CHAPTER III

BASIC KNOWLEDGES

In order to study the phenomena of fluid flow past the parallel inclined flatplates in a square duct , some backgrounds of basic knowledges were required.They could be devided to three parts of the concerning theories. Beginning from the fluid flow past immersed bodies which are the flat plates for this study. Then the boundary-layer would be separated and formed the wake in downstream direction. After ultimately developing leads to the fully developed flow.

3.1 FLOW PAST IMMERSED BODIES

When a body immersed in a streaming flow , the conclusion that the resistance with which a fluid opposes the motion of a body immersed in it in virtue of its inertia must be proportional to the area of section of the body at right angles to the direction of motion (A) and also proportional to the density of the fluid (ρ) and to the square of the velocity (V) , dates back to Newton. This result may be explained by the following very simple argument. Each second the body must move a mass of fluid $M = \rho AV$ out of its ways , and in doing so imparts a velocity , which may be taken as proportional to its own velocity , to each element of the fluid. The resistance is equal to the momentum imparted to the fluid per second and is therefore proportional to $MV = \rho AV^2$ or $R = \rho AV^2 \times$ constant.

In Newton's theory the laws of collision of elastic bodies are applied to the resistance of a fluid. The detailed results , however , have proved untenable. (Prandtl , 1952)

The Newtonian conception of fluid resistance has been replaced by thehydrodynamical conception , according to which the resistance consists of the pressure differences and frictional stresses arising from the fluid flowing round the body In this hydrodynamical principles , the resistance is proportional to the dynamic pressure corresponding to the velocity (V) as proportional to (1 / 2) ρV^2 and the area exposed to it

$$R \propto (1/2)\rho V^2 \times A \quad \text{or}$$

$$R = (1/2)\rho A V^2 \times \text{constant} \qquad (3.1)$$

where this constant is called *resistance coefficient* which is dimensionless.

The resistance or the resultanant force on the body is composed of two components , one parallel to the direction of flow called *drag force* (D) and one perpendicular to the direction of flow called *lift force* (L) see figure 3.1.

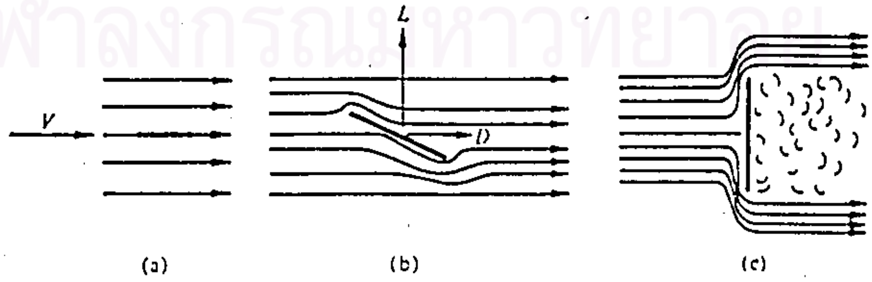


Figure 3.1 Flat plates.

Similarity of equation (3.1)

$$D = (1/2)\rho AV^2 \times C_D \qquad (3.2)$$

and

$$L = (1/2)\rho AV^2 \times C_L \qquad (3.3)$$

where C_D is the *drag coefficient.*

C_L is the *lift coefficient.*

Furthermore , both C_D and C_L on a given body are solely functions of the Reynolds number. (Granet , 1971)

Figure 3.2 illustrates the drag coefficient and the lift coefficient on a flat plate at any other inclination α. (Shames , 1982) These curves show no dependence on Reynolds number and that at an angle of zero degrees when a flat plate is parallel to the flow the drag coefficient is given as a single value. Since the condition of the test are not specifically given and the Reynolds number is also not given , this curve should be used for design with caution. However , the qualitative trend shows an increase in both drag and lift coefficients as the angle a flat plate makes with the stream increases , at some finite angle , approximately 15 degrees , C_L starts to decrease , but C_D continues to increase.

The purpose of this study will focus on the downstream direction behind the parallel inclined flat plates which are installed in a square duct so the drag force (D) only will be considered.

Total drag force on a body consists of the *skin-friction drag* and the *pressure drag.*

Skin-friction drag is due to viscous shear forces produced at the body surface predominantly in those regions to which the boundary layer is attached. On any element of area dA ,

Component of shear force in flow direction = $\tau_w.\cos \alpha.dA$

where τ_w is the local viscous shear stress at the body surface.

