dt\vec{f}(t).\vec{r}(t)$. From Feynman and Hibbs (1965), the path integral of equation (A.1) can be carried out exactly as

$$<\exp\{ (i/\hbar)\int^T dt\, \vec{f}(t).\vec{r}(t) \}>_o = \exp\{ (i/\hbar)[S'_{ocl}[\vec{r}'',\vec{r}'] - S_{ocl}[\vec{r}'',\vec{r}']] \}, \quad (A.2)$$

where $S'_{ocl}[\vec{r}'',\vec{r}']$ and $S_{ocl}[\vec{r}'',\vec{r}']$ are the corresponding classical actions of $L'_o(\vec{r},\dot{\vec{r}};t)$ and $L_o(\vec{r},\dot{\vec{r}};t)$ respectively. This means that the the path integral of equation (A.1) can be reduced to an exponential function. Once the classical action $S'_{ocl}[\vec{r}'',\vec{r}']$ has been obtained, the classical action $S_{ocl}[\vec{r}'',\vec{r}']$ can be obtained from it by setting $\vec{f}(t)$ identically zero. To obtain the classical action $S'_{ocl}[\vec{r}'',\vec{r}']$ we need to find the classical path which can be obtained by making a variation on $S'_o[\vec{r}(t)]$. In order to make a variation on $S'_o[\vec{r}(t)]$, we begin with the Lagrangian $L'_o(\vec{r},\dot{\vec{r}};t)$ which corresponds to the the action $S'_o[\vec{r}(t)]$,

$$L'_o(\vec{r},\dot{\vec{r}},t) = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + \frac{m\Omega}{2}(x\dot{y} - y\dot{x}) + \vec{f}(t).\vec{r}(t), \qquad \text{(A.3)}$$

where $\vec{r} = \vec{r}(x,y)$ is the position vector of an electron in the xy-plane. Clearly, from (A.3) the Lagrangian $L_o(\vec{r},\dot{\vec{r}},t)$ which corresponds to $S_{ocl}[\vec{r}\,'',\vec{r}\,']$ can be obtained by letting $\vec{f}(t)$ equal to zero.

We now wish to evaluate the classical action of an electron whose Lagrangian is given by (A.3). After applying the variation on $S'_o[\vec{r}(t)]$,

where, $$S'_o[\vec{r}(t)] = \int_0^T L'_o(\vec{r},\dot{\vec{r}},t)dt\,, \qquad \text{(A.4)}$$

we get the equations of motion,

$$\frac{d}{dt}\frac{\partial}{\partial \dot{x}} L'_o(\vec{r},\dot{\vec{r}},t) - \frac{\partial}{\partial x} L'_o(\vec{r},\dot{\vec{r}},t) = 0 \qquad \text{(A.5)}$$

$$\frac{d}{dt}\frac{\partial}{\partial \dot{y}} L'_o(\vec{r},\dot{\vec{r}},t) - \frac{\partial}{\partial y} L'_o(\vec{r},\dot{\vec{r}},t) = 0, \qquad \text{(A.6)}$$

with the boundary conditions $\vec{r}(0) = \vec{r}\,' = (x',y')$ and $\vec{r}(T) = \vec{r}\,'' = (x'',y'')$.

In principle, by substituting Eq. (A.3) into Eq. (A.5) and (A.6) one can get the exact solution of the classical path $\vec{r}_{cl}(t)$ of an electron directly. However, in real practice this is rather complicated to perform. To avoid such a complication we will solve (A.5) and (A.6) in the following way.

By using the 2x2 matrix introduced by Papadopoulos (30), $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ which has the property $J^2 = -I = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the Lagrangian in Eq. (A.3) can be rewritten in the form

$$L'_0(\vec{r},\dot{\vec{r}},t) = \frac{m}{2}\dot{r}_\perp^2 - \frac{m\Omega}{2}\tilde{r}_\perp J\,\dot{r}_\perp + \tilde{r}_\perp f_\perp, \qquad \text{(A.7)}$$

where $r_\perp = \begin{pmatrix} X \\ Y \end{pmatrix}$, $f_\perp = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$, $\tilde{r}_\perp$ and $\tilde{f}_\perp$ are the transpose of $r_\perp$ and $f_\perp$ respectively. The equations of motion (A.5) and (A.6) are then reduced to one equation as

$$(D^2 + \Omega J D)\, r_\perp = \frac{f_\perp}{m}, \qquad \text{(A.8)}$$

with the boundary conditions $r_\perp(0) = r_\perp' = \begin{pmatrix} X' \\ Y' \end{pmatrix}$ and $r_\perp(T) = r_\perp'' = \begin{pmatrix} X'' \\ Y'' \end{pmatrix}$,where the differential operator $D = \frac{d}{dt}$. The solution of Eq.(A.8) is exactly and we obtain the classical path of an electron

