CHAPTER IV
RESULTS AND DISCUSSION

To deal with HEN synthesis and retrofit problem systematically, a superstructure model develop Yee and Grossmann (1990), Synheat model, is often used. In this work, the modified Synheat model is developed for both grass-root and retrofit design. The model is MINLP often involving highly non-convex terms; therefore, a new systematic initialization strategy is implied to provide good initial values for simultaneous synthesis and retrofit of HENs. The remaining in chapter IV is structured as follows. In section 4.2, the modified Synheat model (model A1 and model A2) with non-isothermal mixing assumption will be presented with allowing the series of matches per branch flow and also stage-bypass. The model formulations and effective initialization strategy for small case of both synthesis and retrofit design will be presented in section 4.3 and 4.4, respectively. Finally, the several examples are solved to demonstrate the significant efficiency of our model in section 4.5.Retrofitting the existing HEN of Crude Distillation Units (CDU) which is one of the most challenging design is also presented in the section 4.5. The objective is to reduce the recent utility consumption and maximize Net Present Value (NPV) with using the existing exchanger. To provide good initial values for simultaneous HENs retrofit, a two-step strategy is applied. The effective model formulations and effective initialization strategy are applied to CDU case-study.

4.1 The Modified Synheat Model

To represent a HEN, a simple stage-wise model first proposed by Yee and Grossmann in 1990 is used due to the low level of non-linearity of the MINLP optimization model. It allows streams entering each stage be able to split up in each stage interval and then are mixed at the end of each interval. However, it does not account for non-isothermal mixing in order to simplify the model with linear heat balances around the exchangers as well as linear heat mixing equations. Parenthetically, the model assuming isothermal mixing causes the restriction of the area trade-offs between the exchangers and the overestimation of the area cost.

4.1.1 Model A1: The Modified Synheat Model with Non-isothermal Mixing

In this work the modified model from the stage-wise model (1990); model A1, is proposed to do HEN design with eliminating the limitation of isothermal mixing assumption. It introduces the non-linear and non-convex constraints accounting for the heat capacity flowrate of each splitting stream and the outlet temperature of each exchanger as presented in Figure 4.1. The objective function minimizing TAC is a concave cost functions from the power of exponential term of exchanger in area costs from work of Björk and Westerlund (2002).

The illustration of the modified model with new continuous temperature and flow variables are respectively presented in Figure 4.1.The usual assumptions are set up as follows.

- Process stream flows and heat capacities are assumed.
- Both heating and cooling utilities are only used to adjust the temperature at the end of superstructure to the target temperature.
- The inlet and outlet temperatures of both utilities are known, but their flows are unknown.
- Overall heat transfer coefficients are calculated from convective heat transfer coefficients which are assumed constant for each stream, and independent of flow rates.
- Heat exchangers are modeled as hot-to-hot matches (1-1 units). Hot-to-hot and cold-to-cold exchangers are not allowed.

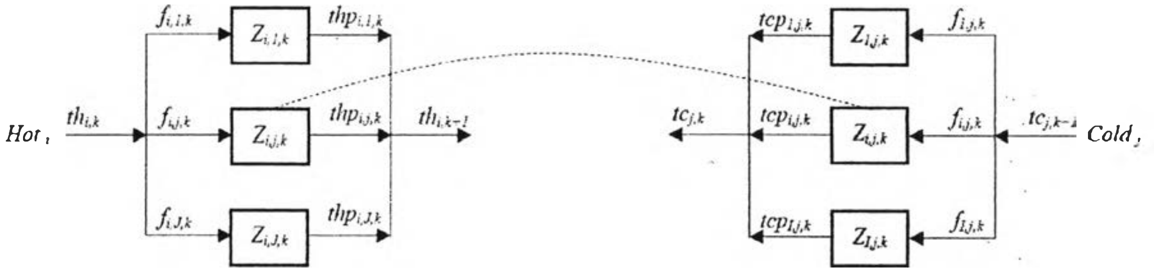

Figure 4.1 The modified Synheat model (model A1) with non-isothermal mixing.

- Single hot and one cold utilities are used for simultaneous approaches for HENS design.

4.1.2 Model A2: The Modified and Extended Synheat Model

In addition to the modified model A1, model A2 allows any branch of splitting stream contains more than one exchanger as depicted in Figure 2. Each of hot or cold streams entering stage K is split into number of sub-streams where each sub-stream is divided into sub-stages, SKs. At each sub-stage SK, any hot splitting stream exchanges heat with any cold splitting stream through several exchanger matches or does not exchange heat a bypass stage. Thus the model requires extra set of temperature variables in each sub-stage for a hot stream at the hot end of exchanger, or for a cold stream at the cold end of exchanger. At last sub-stage; SK of each stage; K, splitting streams merge to form the main stream. To overcome the area trade-off restriction caused by the assumption of isothermal mixing, the outlet temperatures of each splitting stream can be varied at last sub-stage. Finally, the target temperature of each main stream is achieved by using utility at last stage K.

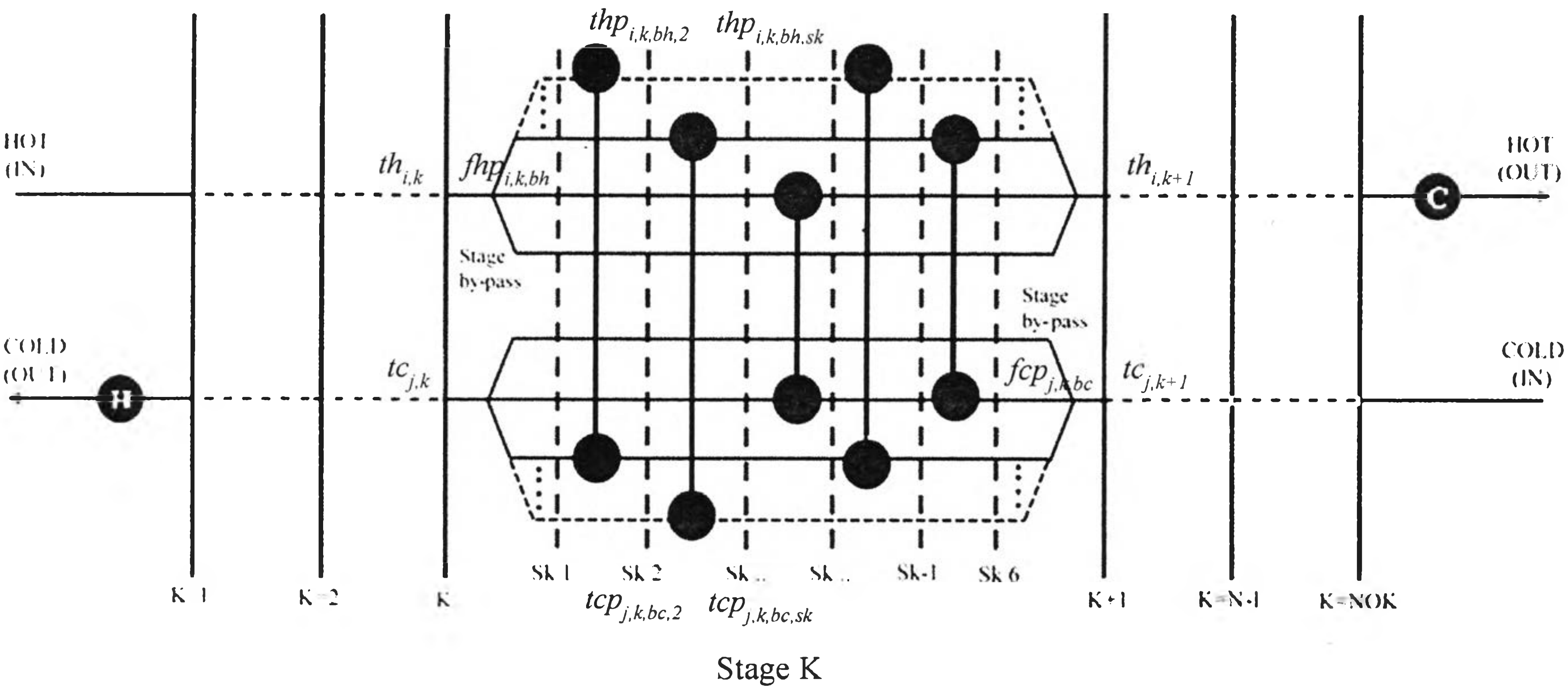

Figure 4.2 The modified and extended Synheat model (model A2) with non-isothermal mixing with allowing several matches per branch of splitting stream.

4.2 HEN Synthesis

HEN is designed by applying mathematical programming formulated by objective function with equality and inequality constraints.

Indices and Index Sets

Let i represent hot streams and j represent cold streams.