α is the inclination of the area dA to the flow direction.

Integrating over the whole surface :

Skin-friction drag (D_F) = $\oint \tau_w.\cos \alpha.dA$

For surface normal to the direction of flow $\alpha = 90^\circ$ and the skin-friction drag will therefore be zero.

Pressure drag , sometimes called *form drag* , is due to the unbalanced pressure which exist between the relatively high pressures on the upstream body surfaces and the lower pressures on the downstream surfaces. On any area dA ,

Component of pressure force in flow direction = $P_L.\sin \alpha.dA$

where P_L is the local static pressure acting on the body surface.

α is the inclination of the area dA to the flow direction.

Integrating over the whole surface :

Pressure drag (D_P) = $\oint P_L.\sin \alpha.dA$

Pressure acting on a surface in line with the flow direction for which $\alpha = 0^\circ$ will not contribute to the pressure drag.

The total drag (D_T) on a body is the sum of the skin-friction drag and the pressure drag.

$$D_T = D_F + D_P$$

However, seldom are both these effects of appreciable magnitude simultaneously. For objects which exhibit no lift, profile drag is synonymous with total drag. The following tabulation will illustrate. (Giles , 1976)

Object	**Skin-friction drag**		**Pressure drag**		**Total drag**
1. Spheres	negligible	+	Pressure drag	=	total drag
2. Cylinders (axis perpendicular to velocity)	negligible	+	Pressure drag	=	total drag
3. Disks and thin plates (perpendicular to velocity)	zero	+	Pressure drag	=	total drag
4. Thin plates (parallel to velocity)	Skin-friction drag	+	negligible to zero	=	total drag
5. Well-streamlined objects	Skin-friction drag	+	small to negligible	=	total drag

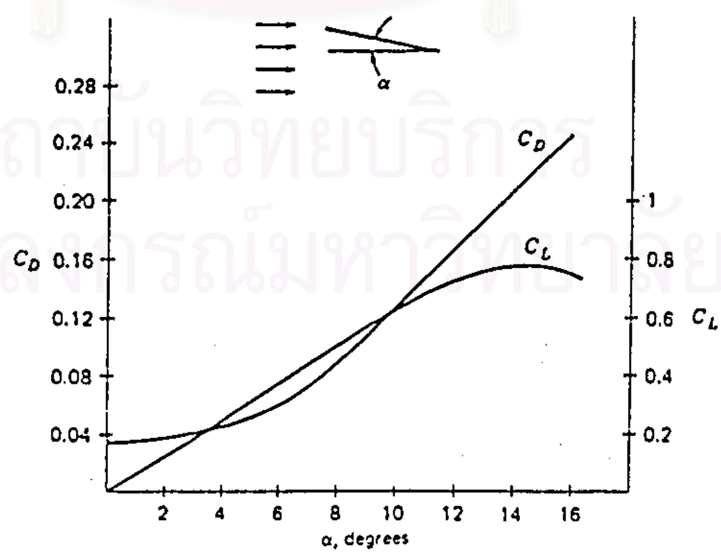


Figure 3.2 Coefficients of drag and lift for a flat plate at varying inclination α.

3.2 BOUNDARY-LAYER SEPARATION AND WAKE FORMATION

When a viscous fluid flows over a solid surface a boundary-layer is formed in which the fluid velocity changes from zero at the solid surface to the free stream velocity at the boundary-layer edge. When a fluid passes over the convex surface of a solid , such as a cylinder , the boundary-layer will tend to separate from this surface just aft of the point of maximum thickness where the surface curvature requires that the fluid should decelerate. As shown in figure 3.3 the fluid velocity in the boundary-layer is increasing in the region upstream of this maximum thickness point and then decreasing downstream of it.

Where the flow is accelerating the boundary-layer thickness reduces slightly but starts increasing rapidly when the flow begins to decelerate. While the fluid outside the boundary-layer is accelerating the pressure gradient *dp/dx* is said to be negative, or *favourable* , and can lead to a reduction in the boundary-layer thickness.