$$\begin{aligned}\vec{r}_{cl}(t) \quad = \quad & \frac{1}{\sin(\Omega T/2)}\, e^{-\Omega J T/2}\Big[\sin\big(\tfrac{\Omega}{2}(T-t)\big) r_\perp' + \sin\big(\tfrac{\Omega T}{2}\big) e^{\Omega J T/2}\, r_\perp''\Big] \\ & - \frac{1}{(m\Omega/2)\sin(\Omega T/2)}\int_0^T \Big[\sin\big(\tfrac{\Omega s}{2}\big)\sin\big(\tfrac{\Omega}{2}(T-t)\big)H(t-s) \\ & + \sin\big(\tfrac{\Omega t}{2}\big)\sin\big(\tfrac{\Omega}{2}(T-s)\big)H(s-t)\Big] e^{\Omega J(s-t)/2} f_\perp(s)\,ds, \qquad \text{(A.9)}\end{aligned}$$

where H represents the Heaviside step function. We now substitute (A.9) into (A.4), and we find the classical action corresponding to the Lagrangian $L'_0(\vec{r},\dot{\vec{r}},t)$,

$$\begin{aligned}S'_{ocl}[\vec{r}'',\vec{r}'] = {} & S_{ocl}[\vec{r}'',\vec{r}'] + \frac{1}{\sin(\Omega T/2)}\int_0^T \tilde{f}_\perp(t) e^{-\Omega J t/2}\Big[\sin\big(\tfrac{\Omega T}{2}\big) e^{\Omega J T/2} r_\perp'' + \sin\big(\tfrac{\Omega}{2}(T-t)\big) r_\perp'\Big] dt \\ & - \frac{1}{\Omega \sin(\Omega T/2)}\int_0^T\!\!\int_0^T \Big[\sin\big(\tfrac{\Omega s}{2}\big)\sin\big(\tfrac{\Omega}{2}(T-t)\big)H(t-s) + \sin\big(\tfrac{\Omega t}{2}\big)\sin\big(\tfrac{\Omega}{2}(T-s)\big)H(s-t)\Big]\tilde{f}_\perp(t) e^{\Omega J(s-t)/2} f_\perp(s)\,ds\,dt\end{aligned}$$

(A.10)

where

$$S_{ocl}[\vec{r}\,'',\vec{r}\,'] = \frac{m}{2}[\frac{\Omega}{2}\cot(\frac{\Omega T}{2})(r_\perp''^2+r_\perp'^2)-\Omega\cot(\frac{\Omega T}{2})\tilde{r}_\perp''r_\perp'-\frac{\Omega}{2}(\tilde{r}_\perp'Jr_\perp''-\tilde{r}_\perp''Jr_\perp')], \quad (A.11)$$

is the classical action corresponding to the Lagrangian $L_0(\vec{r},\dot{\vec{r}},t)$. From Eq. (A.11), after applying the notation $\tilde{r}\,'= (x' \;\; y')$, $\tilde{r}\,''= (x'' \;\; y'')$ and $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, we then obtain the classical action in the usual representation as

$$S_{ocl}[\vec{r}\,'',\vec{r}\,'] = \frac{m}{2}[\frac{\Omega}{2}\cot(\frac{\Omega T}{2})\{(x''-x')^2+(y''-y')^2\}+\Omega(x'y''-x''y')]. \quad (A.12)$$

From Eq. (A.1) and (A.10), we finally get the generating functional

$$<e^{(i/\hbar)\int_0^T dt\vec{f}(t).\vec{r}(t)}>_0=\exp[\frac{i}{\hbar}\{\frac{1}{\sin(\Omega T/2)}\int_0^T\tilde{f}_\perp(t)e^{-\Omega JT/2}[\sin(\frac{\Omega T}{2})e^{\Omega JT/2}r_\perp''+\sin(\frac{\Omega}{2}(T-t))r_\perp']dt$$

$$+\frac{1}{\Omega\sin(\Omega T/2)}\int_0^T\int_0^T[\sin(\frac{\Omega s}{2})\sin(\frac{\Omega}{2}(T-t))H(t-s)$$

$$+\sin(\frac{\Omega t}{2})\sin(\frac{\Omega}{2}(T-s))H(s-t)]\tilde{f}_\perp(t)e^{\Omega J(s-t)/2}f_\perp(s)dsdt\}]$$

(A.13)