H	$\{ i \mid i$ is hot process stream$\}$
C	$\{ j \mid j$ is cold process stream$\}$
K	Stage or temperature location; stages are numbered from 1 to ST with descending temperature; for stage k there are two temperature locations, $k$at inlet and $k+1$ at outlet
SK	Sub-stage within each main stage k; stages are numbered from 1 to $STSK$with descending temperature as K
BH	Branch of hot splitting stream
BC	Branch of cold splitting stream
HU	$\{ hu \mid hu$ is heatingutility$\}$
CU	$\{ cu \mid cu$ is cooling utility$\}$

Model parameters

$TH_{i,IN}$	Supply temperature of hot process stream i
$TH_{i,OUT}$	Target temperature of hot process stream i
$TC_{j,IN}$	Supply temperature of cold process stream j
$TC_{j,OUT}$	Target temperature of cold process stream j
$TCUcu_{,IN}$	Inlet temperature of cooling utilitycu
$TCU_{cu,OUT}$	Outlet temperature of cooling utilitycu
$THU_{hu,IN}$	Inlet temperature of heating utility hu
$THU_{hu,OUT}$	Outlet temperature of heating utility hu
FH_i	Heat capacity of hot process stream i
FC_i	Heat capacity of cold process stream j
$U_{i,j}$	Overall heat transfer coefficient of heat exchanger of process stream $i-j$

$UCU_{i,cu}$	Overall heat transfer coefficient of cooling utilitycu and hot process stream i
$UHU_{j,hu}$	Overall heat transfer coefficient of heating utilityhu and cold process stream j
CCU_i	Cost of cooling utilitycu
CHU_j	Cost of heating utility hu
$CFHX_{i,j}$	Fixed charges for exchanger $i-j$
$CFCU_{i,cu}$	Fixed charges for cooling utilitycu
$CFHU_{j,hu}$	Fixed charges for heating utility hu
$ACHX_{i,j}$	Area cost coefficient of heat exchanger $i-j$
$ACCU_{i,cu}$	Area cost coefficient of cooling utilitycu
$ACHU_{j,hu}$	Area cost coefficient of heating utility hu
$\Omega_{i,j}$	Upper bound of heat content for heat exchanger $\Omega_{i,j} = \min\{ FH_i(TH_{i,IN} - TH_{i,OUT}), FC_j(TC_{j,OUT} - TC_{j,IN}) \}$
Ω_i	Upper bound of heat content for cooling utility and hot process stream i $\Omega_i = FH_i(TH_{i,IN} - TH_{i,OUT})$
Ω_j	Upper bound of heat content for heating utility and cold process stream j $\Omega_j = FC_j(TC_{j,OUT} - TC_{j,IN})$
Γ	Upper bound for temperature difference $\Gamma = \max\{ 0, (TC_{j,IN} - TH_{i,IN}), (TC_{j,IN} - TH_{i,OUT}),$ $(TC_{j,OUT} - TH_{i,IN}), (TC_{j,OUT} - TH_{i,OUT}) \}$
$dtcup_{i,cu}$	Temperature approach in cold end of cooling utility cu $dtcup_{i,cu} = TH_{i,OUT} - TCU_{cu,IN}$
$dthup_{j,hu}$	Temperature approach in hot end of heating utility hu $dthup_{j,hu} = THU_{hu,IN} - TC_{j,OUT}$
$EMAT$	Exchanger minimum-approach temperature
β	Exponent for area costs of heat exchanger $i-j$, hot and cold utility

ST	Number of stage (often chosen as maximum between number of hot and cold streams)
$STSK$	Number of sub-stage

Model (Free) variable

TAC	Total cost associated in heat exchanger network

Model (positive) variables

$ai_{i,j,k}$	Area for heat exchanger $i-j$ in stage k raised to the power of β (for model A1)
$a_{i,j,k,bh,bc,sk}$	Area for heat exchanger $i-j$ in stage k raised to the power of β (for model A2)
$acui_{i,cu}$	Area for cooling utilitycu raised to the power of β (for model A1)
$acu_{i,cu}$	Area for cooling utility cu raised to the power of β (for model A2)
$ahui_{j,hu}$	Area for heating utility hu raised to the power of β (for model A1)
$ahu_{j,hu}$	Area for heating utilityhu raised to the power of β (for model A2)
$th_{i,k}$	Temperature of hot process stream i at "hot end" of stage k
$tc_{j,k}$	Temperature of cold process stream j at "hot end" of stage k
$qi_{i,j,k}$	Heat exchanged between hot process stream $i$and cold process stream j in stage k (for model A1)
$q_{i,j,k,bh,bc,sk}$	Heat exchanged between branch BH of hot process stream $i$and branch BC of cold process stream j in sub-stage sk in stage k (for model A2)
$qcui_{i,cu}$	Heat exchanged between cold utility$cu$and hot process stream i (for model A1)
$qcu_{i,cu}$	Heat exchanged between cold utilitycu and hot process stream i (for model A2)
$qhui_{j,hu}$	Heat exchanged between hot utility hu and cold process stream j (for model A1)
$qhu_{j,hu}$	Heat exchanged between hot utility hu and cold process stream j (for model A2)
$f_{i,j,k}$	Fractional flow of hot process stream $i$exchanged with coldprocess stream $j$in stage k (for model A1)

$g_{i,j,k}$	Fractional flow of cold process stream $j$exchanged with hotprocess stream $i$in stage k (for model A1)
$fhp_{i,k,bh}$	Fractional flow of branch bh of hot process stream i stage k (for model A2)
$fcp_{j,k,\mathrm{bc}}$	Fractional flow of branch $bc$of cold process stream $j$in stage k (for model A2)
$thpi_{i,j,k}$	Temperature of fractional hot process stream i at "cold end" of heat exchanger at the stage k(for model A1)
$tcpi_{i,j,k}$	Temperature of fractional cold process stream $j$at "hot end" of heat exchanger at the stage k (for model A1)
$thp_{i,k,bh,sk}$	Temperature of fractional hot process stream i at "cold end" of heat exchanger at the stage k(for model A2)
$tcp_{j,k,bc,sk}$	Temperature of fractional cold process stream $j$at "hot end" of heat exchanger at the sub-stage sk in stage k (for model A2)
$dthi_{i,j,k}$	Temperature approach for match $i-j$ at hot end of heat exchanger (for model A1)
$dtci_{i,j,k}$	Temperature approach for match $i-j$ at cold end of heat exchanger (for model A1)
$dtcui_{i,cu}$	Temperature approach for match between cooling utility cu and hot process stream $i$before heat exchanger (for model A1)
$dthui_{j,hu}$	Temperature approach for match of heating utility huandcoldprocess stream $j$after heat exchanger (for model A1)
$dth_{i,j,k,bh,bc,sk}$	Temperature approach for match $i-j$at hot end of heat exchanger in sub-stage sk and stage k (for model A2)
$dtc_{i,j,k,bh,bc,sk}$	Temperature approach for match $i-j$at cold end of heat exchanger in sub-stage sk and stage k (for model A2)
$dtcu_{i,cu}$	Temperature approach for match between cooling utility cu and hot process stream i before heat exchanger (for model A2)
$dthu_{j,hu}$	Temperature approach for match of heating utility hu andcoldprocess stream j after heat exchanger (for model A2)

Model (binary) variables

In order to define the existence or non-existence of a match in a HEN it is necessary to use binary variables. Due to the binary variable and nonlinearities, the model is of the MINLP type.

$zi_{i,j,k}$	Existence of an exchanger for match $i - j$in stage k (for model A1)
$zcui_{i,cu}$	Existence of an exchanger for match between cooling utilitycu and hot process stream i (for model A1)
$zhui_{j,hu}$	Existence of an exchanger for match between heating utility hu and cold process stream j (for model A1)
$z_{i,j,k,bh,bc,sk}$	Existence of an exchanger for match $i - j$ in sub-stage sk and stage k (for model A2)
$zcu_{i,cu}$	Existence of an exchanger for match between cooling utility cu and hot process stream i (for model A2)
$zhu_{j,hu}$	Existence of an exchanger for match between heating utility hu and cold process stream j (for model A2)

4.2.1 Model A1 Formulation for HEN Synthesis

The HENS objective of minimum TAC is given by

$$\begin{aligned} Minimize\ (TAC) = \ & \{ [\sum_{i,j,k}(zi_{i,j,k}CFHX_{i,j}) + \sum_{i,cu}(zcui_{i,cu}CFCU_{i,cu}) \\ & + \sum_{j,hu}(zhui_{j,hu}CFHU_{j,hu})] + [\sum_{i,j,k}(ai_{i,j,k}ACHX_{i,j}) \\ & + \sum_{i,cu}(acui_i, cuACCU_{i,cu}) + \sum_{j,hu}(ahui_{j,hu}ACHU_{j,hu})\ ] \\ & +[\sum_{i,cu}(qcui_{i,cu}CCU_{i,cu}) + \sum_{j,hu}(qhui_{j,hu}CHU_{j,hu})\ ]\ \}; \\ & i \in I, j \in J, k \in ST,\ cu \in CU, hu \in HU \end{aligned} \tag{4.1}$$

- Exchange area equations

$$\frac{qi_{i,j,k}}{ai_{i,j,k}{}^{\frac{1}{\beta}}} - \frac{2}{3}U_{i,j}dthi_{i,j,k}{}^{1/2}dtci_{i,j,k}{}^{1/2}$$

$$- \frac{1}{6}U_{i,j}dthi_{i,j,k} - \frac{1}{6}U_{i,j}dtci_{i,j,k} \qquad \leq \qquad 0\ ;$$