At , or about , the point of maximum thickness however the streamwise pressure gradient *dp/dx* reduces to zero and downstream of this point the pressure gradient becomes positive , or *adverse.* Adverse pressure gradients oppose the flow , reducing the velocities in the boundary-layer and increasing its thickness.

These two effects combine to reduce the velocity gradient *du/dy* at the wall. At the separation point *du/dy* is zero. Thereafter a reversed flow region occurs producing vortices and subsequent wake formation.

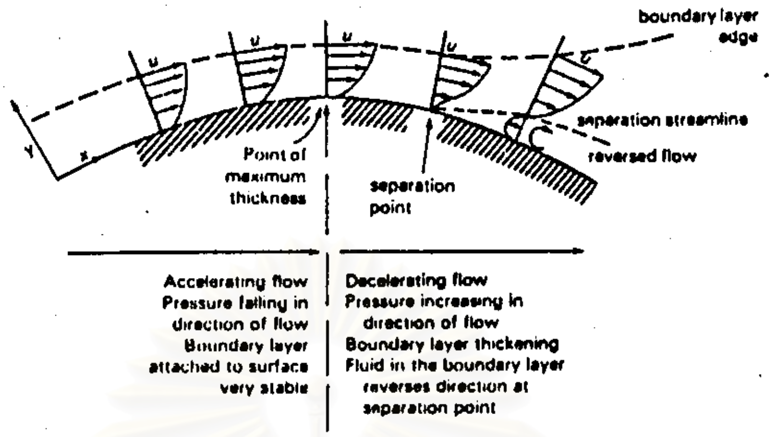


Figure 3.3 Boundary-layer separation.

The near wake , immediately downstream of the solid body , is in general a region where the static pressure and/or the flow velocity are lower than in the undisturbed stream. Near the solid body the static pressure in the wake is close to that at the separation point. This low pressure leads to a pressure difference between the high pressures over the upstream surfaces and the low pressures on the downstream surfaces in the near wake thus causing a pressure drag force on the body in the direction of fluid flow.

The structure of the wake depends on the Reynolds number of the flow and the detailed shape of the body. Immediately aft of the body , following boundary-layer separation , strong vortices will be formed which may detach themselves at regular intervals of time forming the Von Karmann *vortex street.*

In the far wake further downstream the static pressure rises to the undisturbed value but the fluid velocity will remain below the undisturbed value for a great distance until the effects of viscosity have obliterated the wake. (Douglas , 1986)

Figure 3.4 shows some of the characteristics of a wake downstream of a prismatic body.

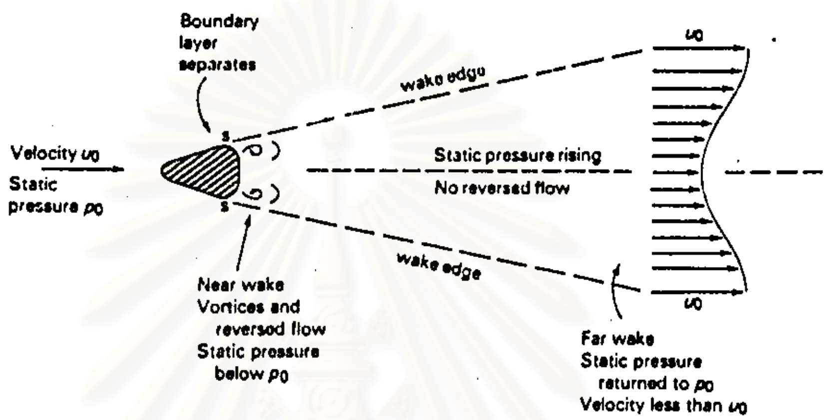


Figure 3.4 Wake formation.

In case of a thin flat plate aligned in the direction of the flow , has only skin-friction drag. There is no adverse pressure gradient and consequently no boundary-layer separation to cause pressure drag. However , when a thin flat plat is aligned perpendicular to a streaming flow , as shown in figure 3.5 , there is zero skin-friction drag and very large pressure drag. There is a shear stress acting on the front surface of the plate , but its direction is perpendicular to that of the streaming flow and hence has no component in the drag direction. The plate presents a very bluff profile to the flow , causing the streamlines to be highly curved.