By taking $\vec{f}(t) = (\delta(t-\tau) - \delta(t-\sigma))\hbar\vec{k}$ or in the matrix representation

$$f_\perp(t) = (\delta(t-\tau) - \delta(t-\sigma))\hbar k_\perp, \quad (A.14)$$

where $k_\perp=\begin{pmatrix} k_x \\ k_y \end{pmatrix}$, and after completing the integration we get the final result

$$<e^{i\vec{k}.(\vec{r}(\tau)-\vec{r}(\sigma))}>_0 = \exp[\frac{i}{\hbar}\{\hbar(k_xA(\tau,\sigma) + k_yB(\tau,\sigma)) - \hbar^2\vec{k}^2C(\tau,\sigma)\}], \quad (A.15)$$

where

$$A(\tau,\sigma) = \frac{1}{4\sin(\Omega T/2)}[(x''-x')\cos\frac{\Omega}{2}(T-(\tau+\sigma))\sin\frac{\Omega}{2}(\tau-\sigma)-(y''-y')\sin\frac{\Omega}{2}(T-(\tau+\sigma))\sin\frac{\Omega}{2}(\tau-\sigma)],$$

(A.16)

$$B(\tau,\sigma) = \frac{1}{4\sin(\Omega T/2)}[(x''-x')\sin\frac{\Omega}{2}(T-((\tau+\sigma))\sin\frac{\Omega}{2}(\tau-\sigma)-(y''-y')\cos\frac{\Omega}{2}(T-(\tau-\sigma))\sin\frac{\Omega}{2}(\tau-\sigma)]$$

(A.17)

and $C(\tau,\sigma) = \frac{1}{m\Omega\sin(\Omega T/2)}\sin\frac{\Omega}{2}(T-(\tau-\sigma))\sin\frac{\Omega}{2}(\tau-\sigma).$ (A.18)

However, in the evaluation of the density of states the end point $\vec{r}\,''$ and the initial point $\vec{r}\,'$ are the same. It therefore follows that

$$<e^{i\vec{k}.(\vec{r}(\tau) - \vec{r}(\sigma))}>_o = \exp\left[- i\hbar \vec{k}^2 \frac{\sin[\Omega(T-(\tau-\sigma))/2]\sin[\Omega(\tau-\sigma)/2]}{m\Omega\sin(\Omega T/2)} \right]. \quad (A.19)$$

APPENDIX B

Limiting Case of the DOS when the Magnetic Field Goes to Zero

From (3.42), we have

$$n(E) \quad = \quad n_o\hbar\Omega(2\pi\Gamma^2)^{-1/2}\sum_{n=0}^{\infty}\exp\left[-\frac{(E-E_n)^2}{2\Gamma^2}\right]. \qquad (B.1)$$

However, it is more convenient to investigate the limiting case of (B.1) when the DOS is expressed in dimensionless units as in (3.46), so we write

$$n(E) \quad = \quad n_o x(2\pi\Gamma'^2)^{-1/2}\sum_{n=0}^{\infty}\exp\left[-\frac{(v-(n+1/2)x)^2}{2\Gamma'^2}\right], \qquad (B.2)$$

where $x = \hbar\Omega/E_L$, $v = E/E_L$, $\Gamma'^2 = \frac{\xi_L'}{1+4/x}$, and $E_L = \hbar^2/2mL^2$. When the magnetic field $B \to 0$, corresponding to $x \to 0$, then $\Gamma'^2 \to \frac{\xi_L' x}{4}$ and (B.2) becomes

$$\begin{aligned} n(E) &= n_o x\left(\frac{2}{\pi\xi_L' x}\right)^{1/2}\sum_{n=0}^{\infty}\exp\left[-\frac{2(v-(n+1/2)x)^2}{\xi_L' x}\right] \\ &= n_o x\left(\frac{2}{\pi\xi_L' x}\right)^{1/2}\sum_{n=0}^{\infty}\exp\left[-\frac{2v^2}{\xi_L' x}+\frac{2xv}{\xi_L' x}-\frac{x}{2\xi_L'}+\left(\frac{4v}{\xi_L' x}-\frac{2}{\xi_L'}\right)nx-\frac{2(nx)^2}{\xi_L' x}\right]. \end{aligned} \qquad (B.3)$$

When $x \to 0$, (B.3) may be rewritten in an integral form as

$$n(E) = n_o x\left(\frac{2}{\pi\xi_L' x}\right)^{1/2}\exp\left[-\frac{2v^2}{\xi_L' x}+\frac{2xv}{\xi_L' x}-\frac{x}{2\xi_L'}\right]\int_0^{\infty}(1/x)\exp\left[-\left(\frac{2}{\xi_L'}-\frac{4v}{\xi_L' x}\right)y-\frac{2y^2}{\xi_L' x}\right]dy.$$