$i \in I, j \in J, k \in ST$ (4.2)

$$\frac{qcui_{i,cu}}{acui_{i,cu}{}^{\frac{1}{\beta}}} - \frac{2}{3}UCU_{i,cu}dtcui_{i,cu}{}^{1/2}dtcup_{i,cu}{}^{1/2}$$

$$- \frac{1}{6}UCU_{i,cu}dtcui_{i,cu} - \frac{1}{6}\ UCU_{i,cu}dtcup_{i,cu} \qquad \leq \qquad 0\ ;$$

$i \in I, cu \in CU$ (4.3)

$$\frac{qhui_{j,hu}}{ahui_{j,hu}{}^{\frac{1}{\beta}}} - \frac{2}{3}UHU_{j,hu}dthui_{j,hu}{}^{1/2}dthup_{j,hu}{}^{1/2}$$

$$- \frac{1}{6}UHU_{j,hu}dthui_{j,hu} - \frac{1}{6}UHU_{j,hu}dthup_{j,hu} \qquad \leq \qquad 0\ ;$$

$j \in J, hu \in HU$ (4.4)

To avoid numerical difficulties, the logarithmic mean temperature difference (LMTD) is approximated according to Peterson (1984) approximation where no logarithmic terms are involved in area equations. Although the modified LMTD of Chen (1987) has been used mostly in the literature because it returns a zero LMTD in case of zero either $dth_{i,j,k}$ or $dtc_{i,j,k}$, its accuracy is less than the Paterson (1984) approximation and its calculated area is underestimate.

- LMTD approximation by Paterson (1984):

$$LMTDi_{i,j,k} \cong \frac{2}{3}\ (dth_{i,j,k}dtc_{i,j,k})^{1/2} + \frac{1}{6}(dth_{i,j,k} + dtc_{i,j,k})$$

$; i \in I, j \in J, k \in ST$ (4.5)

The constraints of the model are as follows:

- Overall energy balance of hot and cold streams

Hot: $$FH_i(TH_{i,IN} - TH_{i,OUT}) \quad = \quad \textstyle\sum_{k,j}(qi_{i,j,k}) + \sum_{cu} qcui_{i,cu} \,; \quad i \in I, j \in J, k \in ST, cu \in CU \qquad (4.6)$$

Cold: $$FC_j(TC_{j,OUT} - TC_{j,IN}) \quad = \quad \textstyle\sum_{k,i}(qi_{i,j,k}) + \sum_{hu} qhui_{j,hu} \,; \quad i \in I, j \in J, k \in ST, \; hu \in HU \qquad (4.7)$$

- Energy balance at each stage

Hot: $$FH_i(th_{i,k} - th_{i,k+1}) \quad = \quad \textstyle\sum_{j}(qi_{i,j,k}) \,; \; i \in I, j \in J, k \in ST \qquad (4.8)$$

Cold: $$FC_j(TC_{j,k} - TC_{j,k+1}) \quad = \quad \textstyle\sum_{i}(qi_{i,j,k}) \,; \; i \in I, j \in J, k \in ST \qquad (4.9)$$

For each stream, an overall heat balance must be performed within each stage.

- Heat exchanged for heat exchanger $i - j$ at stage k

Hot: $$qi_{i,j,k} \quad \leq \quad f_{i,j,k} FH_i(th_{i,k} - thpi_{i,j,k}) ; \; i \in I, j \in J, k \in ST \qquad (4.10)$$

Cold: $$qi_{i,j,k} \quad \leq \quad g_{i,j,k} FC_j(tcpi_{i,j,k} - tc_{j,k+1}) ; \; i \in I, j \in J, k \in ST \qquad (4.11)$$

To ensure the feasible temperature of stream exiting the exchanger, the equation relating fractional flow, temperature of stream exiting the exchanger and heat duty is required and able to relate to an inequality constraint in an appropriate way.

- Heat load of cooling utility and heating utility

Cold utility: $$\textstyle\sum_{cu} qcui_{i,cu} \quad = \quad FH_i(th_{i,k} - TH_{i,OUT}) ; \quad i \in I, j \in J, k \in ST_{last}, \; cu \in CU \qquad (4.12)$$

Hot utility: $\sum_{hu} qhui_{j,hu} \quad = \quad FC_j(TC_{j,OUT} - tc_{j,k});$
$i \in I, j \in J, k \in ST_{first}, hu \in HU$ (4.13)

The cooler is located at the lowest temperature region where is the end of the superstructure to cool down the hot process stream. The heater is placed at the highest temperature region to heat up the cold process stream at the end of superstructure.

- Assignments of inlet and outlet temperatures to the superstructure

Hot: $th_{i,k} \quad = \quad TH_{i,IN};\ i \in I, j \in J, k \in ST_{first}$ (4.14)

Cold: $tc_{j,k} \quad = \quad TC_{j,IN};\ i \in I, j \in J, k \in ST_{last}$ (4.15)

The temperature of stream entering to the superstructure should be equal to the supply temperature of that stream. In other words, the temperature of hot stream with $k = ST_{first}$ equals to its supply temperature and the temperature of cold stream with $k = ST_{last}$ equals to supply its supply temperature.

- Outlet temperature of each stage k

Hot: $th_{i,k+1} \quad = \quad \sum_j(f_{i,j,k} thpi_{i,j,k});\ i \in I, j \in J, k \in ST$ (4.16)

Cold: $tc_{j,k} \quad = \quad \sum_j(g_{i,j,k} tcpi_{i,j,k});\ i \in I, j \in J, k \in ST$ (4.17)

The temperature of split stream before entering the exchanger is the same as the temperature of stream entering each stage whereas the outlet temperature of one exchanger is varied. As a consequence, it is not capable of compute the split stream flow from the temperature of stream exiting the stage as the isothermal- mixing does. Hence, the additional variables for temperatures at outlet of each exchanger are introduced to model and the outlet temperatures of each stage are computed by these below equations.

- Constraints ensuring feasibility of temperatures in the super-structure.

Hot: $th_{i,k} \geq TH_{i,OUT}\ ;\ i \in I, j \in J, k \in ST_{last}$ (4.18)

$th_{i,k} \geq th_{i,k+1}\ ;\ i \in I, j \in J, k \in ST$ (4.19)

Cold: $tc_{j,k} \leq TC_{j,OUT}\ ;\ i \in I, j \in J, k \in ST_{first}$ (4.20)

$tc_{j,k} \geq tc_{j,k+1}\ ;\ i \in I, j \in J, k \in ST$ (4.21)

In contrast to the assignment of inlet temperature to the superstructure in equation (4.14) and (4.15), the outlet temperature for some or all streams will be heated up or cooled down by heating or cooling utility. Thus, the inequality relation is required to apply as equation (4.18) and (4.21). Ensuring the feasible temperatures in the superstructure, the constraints of equation (4.19) and equation (4.22) impose that the outlet temperature of any stage is equal to or less than that of previous one. Because heat load in a stage can be zero in case of no heat exchange.

- Logical constraints on heat loads

Heat load: $qi_{i,j,k} \leq \Omega_{i,j} zi_{i,j,k};\ i \in I, j \in J, k \in ST$ (4.22)

Cold utility load: $qcui_{i,cu} \leq \Omega_i zcui_i;\ i \in I, j \in J, k \in ST,\ cu \in CU$ (4.23)

Hot utility load: $qhui_j \leq \Omega_j zhui_j;\ i \in I, j \in J, k \in ST,\ hu \in HU$ (4.24)

The binary variable is introduced to define the existence of match. If the hot stream i matches cold stream j in stage$k$, the binary variable equals to one and the heat load should be less than the upper bound of heat content for heat exchanger($\Omega_{i,j}$).

- Constraints for flowrate of splitting stream of hot stream i ($f_{i,j,k}$) and cold stream j ($g_{i,j,k}$) are shown below

Hot: $$f_{i,j,k} \leq FH_i \; ; \; i \in I, j \in J, k \in ST \qquad (4.25)$$

$$\sum_j f_{i,j,k} \leq FH_i \; ; \; i \in I, j \in J, k \in ST \qquad (4.26)$$

Cold: $$g_{i,j,k} \leq FC_j ; \; i \in I, j \in J, k \in ST \qquad (4.27)$$

$$\sum_i g_{i,j,k} \leq FC_j ; \; i \in I, j \in J, k \in ST \qquad (4.28)$$

If a splitting stream bypasses stage k (no exchanger exists in that stage k for that split stream), then the summation of splitting stream flowrate stream must be less than the main stream flowrate. Otherwise, the summation must be equal to the main stream flowrate.

- Approach temperatures at temperature locations which are the thermodynamic constraints for matches.