Point *a* is a stagnation point. From the stagnation point to the top and bottom edges of the plate , point *b* and *c* , the flow accelerates and the pressure falls. The pressure gradient is favourable , and there is no boundary-layer separation. However , if the streamline pattern is to be the same on the back surface as the front surface , the streamlines at the edge must curve very sharply with an accompanying large increase in pressure. The adverse pressure gradient is so large that regardless of whether the boundary-layer flow is laminar or turbulent , it separates right at the edges , resulting in a large low-pressure wake behind the plate. Since the location of boundary-layer separation is unaffected by whether the flow is laminar or turbulent. The drag coefficient is independent of Reynolds number. Furthermore , surface roughness is unimportant , since there is no skin-friction drag and inducing turbulence does not move the separation point.

A fixed boundary-layer separation point is characteristic of bodies with sharp corners. Except for skin-friction contributions , the drag coefficient is nearly independent of the Reynolds number. (Mironer , 1979)

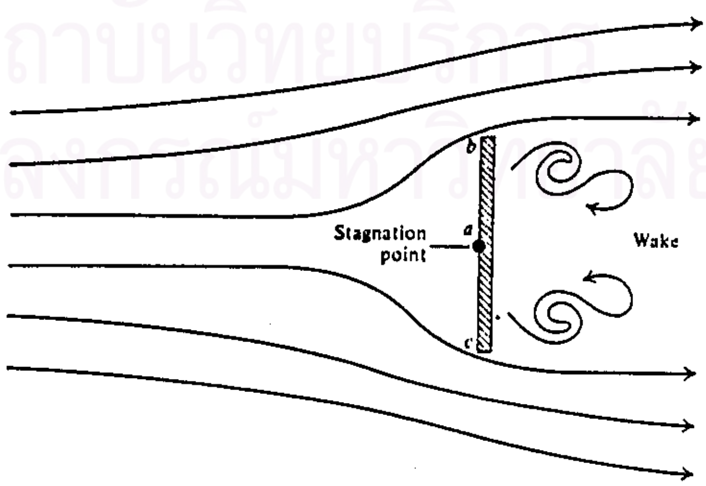


Figure 3.5 Flat plate normal to streaming flow.

3.3 DEVELOPING AND FULLY DEVELOPED FLOW

Flows completely bounded by solid surfaces are called internal flows. Thus internal flows include flows through pipes , ducts , nozzles , diffusers , sudden contractions and expansions , valves and fittings. In many practical cases the flow in pipes is not fully developed , but rather in the process of developing and hence eventually reaching the fully developed state.

One typical example of such a developing flow is the flow in the entrance region of a pipe. At the entrance section of the pipe the flow is assumed to have constant velocity over the whole cross section ; however , fluid in contact with the pipe wall has zero velocity because of the no-slip condition. The velocity gradient near the pipe wall is associated with a retarding shear stress on the fluid. A boundary-layer of slower-moving fluid is established at the wall. The boundary-layer grows in thickness as the fluid proceeds downstream.

The fluid outside the boundary-layer , in the central region of the pipe , is called the *core flow*. The fluid velocity in the boundary-layer is reduced , which means that the core fluid must accelerate as it moves downstream to maintain constant mass flow at all cross section.

Sufficiently far from the pipe entrance , the boundary-layer developing on the pipe wall reaches the pipe centerline and the flow becomes entirely viscous. The velocity profile shape changes slightly after the inviscid core disappears. When the profile shape no longer changes with increasing distance along downstream direction , the flow is *fully developed*. The distance downstream from the entrance to the location at which fully developed flow begins is called the *entrance length* , L_e , or *development length*.

The actual shape of the fully developed velocity profile depends on whether the flow is laminar or turbulent. In figure 3.6 the profiles are shown qualitatively for laminar and turbulent flow.