(B.4)

Here we take $nx = y$ as a variable of integration. Using the integral formula (43),

$$\int_0^{\infty}\exp\left[-\gamma y-\frac{y^2}{4\beta}\right]dy \quad = \quad \sqrt{\pi\beta}\,\exp[\beta\gamma^2][1-\varnothing(\gamma\sqrt{\beta})], \qquad (B.5)$$

where $$\emptyset(x) = (2/\sqrt{\pi})\int_0^x \exp[-t^2]dt \qquad (B.6)$$

is the probability integral, then we obtain

$$n(E) = (1/2)n_o[1 - \emptyset((2- \frac{4v}{x})\sqrt{\frac{x}{8\xi_L'}})]. \qquad (B.7)$$

When $x \to 0$, (B.8) can be reduced to

$$n(E) = (1/2)n_o[1 - \emptyset(- \frac{2v}{\sqrt{2\xi_L' x}})]. \qquad (B.8)$$

Using the formula (43)

$$1 - \emptyset(\frac{z}{\sqrt{2}}) = \exp[-\frac{z^2}{4}]\sqrt{\frac{2}{\pi}}\, D_{-1}(z), \qquad (B.9)$$

where $D_{-1}(z)$ is the parabolic cylinder function, then (B.8) becomes

$$n(E) = (1/2)n_o \exp[-\frac{v^2}{\xi_L' x}]\sqrt{\frac{2}{\pi}} D_{-1}(-\frac{2v}{\sqrt{\xi_L' x}}). \qquad (B.10)$$

After applying the asymptotic expansion of the parabolic cylinder function $D_p(z)$ when $z \to -\infty$ (43),

$$D_p(z) = -\frac{\sqrt{2\pi}}{\Gamma(-p)} e^{i\pi p} \exp[\frac{z^2}{4}]\, z^{-p-1} (1 + \frac{(p+1)(p+2)}{2z^2} + \ldots\ldots), \qquad (B.11)$$

we finally obtain

$$n(E) = n_o, \qquad (B.12)$$

where $n_o = m/\pi\hbar^2$.

APPENDIX C

The Transformation of Eq. (3.35) into Eq. (3.51) and Eq. (3.54)

For a uniform system, the DOS is related to the diagonal elements of $\bar{K}$ by

$$n(E) = (S/\pi\hbar)\int_{-\infty}^{\infty} dT\, \bar{K}(0,0;T)\, e^{iET/\hbar}$$
$$= (S/\pi\hbar)\,\mathrm{Re}\int_{0}^{\infty} dT\, \bar{K}(0,0;T)\, e^{iET/\hbar} \qquad (C.1)$$

From (3.35), (3.36), (C.1), with $K_o(0,0;T) = (\frac{m}{2\pi i\hbar T})(\frac{\Omega T}{2\sin(\Omega T/2)})$, we obtain

$$n(E) = (S/\pi\hbar)\,\mathrm{Re}\int_{0}^{\infty} dT(\frac{m}{2\pi i\hbar T})(\frac{\Omega T}{2\sin(\Omega T/2)})\exp[\, iET/\hbar - \frac{\xi_L T}{2\hbar^2}\int_0^T dy\,(1 + \frac{8i\sin(\Omega(T-y)/2)\sin(\Omega y/2)}{x\sin(\Omega T/2)})^{-1}]. \qquad (C.2)$$

Using the formula

$$\frac{1}{\sin(\Omega T/2)} = \frac{2i}{e^{i\Omega T/2} - e^{-i\Omega T/2}}$$
$$= 2i\, e^{-i\Omega T/2}\frac{1}{1-e^{-i\Omega T}}$$
$$= 2i\sum_{n=0}^{\infty} e^{-i(n+1/2)\Omega T}, \qquad (C.3)$$

then (C.2) becomes

$$n(E) = n_o(\Omega/\pi)\sum_{n=0}^{\infty}\mathrm{Re}\int_0^{\infty} dT\, e^{i(E-\hbar\Omega(n+1/2))T/\hbar + f(T)}, \qquad (C.4)$$

$$\text{with}\quad f(T) = -\frac{\xi_L T}{2\hbar^2}\int_0^T dy\,(1 + \frac{8i\sin(\Omega(T-y)/2)\sin(\Omega y/2)}{x\sin(\Omega T/2)})^{-1}. \qquad (C.5)$$