Hot end: $$dthi_{i,j,k} \leq th_{i,k} - tcpi_{i,j,k} + \Gamma\left(1 - zi_{i,j,k}\right) ; \; i \in I, j \in J, k \in ST \qquad (4.29)$$

Cold end: $$dtci_{i,j,k} \leq thpi_{i,j,k} - tc_{j,k+1} + \Gamma\left(1 - zi_{i,j,k}\right) ; i \in I, j \in J, k \in ST \qquad (4.30)$$

Cold utility: $$dtcui_{i,cu} \leq th_{i,k} - TCU_{cu,OUT} + \Gamma\left(1 - zcui_{i,cu}\right) ;$$
$$i \in I, k \in ST_{last}, cu \in CU \qquad (4.31)$$

Hot utility: $$dthui_{j,hu} \leq THU_{hu,out} - tc_{j,k} + \Gamma\left(1 - zhui_{j,hu}\right) ;$$
$$j \in J, k \in ST_{first}, hu \in HU \qquad (4.32)$$

Temperature approaches have to be calculated not only for the case of existing matches but also no-existing matches. Accordingly, the logical condition is introduced to ensure that temperatures of matches yield positive temperature differences at both sides and deactivate the thermodynamic conditions for non-existing matches. However, unlike isothermal mixing, temperature approaches at hot and cold ends of exchanger have to be calculated in the model.

- Minimum approach-temperature constraints

$$dthi_{i,j,k}, dtci_{i,j,k}, dtcui_{i,cu}, dthui_{j,hu} \geq EMAT\,;$$
$$i \in I, j \in J, k \in ST, cu \in CU, hu \in HU \tag{4.33}$$

Besides, the temperature approaches should be larger than the given $EMAT$value which is the minimum approach-temperatureof exchanger to ensure the positive approach temperature for existing matches in stagewise superstructure.

- Variables bounds

$$TMIN \leq th_{i,k}, thpi_{i,j,k} \leq TMAX\,;\; i \in I, k \in ST \tag{4.34}$$

$$TMIN \leq tc_{j,k}, tcpi_{i,j,k} \leq TMAX\,;\; j \in J, k \in ST \tag{4.35}$$

In contrast to the original model limiting the sub-stream.temperature to be within the initial and final temperatures of their parent stream, the bounds of the branch stream temperature are improved by the maximum and minimum temperatures in the HEN. Thus the bounds, as we show later in section 4.5, result in better HEN solutions.

4.2.2 Model A2 formulation for HEN synthesis

$$\begin{aligned} Minimize\,(TAC) = \{[& \sum_{i,j,k,bh,j,bc,sk} (z_{i,j,k,bh,bc,sk} CFHX_{i,j}) \\ & + \sum_{i,cu} (zcu_{i,cu} CFCU_{i,cu}) + \sum_{j,hu} (zhu_{j,hu} CFHU_{j,hu})] \\ & + [\sum_{i,j,k,bh,j,bc,sk} (a_{i,j,k,bh,bc,sk} ACHX_{i,j}) \end{aligned}$$

$$+\sum_{i,cu}(acu_{i,cu}ACCU_{i,cu}) + \sum_{j,hu}(ahu_{j,hu}ACHU_{j,hu})]$$

$$+\,[\sum_{i,cu}(qcu_{i,cu}CCU_{i,cu}) + \sum_{j,hu}(qhu_{j,hu}CHU_{j,hu})\,]\,\}\,;$$

$$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK, cu \in CU, hu \in HU \qquad (4.36)$$

- Exchange area calculations

$$\frac{q_{i,j,k,bh,bc,sk}}{a_{i,j,k,bc,bh,sk}^{\frac{1}{\beta}}} - \frac{2}{3}U_{i,j}dth_{i,j,k,bh,bc,sk}{}^{1/2}dtc_{i,j,k,bh,bc,sk}{}^{1/2}$$
$$-\frac{1}{6}U_{i,j}dth_{i,j,k,bh,bc,sk} - \frac{1}{6}U_{i,j}dtc_{i,j,k,bh,bc,sk} \qquad \leq \qquad 0\,;$$

$$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK \qquad (4.37)$$

$$\frac{qcu_i}{acu_i^{\frac{1}{\beta}}} - \frac{2}{3}UCU_i dtcu_i{}^{1/2}dtcup_i{}^{1/2} - \frac{1}{6}UCU_i dtcu_i - \frac{1}{6}\,UCU_i dtcup_i \;\leq\; 0\,;$$

$$i \in I \qquad (4.38)$$

$$\frac{qhu_j}{ahu_j^{\frac{1}{\beta}}} - \frac{2}{3}UHU_j dthu_j{}^{1/2}dthup_j{}^{1/2} - \frac{1}{6}UHU_j dthu_j - \frac{1}{6}UHU_j dthup_j \;\leq\; 0\,;$$

$$j \in J \qquad (4.39)$$

- LMTD approximation by Paterson (1984) is shown below;

$$LMTD_{i,j,k,bh,bc,sk} \cong \frac{2}{3}\,(dth_{i,j,k,bh,bc,sk}dtc_{i,j,k,bh,bc,sk})^{1/2}$$
$$+\frac{1}{6}(dth_{i,j,k,bh,bc,sk} + dtc_{i,j,k,bh,bc,sk})$$

$$;\; i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK \qquad (4.40)$$

The equality/inequality constraints are shown below;

- Overall heat balance for each stream

Hot: $FH_i(TH_{i,IN} - TH_{i,OUT}) \;=\; \sum_{k,bh,j,bc,sk}(q_{i,j,k,bh,bc,sk}) + \sum_{cu} qcu_{i,cu}$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK, cu \in CU, hu \in HU$ (4.41)

Cold: $FC_j(TC_{j,OUT} - TC_{j,iIN}) \;=\; \sum_{k,bc,i,bh,sk}(q_{i,j,k,bh,bc,sk}) + \sum_{hu} qhu_{j,hu}$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK, cu \in CU, hu \in HU$ (4.42)

- Heat balance at each stage; K, for each stream

Hot: $FH_i(th_{i,k} - th_{i,k+1}) \;=\; \sum_{bh,j,bc,sk}(q_{i,j,k,bh,bc,sk})$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.43)

Cold: $FC_j(tc_{j,k} - tc_{j,k+1}) \;=\; \sum_{bc,i,bh,sk}(q_{i,j,k,bh,bc,sk})$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.44)

- Heat balance at each sub-stage; SK, on any splitting stream

Hot: $\sum_{j,bc} q_{i,j,k,bh,bc,sk} = fhp_{i,k,bh}(thp_{i,k,bh,sk} - thp_{i,k,bh,sk+1})$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.45)

Cold: $\sum_{i,bh} q_{i,j,k,bh,bc,sk} = fcp_{j,k,bc}(tcp_{j,k,bc,sk} - tcp_{j,k,bc,sk+1})$;
$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.46)

- Heat balance for cold and hot utility

Hot Utility: $\sum_{cu} qcu_{i,cu} \;=\; FH_i(th_{i,k} - TH_{i,OUT})$;
$i \in I, k = ST_{last}, cu \in CU$ (4.47)

Cold Utility: $\sum_{hu} qhu_{j,hu} \;=\; FC_j(TC_{j,OUT} - tc_{j,k})$;

$$j \in J, k = ST_{first}, hu \in HU \qquad (4.48)$$

- Assignments of temperature

Hot: $$TH_{i,IN} = th_{i,k} \; ; \; i \in I, k = ST_{first} \qquad (4.49)$$

$$FH_i th_{i,k} = \textstyle\sum_{bh}\left(fhp_{i,k,bh} thp_{i,k,bh,sk}\right) + \left(FH_i - \sum_{bh}\left(fhp_{i,k,bh}\right)\right) th_{i,k} \; ;$$
$$i \in I, k \in ST, bh \in BH, sk = STSK_{first} \qquad (4.50)$$

$$FH_i th_{i,k+1} = \textstyle\sum_{bh}\left(fhp_{i,k,bh} thp_{i,k,bh,sk}\right) + \left(FH_i - \sum_{bh}\left(fhp_{i,k,bh}\right)\right) th_{i,k} \; ;$$
$$i \in I, k \in ST, bh \in BH, sk = STSK_{last} \qquad (4.51)$$

$$th_{i,k} = thp_{i,k,bh,sk} \; ; \; i \in I, k \in ST, bh \in BH, sk = STSK_{first} \qquad (4.52)$$

Cold: $$TC_{j,IN} = tc_{j,k} ; \quad j \in J, k = ST_{last} \qquad (4.53)$$

$$FC_i tc_{j,k+1} = \sum_{bc}\left(fcp_{j,k,bc} tcp_{j,k,bc,sk}\right) + \left(FC_j - \sum_{bc}\left(fcp_{j,k,bc}\right)\right) tc_{j,k+1} \; ;$$
$$j \in J, k \in ST, bc \in BC, sk = STSK_{last} \qquad (4.54)$$

$$FC_j tc_{j,k} = \sum_{bc}\left(fhp_{j,k,bc} tcp_{j,k,bc,sk}\right) + \left(FC_j - \sum_{bc}\left(fcp_{j,k,bc}\right)\right) tc_{i,k+1} \; ;$$
$$j \in J, k \in ST, bc \in BC, sk = STSK_{first} \qquad (4.55)$$

$$tc_{j,k+1} = tcp_{j,k,bc,sk} \; ; \; j \in J, k \in ST, bc \in BC, sk = STSK_{last} \qquad (4.56)$$

By means of the non-isothermal mixing, the mixer at the outlet from the heat exchanger and the splitter at the inlet stream to the heat exchanger are introduced to the model. Heat balance equations for splitter and mixer are defined in the equation (4.50) and (4.51) for hot process stream, and (4.54) and (4.55) for cold process stream, respectively. The first termof them is the summation of splitting stream flowrates where the splitting streams pass through the exchangers. The second one is flowrates of stage-bypass streams. It is necessary to notice that for equation (4.52) the inlet substage temperatures of hot process splitting stream located at the first substage ($sk = STSK_{first}$) should be equal to the temperature of that main stage. Whereas, for equation (4.56) the inlet substage temperatures of cold process splitting stream placed at the last substage ($sk = STSK_{last}$) are assigned to the outlet temperatures from the $k+1$ stage.