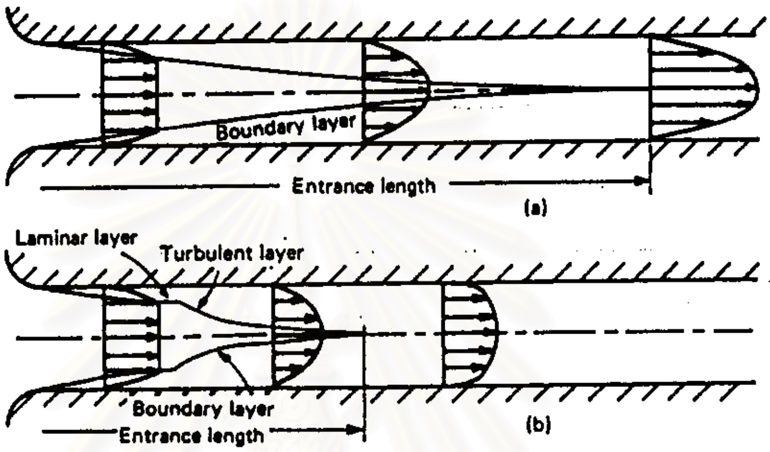


Figure 3.6 Growth of boundary-layer in a pipe (not to scale).
(a) Laminar flow. , (b) Turbulent flow.

The distinction between laminar and turbulent flow in a pipe applies only to the fully developed flow condition. In the development zone , the core flow is irrotational and thus neither laminar nor turbulent. The flow in the boundary-layer is laminar or turbulent. If the fully developed pipe flow is laminar ($Re < 2,300$) , the boundary-layer is laminar , but if the fully developed pipe flow is turbulent ($Re \geq 4,000$) , the boundary-layer is laminar near the pipe entrance , undergoes transition and is turbulent as it approaches the fully developed condition.

By dimensional analysis

$$\frac{L_e}{D} = f(Re)$$

where D is pipe diameter.

The function f is different for laminar and turbulent flow. Analytic and experimental investigations have demonstrated the validity of the following correlations. (Gerhart ,Gross and Hochstein , 1992)

Laminar flow $$\frac{L_e}{D} \approx 0.06\,Re \qquad (3.4)$$

and

Turbulent flow $$\frac{L_e}{D} \approx 4.4\,(Re)^{1/6} \qquad (3.5)$$

The longest practical entrance length corresponds to laminar flow with Re $\approx$ 2,300.

$$\left(\frac{L_e}{D}\right)_{max} \approx 140$$

In many engineering pipe flows , Re is between 10^4 and 10^5. For this case , the flow is turbulent and the entrance length for turbulent pipe flow is found experimentally to be considerably less than those required for laminar flow , due to the large amount of mixing caused by the turbulence. It is found experimentally that the wall shear stress and axial pressure gradient attain their fully developed mean values in entrance length in smooth pipe of less than 25 diameters. However , the detailed structure of the turbulence requires considerably longer entrance length to become fully developed. (Mironer , 1979) Thus , typical engineering flow

$$\frac{L_e}{D} \approx 25$$

With many pipes in engineering applications being hundreds or thousands of diameters long , the flow is fully developed over most of their lengths.

Many flows of technical interest take place in pipes or ducts of noncircular cross section. In many air- and gas-handling systems , such as power plant air and flue gas duct , commonly have rectangular ducts. The empirical correlations for pipe flow also may be used for computations involving noncircular ducts , provided their cross sections are not too exaggerated. (Fox and Mcdonald , 1994)

Flows through noncircular cross sections may be very complex , with regions of recirculating flow. For turbulent flows , it has been found that introducing an equivalent diameter for the noncircular cross section and then simply using it as the characteristic dimension in the Reynolds number. This equivalent diameter , called the *hydraulic diameter* , D_h , is defined as

$$D_h = \frac{4 \times \text{cross-sectional area of flow}}{\text{perimeter wetted by flow}} \qquad (3.6)$$

Expressions for calculating the hydraulic diameter of a few common shapes are given in table 3.1. (Gerhart , Gross and Hochstein , 1992)

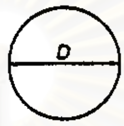
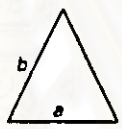
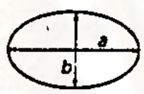
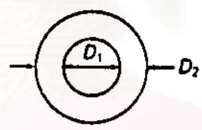
Shape		Hydraulic Diameter $D_h = \frac{4A}{P}$
Circle		D
Rectangle		$\frac{2ab}{(a+b)}$
Isosceles triangle		$\frac{\sqrt{4a^2b^2 - a^4}}{a + 2b}$
Ellipse		$\approx \frac{2\sqrt{2}ab}{\sqrt{a^2 + b^2}}$
Concentric annulus		$D_2 - D_1$

Table 3.1 Hydraulic diameter of common geometric shapes.