Applying the identity of a trigonomic function,

$$\sin A\sin B = (1/2)[\cos(A-B) - \cos(A+B)], \qquad (C.6)$$

and letting $\Omega(T-y)/2 = y'$, we get

$$f(T) = -\frac{\xi_L T}{2\hbar^2}\left(\frac{x\sin(\Omega T/2)}{2i\Omega}\right)\int_0^{\frac{\Omega T}{2}} \frac{dy'}{(x/4i)\sin(\Omega T/2)-\cos(\Omega T/2)+\cos y'}. \quad (C.7)$$

We then define $\xi_L' = \xi_L/E_L^2$, $\nu = E/E_L$ and $x = \hbar\Omega/E_L$, and from (C.4) and (C.7) we get

$$n(E) = (2/\pi)n_0 \sum_{n=0}^{\infty}\int_0^{\infty} dt \, \mathrm{Re}\, e^{2i(\nu/x-(n+1/2)t)+f'(t)}, \quad (C.8)$$

$$\text{where } f'(t) = -\frac{\xi_L' t}{2ix}\sin t \int_0^t \frac{dy}{(x/4i)\sin t-\cos t+\cos y}. \quad (C.9)$$

The integration in (C.9) can be performed analytically. We take $t = \pi N + \theta$ $(-\pi/2 \leq \theta \leq -\pi/2)$ so that (C.8) becomes,

$$n(E) = n_0 \sum_{n=0}^{\infty} K(\nu - x(n+1/2)), \quad (C.10)$$

$$\text{where } K(\nu) = (2/\pi)\int_0^{\infty} dt \, \mathrm{Re}\, e^{2i\nu t/x + f'(t)}, \quad (C.11)$$

$$\text{with } f'(t) = -(\pi N+\theta)\xi'_L \frac{\sin\theta}{2ix\sqrt{a^2-1}}\left[\pi N + 2\tan^{-1}\left(\frac{\sqrt{a-1}}{\sqrt{a+1}}\tan(\theta/2)\right)\right], \quad (C.12)$$

$$\text{and } a = (x/4i)\sin\theta - \cos\theta, \; |a - \sqrt{a^2-1}| < 1. \quad (C.13)$$

APPENDIX D

Computer Programs

All numerical values given in Fig. 15 were evaluated on IBM PS/2 computer using programs written in FORTRAN IV language. This appendix gives the list of main programs and subprograms for solving all values of density of states.

There are three subprograms and one main program used to evaluate the numerical results. The representation of all input variables in the programs are given as follows.

F(THETA)	routine name
F	a function that depends on theta (θ)
X	a ratio of cyclotron energy to localization energy of an electron ($\hbar\Omega/E_L$)
XIL	a fluctuation parameter (ξ_L')
E	an energy. For an input, it is an initial value of E.
G(THETA)	routine name
G	an oscillation function of integration
A	a lower limit of integration
B	an upper limit of integration
XI	a quadrature point of the Legendre-Gauss quadrature
YI	changing the variable of XI
WI	a quadrature coefficient of the Legendre-Gauss quadrature
SUM	The density of states.

The subprograms and main program for case x = 2 are listed respectively as follows.