- Temperature feasibility constraints

Hot: $$th_{i,k} \geq th_{i,k+1} \ ; \ i \in I, k \in ST \tag{4.57}$$

$$thp_{i,k,bh,sk} \geq thp_{i,k,bh,sk+1} \ ; \ i \in I, k \in ST, bh \in BH, sk \in STSK \tag{4.58}$$

$$th_{i,k} \geq TH_{i,OUT} \ ; \ i \in I, k \in ST_{last} \tag{4.59}$$

Cold: $$tc_{j,k} \geq tc_{j,k+1} \ ; \ j \in J, k \in ST \tag{4.60}$$

$$tcp_{j,k,bc,sk} \geq tcp_{j,k,bc,sk+1} \ ; \ j \in J, k \in ST, bc \in BC, sk \in STSK \tag{4.61}$$

$$tc_{i,k} \leq TC_{j,OUT} \ ; \ j \in J, k \in ST_{first} \tag{4.62}$$

The constraints enforce that the outlet temperatures from the both main stage and substage decrease. With reason of no heat exchange in the stage and substage, the equalities should be introduced.

- Logical constraints for heat loads are shown below

Heat load: $$q_{i,j,k,bh,bc,sk} - \Omega_{i,j} z_{i,j,k,bh,bc,sk} \leq 0 \ ; \ i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK \tag{4.63}$$

Cold utility load: $qcu_{i,cu} - \Omega_i zcu_{i,cu} \leq \ 0 \ ; \ i \in I, \ cu \in CU$ (4.64)

Hot utility load: $qhu_{j,hu} - \Omega_j zhu_j \leq \ 0; \ j \in J, hu \in HU$ (4.65)

- Maximum matching and Flow constraints are shown below

Hot: $\sum_{j,bc} z_{i,j,k,bh,bc,sk} \leq \ 1;$

$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.66)

$0 \ \leq fhp_{i,k,bh} \leq FH_i \ ; \ i \in I, k \in ST, bh \in BH$ (4.67)

Cold: $\sum_{i,bh} z_{i,j,k,bh,bc,sk} \leq \ 1 \ ;$

$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.68)

$0 \ \leq fcp_{j,k,bc} \leq FC_j \ ; \ ij \in J, k \in ST, bc \in BC$ (4.69)

It is necessary to impose that only one match can exist in each substage. However, the maximum number of matches depends on the defined number of substages.

- Approach temperature constraints are shown below

Hot end: $dth_{i,j,k,bh,bc,sk} \leq thp_{i,k,bh,sk} - tcp_{j,k,bc,sk} + \Gamma(1 - z_{i,j,k,bh,bc,sk}) \ ;$

$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.70)

Cold end: $dtc_{i,j,k,bh,bc,sk} \leq thp_{i,k,bh,sk+1} - tcp_{j,k,bc,sk+1} + \Gamma(1 - z_{i,j,k,bh,bc,sk});$

$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$ (4.71)

Cold Utility: $dtcu_{i,cu} \leq th_{i,k} - TCU_{cu,out} + \Gamma(1 - zcu_{i,cu});$

$i \in I, k \in ST_{last}, \ cu \in CU$ (4.72)

Hot Utility: $$dthu_{j,hu} \le THU_{hu,out} - tc_{j,k} + \Gamma\left(1 - zhu_{j,hu}\right); \quad j \in J, k \in ST_{first}, hu \in HU \qquad (4.73)$$

$$dth_{i,j,k,bh,bc,sk}, dtc_{i,j,k,bh,bc,sk}, dtcu_{i,cu}, dthu_{j,hu} \ge EMAT;$$
$$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK, \ cu \in CU, hu \in HU \qquad (4.74)$$

The total annual cost of the HEN is defined by Equation (4.1) and (4.38). The first term defines the fixed charges for exchanger $i - j$, cooling utility and heating utility, the second defines the cost of utilities usage and the last one defines the cost depending on area of all heat exchangers. The given objective function is a linear equation because the areas $a_{i,j,k}$, acu_i and ahu_jtreated explicitly as variables substituted in objective function are raised to the power of β . Finally, the TAC will be minimized as an objective goal of HEN synthesis. Equation (4.2) to (4.4) and (4.39) to (4.41) are area equations containing a term of logarithmic mean-temperature difference (LMTD) which is approximated by the Paterson (1984) approximation. Although the modified LMTD of Chen (1987) has been used mostly in the literature because of LMTD of zero when either$dthi_{i,j,k}$,$dth_{i,j,k,bh,bc,sk}$ in equation (4.5) and (4.42) or$dtci_{i,j,k}$, $dtc_{i,j,k,bh,bc,sk}$in equation (4.5) and (4.42) of zero, its accuracy is less than the Paterson (1984) approximation and its calculated area is underestimate. Equations (4.6) to (4.11), and (4.43) to (4.48) are heat balances of hot and cold process streams for both models. The overall heat balances (equation (4.6), (4.7), (4.43) and (4.44)) should be ensure that the target temperature will be reached by process streams match and utility match. Heat loads of utilities are defined by equations (4.12), (4.13), (4.49) and (4.50). The equalities in equation (4.14), (4.15), (4.54) and (4.55) are the assignment of the inlet stream temperature to the superstructure of hot and cold stream.

4.2.3 Solution Strategy

The conceptual design is generally based on three key characteristic variables in order of relevance:

- Use of energy resources that related to the utility requirement

- Number of exchanger to transfer heat that related to network complexity and the investment requirement
- Heat transfer area, that relates to the size of exchanger (area cost in the investment cost)

First, the energy targeting is achieved by the calculation of the minimum heating and cooling requirements. The number of units that must be installed is a measure of the network complexity and an important factor for the estimate the required investment for the HEN. HEN with more units tends to be more expensive than the HEN with a small number of units. Consequently, the number of units is an important characteristic variable for targeting. To minimize the number of exchangers for a given balance with fixed utilities, the binary variables must be added to represent the presence of a match between any pair of streams. This result in a MILP problem. Area targeting estimates the minimum total amount of area required to transfer heat from a set of hot and cold streams. The transfer can be initialized from the transferred duty, the effective temperature difference and the heat transfer coefficient of the streams. The duty and the LMTD are determined from previous step. Heat transfer area targets are generally used to calculate network cost.

Although the model A2 looks promising, the problem of the model A2 occurs when it handles the effect of high non-convexities which prevent model from finding the feasible solution. This difficulty needs to be resolved by applying the effective strategy. The strategy is developed from three major steps sequential HEN synthesis of *Floudas et al. (1986)* which predicts the minimum utility cost, number of units and matches and the HEN structure, respectively. The sequential approaches based on mathematical programming address some part of the solution of the previous subproblem as parameters in the sequential subproblem to find the final solution. However, the sequential procedure does not trade-offs factors affecting the total annualized cost (TAC) simultaneously and may give suboptimal solutions. Therefore, this work developed the symmetric simultaneous approach. This strategy consists of two main steps; initialization and design steps as shown in Figure 4.3.The design steps consists of four more steps; second, third, fourth, and fifth steps. The initialization step uses model A1 and the design step uses model A2. For the first step, model A1, which is simpler to solve by MINLP, generates feasible solution, which will be used as initial values for variables in the next design step using model A2. The initialized variables consist of the binary variables of heat exchanger locations as well as continuous variables of heat load distribution and temperature of each stage. The objective of this first step is to minimize total annual cost. The second step formulates the NLP model to maximize the heat recovery or minimize the utility consumption by solving flow of splitting stream with fixed binary variables from first step. For the third step, the MILP model is used to generate a topology of HEN with the goal of minimum utility consumption and fixed cost. To provide better feasible solution, the area of heat exchanger in network and the flow rate of splitting stream are optimized in the fourth step using NLP. The better HEN design is done at the fifth step using MINLP. This MINLP model simultaneously synthesizes effective HEN where the main objective is to minimize total annual cost composing of capital and operational expenses.

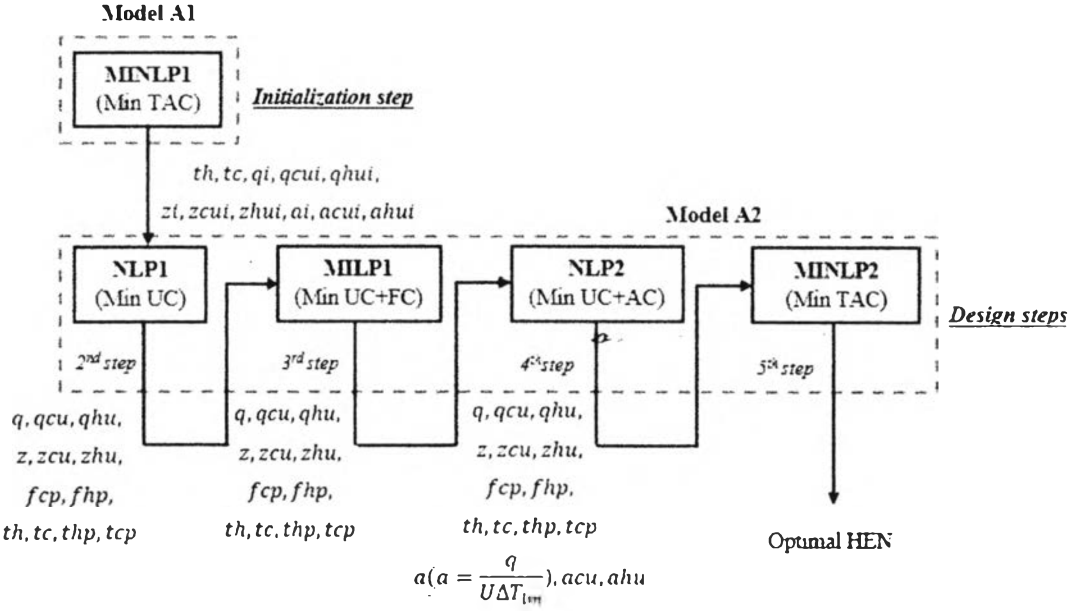

Figure 4.3 The HEN synthesis strategy.

4.3 HEN Retrofit

In contrast to synthesis design, retrofit design is no standard problem formulation potentially to revamp any existing networks. It is commonly categorized into two parts, structure and parameter modifications. This work uses the MINLP model based on the modified and extended Synheat model that proposed for HEN synthesis in the section 4.2.

4.3.1 Model Formulation for HEN Retrofit

The aim of the retrofitted network performing non-isothermal mixing is to minimize costs of additional area as well as structural changes involving reuse of non-profitable exchanger are addressed rigorously;

$$\begin{aligned}
Minimize\ (\ Retrofit\ Cost\ ) = \{ [& \sum_{i,j,k,bh,j,bc,sk} \left(z^{new}_{i,j,k,bh,bc,sk} CFHX^{new} \right) \\
& + \sum_{i,cu} \left(zcu^{new}_{i,cu} CFCU^{new} \right) \\
& + \sum_{j,hu} \left(zhu^{new}_{j,hu} CFHU^{new} \right)] \\
& + [\sum_{i,j,k,bh,j,bc,sk} \left((a^{add}_{i,j,k,bh,bc,sk})^{1/\beta} ACHX^{add} \right) \\
& + \sum_{i,cu} \left((acu^{add}_{i,cu})^{1/\beta} ACCU^{add} \right) \\
& + \sum_{j,hu} \left((ahu^{add}_{j,hu})^{1/\beta} ACHU^{add} \right)] \\
& + [\sum_{i,cu} \left(qcu^{add}_{i,cu} CCU^{add} \right) \\
& + \sum_{j,hu} \left(qhu^{add}_{j,hu} CHU^{add} \right)] \};
\end{aligned}$$

$$i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK, cu \in CU, hu \in HU \qquad (4.75)$$

In addition to synthesis, additional constraints are required in retrofit formulation.

- Constraints for additional areas

$$\begin{aligned}
& a_{i,j,k,bh,bc,sk}{}^{1/\beta} - AHX^{EXIST} z_{i,j,k,bh,bc,sk} \le a^{add}_{i,j,k,bh,bc,sk} \\
& \le AREA^{max} - a_{i,j,k,bh,bc,sk}{}^{1/\beta} + AHX^{EXIST} z_{i,j,k,bh,bc,sk}; \\
& i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK
\end{aligned} \qquad (4.76)$$

$$acu_{i,cu}{}^{1/\beta} - ACU^{EXIST} zcu_{i,cu} \leq acu^{add}_{i.cu}$$

$$\leq AREA^{max} - acu_{i,cu}{}^{1/\beta} + ACU^{EXIST} zcu_{i}, cu;\ i \in I, cu \in CU \qquad (4.77)$$

$$ahu_{j,hu}{}^{1/\beta} - AHU^{EXIST} zhu_{j,hu} \leq ahu^{add}_{j,hu}$$

$$\leq AREA^{max} - ahu_{j,hu}{}^{1/\beta} + AHU^{EXIST} zhu_{j,hu};\ j \in J, hu \in HU \qquad (4.78)$$

The existing areas of heat exchangers AHX^{EXIST}, coolers ACU^{EXIST}, heaters AHU^{EXIST} can be used or extended by additional areas $a^{add}_{i,j,k,bh,bc,sk}$, $acu^{add}_{i,cu}$ and $ahu^{add}_{i.hu}$. The additional areas of $a^{add}_{i.j,k,bh,bc,sk}$, $acu^{add}_{i.cu}$ and $ahu^{add}_{i.hu}$ are the variables that give the heat transfer area of the required matches in the retrofit. These constraints are to determine the new area for each required matches in the retrofit. The variables of $a^{add}_{i,j,k,bh,bc,sk}$, $acu^{add}_{i,cu}$ and $ahu^{add}_{i,hu}$ will be minimized in the objective function.

- Constraints for additional/removal heat exchange of hot and cold utility

$$qcu^{add}_{i,cu} \leq qcu_{i,cu} + QCU^{EXIST} ZCU_{i,cu}{}^{EXIST};\ i \in I \qquad (4.79)$$

$$qhu^{add}_{j,cu} \leq qhu_{j,cu} + QHU^{EXIST} ZHU_{j,hu}{}^{EXIST};\ j \in J \qquad (4.80)$$

The utilities consumption for cooler and heater should be decreased.

- Constraints for the new process exchanger/ hot and cold utility which are the 0-1 binary variables

$$z_{i,j,k,bh,bc,sk} = z^{new}_{i,j,k,bh,bc,sk} + Z^{EXIST}_{i,j,k,bh,bc,sk};\ i \in I, j \in J, k \in ST,$$

$$bh \in BH, bc \in BC, sk \in STSK \qquad (4.81)$$

$$z^{new}_{i,j,k,bh,bc,sk} + Z^{EXIST}_{i,j,k,bh,bc,sk} \leq 1;\ i \in I, j \in J, k \in ST,$$

$$bh \in BH, bc \in BC, sk \in STSK \qquad (4.82)$$

$$zcu_{i,cu} = zcu_{i,cu}^{new} + ZCU_{i,cu}^{EXIST}; \; i \in I, cu \in CU \qquad (4.83)$$

$$zcu_{i,cu}^{new} + zcu_{i,cu}^{EXIST} \leq 1 \; ; \; i \in I, cu \in CU \qquad (4.84)$$

$$zhu_{j,hu} = zhu_{j,hu}^{new} + ZHU_{j,hu}^{EXIST}; \; j \in J, hu \in HU \qquad (4.85)$$

$$zhu_{j,hu}^{new} + zhu_{j,hu}^{EXIST} \leq 1 \; ; \; j \in J, hu \in HU \qquad (4.86)$$

The binary variables of retrofit match can be divided into two binary variables which are the existing or a new match, but only one is chosen.

4.3.2 Solution Strategy

The solution strategy for retrofit is as same as one for synthesis. However, it requires more constraints in each step and the main goal of the initialization step is to find a feasible solution for retrofit while minimizing the costs of utility consumptions, additional area and new exchanger installations.

4.4 Examples

4.4.1 Synthesis and Retrofit HEN

Three examples are given in this section. First two examples and later one illustrate the proposed model with initialization strategies for synthesis and retrofit problem, respectively. The problems were implemented in GAMS 24.2.1 solved by DICOPT as an MINLP using CONOPT 3 and CPLEX 12.6 as nonlinear programming (NLP) solver and MILP solver, respectively. The input data for example 1, 2 and 3 are summarized in Table 4.1, 4.2 and 4.3, respectively.