(subprogram 1)

```
      FUNCTION  F(THETA)
      COMPLEX*16  F,Z,AXT,AXTM1,AXTP1,RAXTP1,RAXTM1,RA2M1
     $,            C1,C2,FAC,ETA
      REAL*8  X,XIL,S,C,A,AS,SS,PI,THETA,RS
      COMMON  /CONSTS/X,XIL,N
C
C     TO  COMPUTE  THE  DISORDER  PART  OF  THE  GREEN'S  FUNCTION
C
      DATA  Z/(0.D0,1.D0)/
      PI=4.D0*DATAN(1.D0)
C
      S=DSIN(THETA)
      C=DCOS(THETA)
      FAC=4.D0*Z/X
      AXT=S/FAC-C
      AXTM1=AXT-1.D0
      RAXTM1=CDSQRT(AXTM1)
C
      IF(DABS(S).LT..001D0)THEN
      SS=S*S
      AS=.5D0*S*(1.D0+.5D0*SS*(1.D0+.25D0*SS*(1.D0+.625D0*SS)))
      A=S*AS
      RS=DSQRT(DABS(S))
      C1=CDSORT(DSIGN(1.D0,S)*(1.D0+AS*FAC)/FAC)
      IF(CDABS(AXT-RS*C1*RAXTM1).GT.1.D0)C1=-C1
      C2=RAXTM1*DSIGN(1.D0,S)*RS/C1/(2.D0-A)
      ETA=C2*C2
      ETA=1.D0-ETA*(1.D0/3.D0-ETA*(.2D0-ETA*(1.D0/7.D0-ETA
     $    *(1.D0/9.D0-ETA*(1.D0/11.D0-ETA/13.D0)))))
      F=DFLOAT(N)*PI+2.D0*C2*ETA
      F=F*RS*DSIGN(1.D0,S)/C1/RAXTM1
C
      ELSE
      AXTP1=AXT+1.D0
      RAXTP1=CDSQRT(AXTP1)
      IF(CDABS(AXT-RAXTM1*RAXTP1).GT.1.D0)RAXTP1=-RAXTP1
      RA2M1=RAXTP1*RAXTM1
      C1=Z*RAXTM1*DTAN(.5D0*THETA)/RAXTP1
      F=DFLOAT(N)*PI-Z*CDLOG((1.D0+C1)/(1.D0-C1))
      F=F*S/RA2M1
C
      ENDIF
C
      F=.5D0*Z*F*XIL*(DFLOAT(N)*PI+THETA)/X
C
      RETURN
      END
```

```
C
C
(subprogram 2)  FUNCTION G(THETA)
                COMPLEX*16 F,Z/(0.D0,1.D0)/
                REAL*8 G,THETA,E,PI,X,XIL
                COMMON /CONSTS/X,XIL,N
                COMMON /ENERGY/E
                PI=4.D0*DATAN(1.D0)
C
                G=1.6D0*2.D0*DREAL(CDEXP(2.D0*Z*E*(DFLOAT(N)*PI+THETA)
               $       /(2.D0*X)+F(THETA)))/PI
C
                RETURN
                END
C
C
C
(subprogram 3)  FUNCTION PQUAD(A,B,FUN)
                REAL*8 PQUAD,A,B,XI,WI,C,D,YI,FUN
                EXTERNAL FUN
C
                PARAMETER(N=48,N2=N*2)
                DIMENSION XI(N),WI(N),YI(N2)
                COMMON /GAUSS/ XI,WI
C
                C=.5D0*(B-A)
                D=.5D0*(B+A)
                DO 2 I=1,N
                YI(I)=C*XI(I)+D
  2             YI(I+N)=-C*XI(I)+D
C
                PQUAD=0.D0
                DO 1 I=1,N
                PQUAD=PQUAD+FUN(YI(I))*WI(I)
  1             PQUAD=PQUAD+FUN(YI(I+N))*WI(I)
                PQUAD=PQUAD*C
C
                RETURN
                END
C
C
C
                PROGRAM MAGNET
(main program)  REAL*8 G,PQUAD,X,W,XIL,E,XI,WI,PI,SUM,PI2,A,H,X0,A0
                EXTERNAL G
                DIMENSION XI(48),WI(48)
                COMMON /CONSTS/X,XIL,N
                COMMON /ENERGY/E
                COMMON /GAUSS/XI,WI
C
                READ(1,100)(XI(I),WI(I),I=1,48)
  100           FORMAT(2D24.17)
C
                PI=4.D0*DATAN(1.D0)
                PI2=.5D0*PI
                X=2.D0
                XIL=81.6D0
```

```
C
      NSTRIP=8
      DO 2 J=-60,150,2
      A0=DFLOAT(J)*.1D0
      E=A0
      SUM=0.D0
 3    N=0
      H=PI2/DFLOAT(NSTRIP)
      DO 5 I=1,NSTRIP
      X0=DFLOAT(I-1)*H
 5    SUM=SUM+PQUAD(X0,X0+H,G)
 1    N=N+1
      H=PI/DFLOAT(NSTRIP)
      A=0.D0
      DO 6 I=1,NSTRIP
      X0=-PI2+DFLOAT(I-1)*H
 6    A=A+PQUAD(X0,X0+H,G)
      SUM=SUM+A
      IF(DABS(A).GT.1.D-6)GOTO 1
      E=E-(2.D0*X)
      IF(E.GT.-4.D0)GOTO 3
      WRITE(6,*)A0,SUM
 2    WRITE(8,*)A0,SUM
      STOP
C
      END
```

Numerical Values of the Density of States

Table 1. Numerical values of the DOS, for $\xi_L' = 1.7$, $\xi_L = 6.8$ meV^2, x =2, and $\hbar\Omega$ =4 meV.