Table 4.1 Stream, cost and solution data for the examples 1 (Biegler *et al.*, 1997)

Stream	**Temperature**		**h**	**F**	**Cost**
	T_{in} *(°C)*	**T_{out} *(°C)***	***(kW °C^{-1} m^{-2})***	***(kW °C^{-1})***	***($ kW^{-1} per year)***
Hot 1	167	77	2.0	22	-
Cold 1	76	157	2.0	20	-
Cold 2	47	95	0.67	7.5	-
HU	227	227	1.0	-	120
CU	27	47	1.0	-	20

Exchanger minimum temperature approach (EMAT) = 1 °C

Heat exchanger cost ($) 6,600+670$a^{0.83}$ (a = area in m^2)

Table 4.2 Stream, cost and solution data for the examples 2 (Björk and Westerlund, 2002)

Stream	**Temperature**		**h**	**F**	**Cost**
	T_{in} *(°C)*	**T_{out} *(°C)***	***(kW °C^{-1} m^{-2})***	***(kW °C^{-1})***	***($ kW^{-1} per year)***
Hot 1	155	30	2.0	8	-
Hot 2	80	40	2.0	15	-
Hot 3	200	40	2.0.	15	-
Cold 1	20	160	2.0	20	-
Cold 2	20	100	2.0	15	-
HU	220	220	2.0	-	120
CU	20	30	2.0	-	20

Exchanger minimum temperature approach (EMAT) = 10 °C

Heat exchanger cost ($) 6,000+600$a^{0.85}$ (a = area in m^2)

Table 4.3 Stream, cost and solution data for the examples 3 (Yee and Grossmann, 1987)

Stream	Temperature		h	F	Cost
	T_{in} (°C)	T_{out} (°C)	(kW °C^{-1} m^{-2})	(kW °C^{-1})	(\$ kW^{-1} per year)
Hot 1	170	60	0.8	30	-
Hot 2	150	30	0.8	15	-
Cold 1	20	135	0.8	20	-
Cold 2	80	140	0.8	40	-
HU	177	177	0.8	-	80
CU	20	40	0.8	-	20

Exchanger minimum temperature approach (EMAT) = 10 °C
Cost of area for a new and existing heat exchanger (\$) 1,300$a^{0.6}$(a = area in m^2)
Fixed cost for a new heat exchanger \$3,000

Example 1, small HEN synthesis case founded in Biegler *et al.* in 1997, consists of one hot process stream, two cold process streams and single hot and cold utility. Figure 4.4 shows our HEN result as three exchangers as same as one from literatures. Our TAC of \$76,327 is same as Huang and Karimi (2012); however, it is slightly less than TAC of \$76,445 from Biegler *et al.* (1997) and TAC of \$76,330 from work of Björk and Westerlund (2002). The difference between our TAC and the first one is due to using the different LMTD approximations (Paterson (1984) and Chen (1987) by Biegler *et al.* (1997)). For the later one, the slight difference between both TACs might be resulted from piece-wise linear approximation from global optimization algorithm used in their work. It means that our proposed model and strategy possible to attain the TAC as global optimization algorithm does in such a small case. When compared to the work of Jongsuwat *et al.* (2014), our work obtained the same number of exchangers. However, their topology, as shown in Figure 4.5, is different from our result with the lower both TAC of \$73,684 and total exchange area of 174 m^2. Because the initialization strategy and model formulation are difference.

The main objective of the second example originally proposed by Björk and Westerlund (2002) is to illustrate the effective HEN synthesis by using proposed model A2 and effective strategy. The original topology with four heat exchangers is depicted in the Figure 4.6. The final structure is shown in Figure 4.8. Comparing to the best isothermal HEN of Björk and Westerlund (2002), the topology and TAC are difference. This is because the isothermal does not allow the temperature different in the mixer.

The network structure obtained in this work is different from all in literatures. The corresponding HEN consists of five heat exchangers with total area of 187.55 m^{2}and TAC of $94,183, which is less than one in literatures from Björk and Westerlund (2002), and Huang and Karimi (2012) with $1,818 and $1,460, respectively. It can be noticed that our proposed model generates the network configuration allowing splitting stream flow through several potential exchangers and trading-off between the number and areas of exchangers affects the optimal TAC. In 2014, the work of Jongsuwat *et al.* (2014) also allows any branch stream passing through the multiple exchangers; however, their model does not account for non-linearity in the area cost. Their work obtained higher TAC of $94,880 and exchange area of 215.5 m^2 as illustrated in Figure 4.7. This is because our proposed strategy and model formulation are difference from their work.

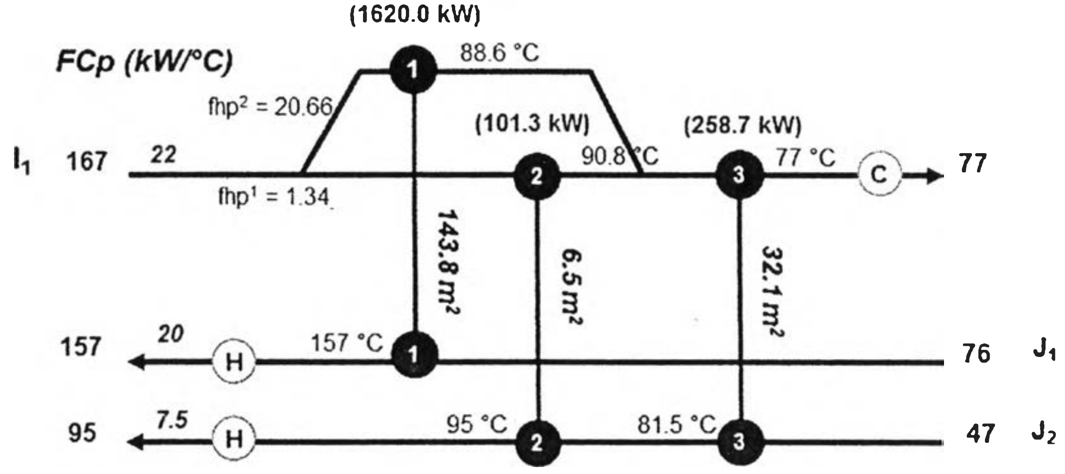

Figure 4.4 HEN result of example 1.

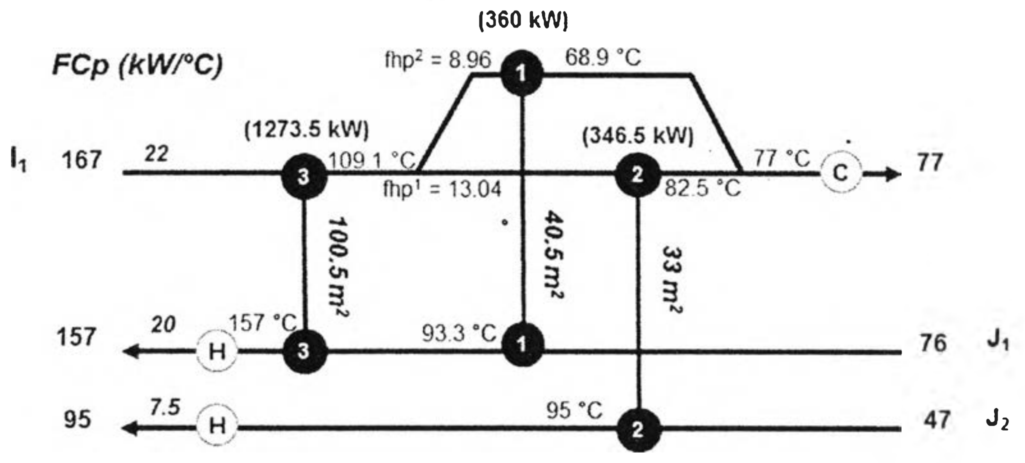

Figure 4.5 HEN from Jongsuwat *et al.* (2014) for example 1.

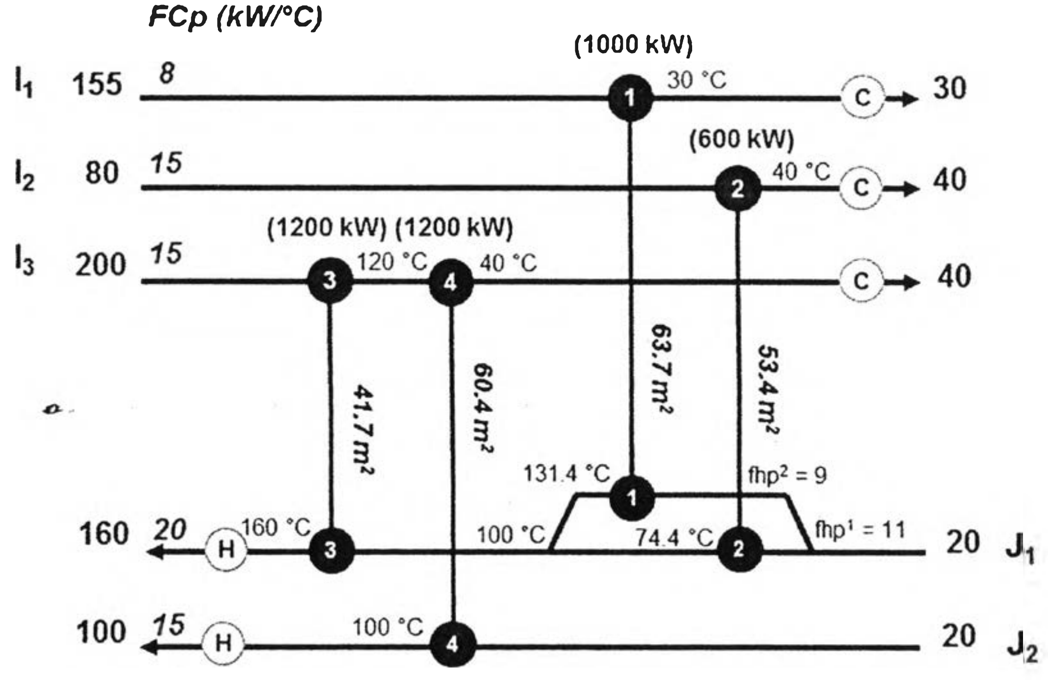

Figure 4.6 HEN from Björk and Westerlund (2002) and Huang and Karimi (2012) for example 2.