E (meV)	n(E) [x 10^{11} cm^{-2}meV^{-1}]
-3.00000000000000	1.103730526600085E-002
-2.80000000000000	1.667290807870338E-002
-2.60000000000000	2.475290593755529E-002
-2.40000000000000	3.610104618857610E-002
-2.20000000000000	5.173280112788474E-002
-2.00000000000000	7.282603535032016E-002
-1.80000000000000	1.007292652605167E-001
-1.60000000000000	1.367757449200459E-001
-1.40000000000000	1.827548966253618E-001
-1.20000000000000	2.397144722302408E-001
-1.00000000000000	3.089011553151590E-001
-8.000000000000000E-001	3.910405113657652E-001
-6.000000000000001E-001	4.862929973986260E-001
-4.000000000000000E-001	5.940474179103215E-001
-2.000000000000000E-001	7.127989760721113E-001
0.000000000000000E+000	8.400241877321616E-001
2.000000000000000E-001	9.721869916773178E-001
4.000000000000000E-001	1.104775537322833
6.000000000000001E-001	1.232481932838318
8.000000000000000E-001	1.349455528573421
1.00000000000000	1.449660921053147
1.20000000000000	1.527274170167615
1.40000000000000	1.577153687809484
1.60000000000000	1.595244737313785
1.80000000000000	1.578995556206269
2.00000000000000	1.527635162477794
2.20000000000000	1.442392730165475
2.40000000000000	1.463186476673024
2.60000000000000	1.367951662137259
2.80000000000000	1.265003918098024
3.00000000000000	1.163856079886930
3.20000000000000	1.074169611354212
3.40000000000000	1.005253279247412
3.60000000000000	9.651826871718294E-001
3.80000000000000	9.600601483218317E-001
4.00000000000000	9.932683959369687E-001
4.20000000000000	1.065019116695938
4.40000000000000	1.172029204674264
4.60000000000001	1.307581693886799
4.80000000000001	1.461859004249892
5.00000000000000	1.622626895399917
5.20000000000000	1.776138027037673
5.40000000000000	1.903314423883274
5.60000000000001	2.005898981938528
5.80000000000001	2.057694518177667

Table 1. Continue

E (meV)	n(E) [x 10^{11} $cm^{-2}meV^{-1}$]
6.000000000000000	2.055549463698015
6.200000000000000	1.995212270221172
6.400000000000000	1.876530034305632
6.600000000000001	1.887418839023617
6.800000000000001	1.727380737569063
7.000000000000000	1.546954922929946
7.200000000000000	1.361852912780732
7.400000000000000	1.188838895457092
7.600000000000001	1.044180583260895
7.800000000000001	9.422335384222934E-001
8.000000000000000	8.939488987021652E-001
8.200000000000001	9.057720958726564E-001
8.400000000000000	9.787080736655678E-001
8.600000000000000	1.107975149264093
8.800000000000001	1.283122355453256
9.000000000000000	1.488770330714611
9.200000000000001	1.705811185254133
9.400000000000000	1.913169755226524
9.600000000000001	2.089652062655906
9.800000000000001	2.216028235033021
10.000000000000000	2.276794403146606
10.200000000000000	2.261750264188096
10.400000000000000	2.166568449760740
10.600000000000000	2.177599920817422
10.800000000000000	1.994362754020378
11.000000000000000	1.769934167729374
11.200000000000000	1.524233116789200
11.400000000000000	1.279624578719065
11.600000000000000	1.058852091674270
11.800000000000000	8.831128126519924E-001
12.000000000000000	7.699724302532236E-001
12.200000000000000	7.317208606470963E-001
12.400000000000000	7.738600384432854E-001
12.600000000000000	8.943002555272044E-001
12.800000000000000	1.083136740617920
13.000000000000000	1.323276378691218
13.200000000000000	1.591754853226373
13.400000000000000	1.861926430713592
13.600000000000000	2.105911638743398
13.800000000000000	2.297482979822088
14.000000000000000	2.414603427508168
14.200000000000000	2.441741818641417
14.400000000000000	2.370814363677039
14.600000000000000	2.386797053172811
14.800000000000000	2.189131370202244
15.000000000000000	1.932996798632939
15.200000000000000	1.641707624907961
15.400000000000000	1.342066374832405
15.600000000000000	1.061878460500541
15.800000000000000	8.276872739133870E-001
16.000000000000000	6.623042445282812E-001
16.200000000000000	5.828219451261568E-001
16.400000000000000	5.986693044294809E-001
16.600000000000000	7.104052648233571E-001
16.800000000000000	9.091099820553518E-001
17.000000000000000	1.176771931768139

Table 2. Numerical values of the DOS, for $\xi_L{}' = 1.7$, $\xi_L = 6.8\ meV^2$, x=1, and $\hbar\Omega$ =2meV.

E (meV)	n(E) [x 10^{11} $cm^{-2}meV^{-1}$]
-3.00000000000000	1.948309466869988E-002
-2.80000000000000	3.113416052572697E-002
-2.60000000000000	4.819628402457191E-002
-2.40000000000000	7.257067874134253E-002
-2.20000000000000	1.058358235204559E-001
-2.00000000000000	1.496748929273378E-001
-1.80000000000000	2.051753968506500E-001
-1.60000000000000	2.725120915876190E-001
-1.40000000000000	3.505413582958968E-001
-1.20000000000000	4.364555769829206E-001
-1.00000000000000	5.256709567818018E-001
-8.000000000000000E-001	6.119891190646465E-001
-6.000000000000001E-001	6.880502320908415E-001
-4.000000000000000E-001	8.191725235546037E-001
-2.000000000000000E-001	8.868481601107944E-001
0.000000000000000E+000	9.368809782378157E-001
2.000000000000000E-001	9.695536010713313E-001
4.000000000000000E-001	9.881927184575626E-001
6.000000000000001E-001	9.986505910328691E-001
8.000000000000000E-001	1.007998557005378
1.00000000000000	1.022753962859851
1.20000000000000	1.046907172747267
1.40000000000000	1.128368365956671
1.60000000000000	1.190603261183189
1.80000000000000	1.256827171924416
2.00000000000000	1.317488312563045
2.20000000000000	1.362861642558059
2.40000000000000	1.386091046104706
2.60000000000000	1.385394399085648
2.80000000000000	1.364850535140668
3.00000000000000	1.333530725005488
3.20000000000000	1.302864784435095
3.40000000000000	1.330929196122029
3.60000000000000	1.351497976269846
3.80000000000000	1.394588497578683
4.00000000000000	1.452496329496900
4.20000000000000	1.511439720230267
4.40000000000000	1.555863475159477
4.60000000000001	1.573274531216908
4.80000000000001	1.558307008519079
5.00000000000000	1.514822297526237
5.20000000000000	1.454924262181539
5.40000000000000	1.394769336950781
5.60000000000001	1.422933535555790
5.80000000000001	1.435090203045406
6.00000000000000	1.479266476253025
6.20000000000000	1.543674171262733
6.40000000000000	1.608602240766226
6.60000000000001	1.652597274224592
6.80000000000001	1.659467487299818
7.00000000000000	1.624009578752387
7.20000000000000	1.554065129479352
7.40000000000000	1.467694094166541

Table 2. Continue

E (meV)	n(E) [x 10^{11} $cm^{-2}meV^{-1}$]
7.600000000000001	1.461238379354076
7.800000000000001	1.440754807692253
8.000000000000000	1.464317884401050
8.200000000000001	1.526052045931910
8.400000000000000	1.606052718319415
8.600000000000000	1.676617993889391
8.800000000000001	1.711387160599861
9.000000000000000	1.694615645547878
9.200000000000001	1.626904097876298
9.400000000000000	1.524827059275914
9.600000000000001	1.489855616233561
9.800000000000001	1.437386563555407
10.000000000000000	1.436347329925598
10.200000000000000	1.489047528648458
10.400000000000000	1.578273530217876
10.600000000000000	1.672559063201460
10.800000000000000	1.736640908482802
11.000000000000000	1.743926392545727
11.200000000000000	1.686096917996436
11.400000000000000	1.575851007603131
11.600000000000000	1.517551500250650
11.800000000000000	1.434614153605268
12.000000000000000	1.406818742770685
12.200000000000000	1.445905328232917
12.400000000000000	1.539341274738266
12.600000000000000	1.653899997683762
12.800000000000000	1.746738988169670
13.000000000000000	1.780639988494503
13.200000000000000	1.737473050875026
13.400000000000000	1.624428982349492
13.600000000000000	1.547003439893551
13.800000000000000	1.435368175404846
14.000000000000000	1.379803826157886
14.200000000000000	1.402114512348077
14.400000000000000	1.495791506929227
14.600000000000000	1.627391092949708
14.800000000000000	1.747681733071909
15.000000000000000	1.809259065209310
15.200000000000000	1.783756431068880
15.400000000000000	1.671763608881084
15.600000000000000	1.578569541689467
15.800000000000000	1.440002602998774
16.000000000000000	1.356360984956241
16.200000000000000	1.359784086318455
16.400000000000000	1.450677604742492
16.600000000000000	1.596542539361983
16.800000000000000	1.742768602925785
17.000000000000000	1.832273136818748

(Note that in Fig. 15a and Fig. 15b, for our numerical results (dotted line), we choose the energy origin of the electorns at 1.07 eV.)

VITA

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