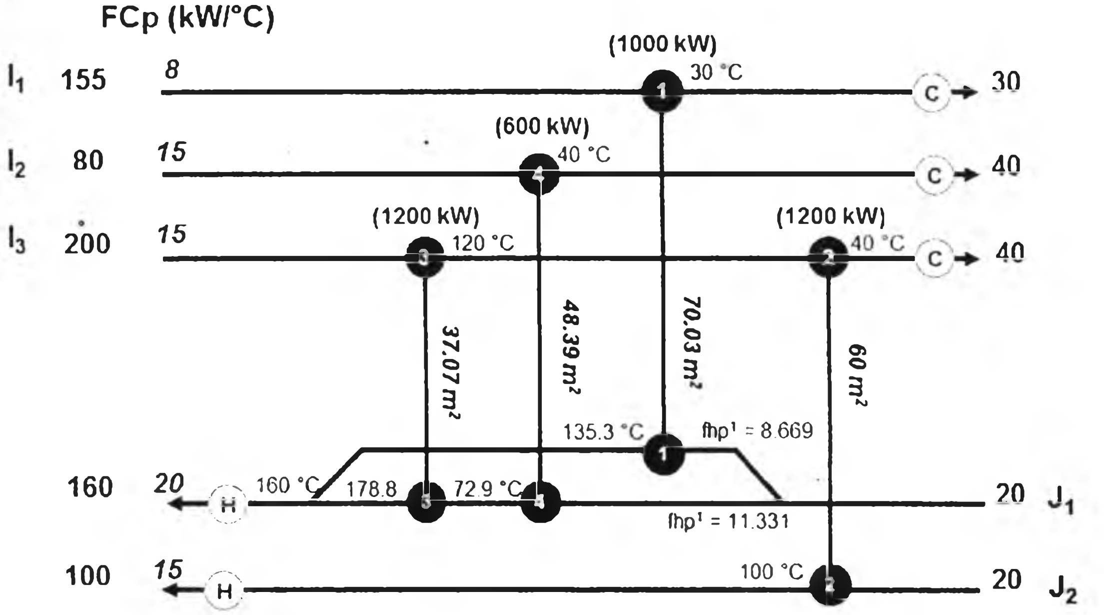

Figure 4.7 HEN from Jongsuwat *et al.* (2014) for example 2.

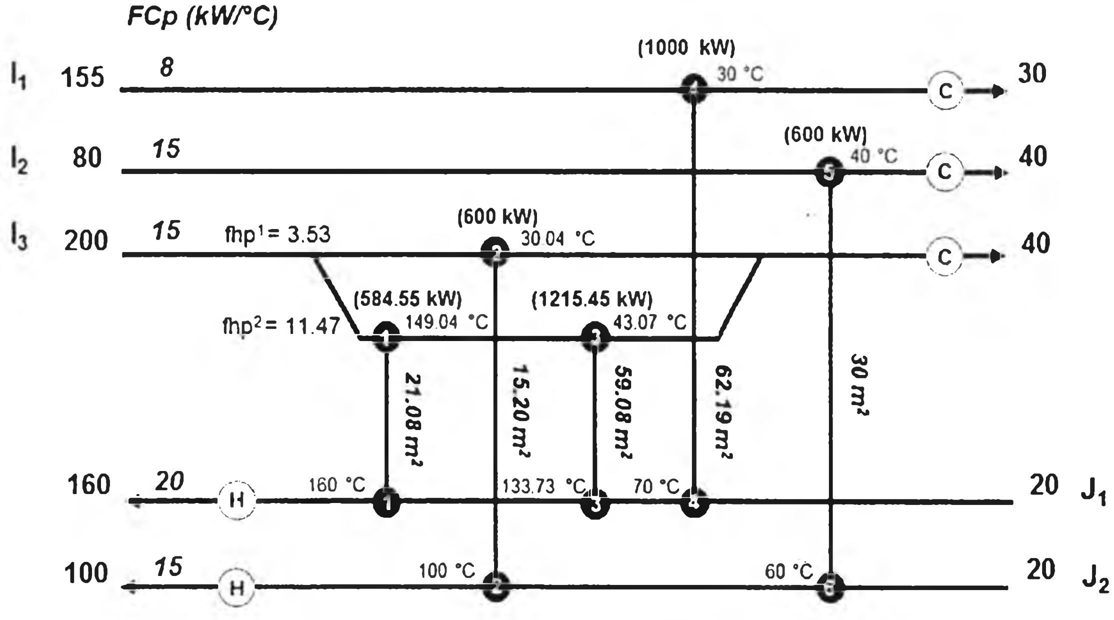

Figure 4.8 HEN result of example 2.

In order to illustrate the retrofit design, the simple example from Yee and Grossmann (1987) is taken. The existing HEN consisting five exchangers, as seen in Figure 4.9, requires 1.5×10^3 kW of steam and 1.9×10^3 KW of cooling water which the utilities cost is about $\$1.58\times10^5$/year. The result of HEN retrofit structure is illustrated in figure 4.11 requiring one more H1-C2 match of exchanger in the first stage. The modification cost is $\$4.36\times10^4$ and utilities cost is about $\$3.52\times10^4$/year which are 270 kW steam and 680 kW of cooling water. The retrofit match and additional area for existing exchangers is shown in table 4.4. The payback period is about 0.357 years. Comparing to the retrofitted HEN of Liu *et al.* (2014), their work contained seven requiring two more H1-C2 and H2-C1 matches of exchanger as seen in Figure 4.10. And 221 KW steam and 621 KW cooling water are needed in the retrofitted HEN and the cost of utilities can reach $\$3.01\times10^4$/year and the total cost of modification is $\$4.79\times10^4$. The payback period is about 0.375 years. Their retrofit model is built on the base of the stagewise superstructure. Hybrid genetic algorithm is used to optimize the retrofit model.

Table 4.4 Heat exchangers and area distribution of retrofitted HEN from Liu *et al.* (2014).

HX No.	**Match(stage)**	**Q(*kW*)**	**$A_{existing}(m^2)$**	**$A_{new}(A_{add})(m^2)$**
1	*H1C1(1)*	705	*38.31*	38.31 (0)
2	*H1C2(1) *NEW*	1,974	-	214.09 (214.09)
3	*H2C1(1) *NEW*	474	-	24.58 (24.58)
4	*H2C2(1)*	426	46.74	46.74 (0)
5	*H2C1(2)*	900	68.72	68.72 (0)
C	*H1CU*	621	40.23	19.25 (0)
H	*HUC1*	221	35.00	5.85 (0)

Table 4.5 Heat exchangers and area distribution of retrofitted HEN from this work

HX No.	Match(stage)	Q(kW)	$A_{existing}(m^2)$	$A_{new}(A_{add})(m^2)$
1	*H1C1(1)*	1,400	*38.31*	87.59 (49.28)
2	*H1C2(1) *NEW*	1,220	-	87.65 (87.65)
3	*H2C2(1)*	900	46.74	67.27 (20.53)
4	*H2C1(2)*	900	68.72	68.72 (0)
C	*H1CU*	680	40.23	9.18 (0)
H	*HUC1*	270	35.00	8.44 (0)

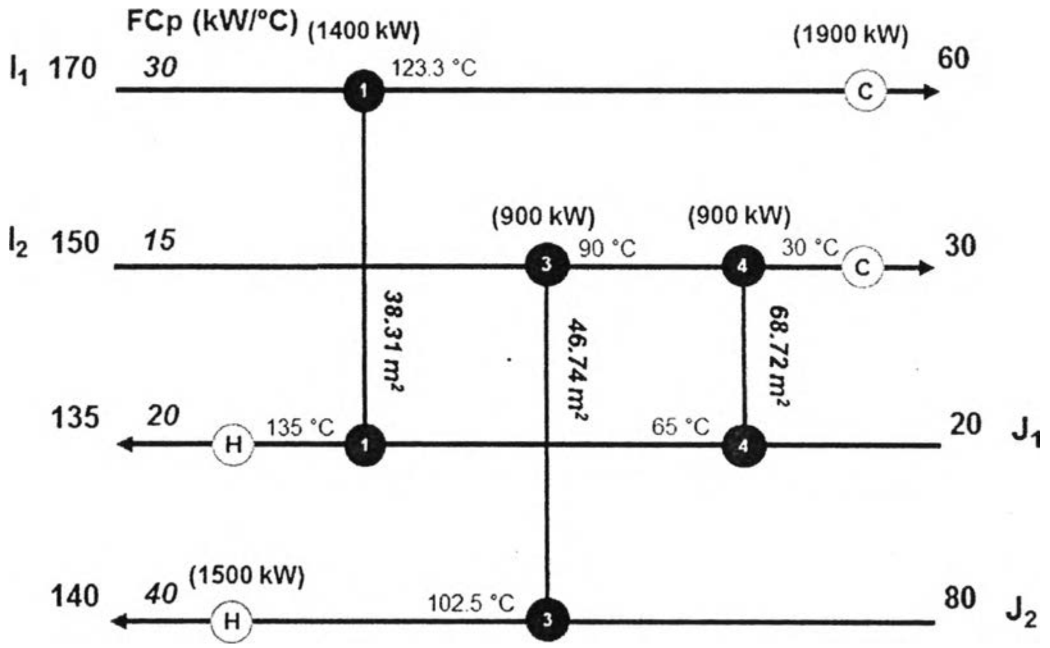

Figure 4.9 Existing HEN of example 3.

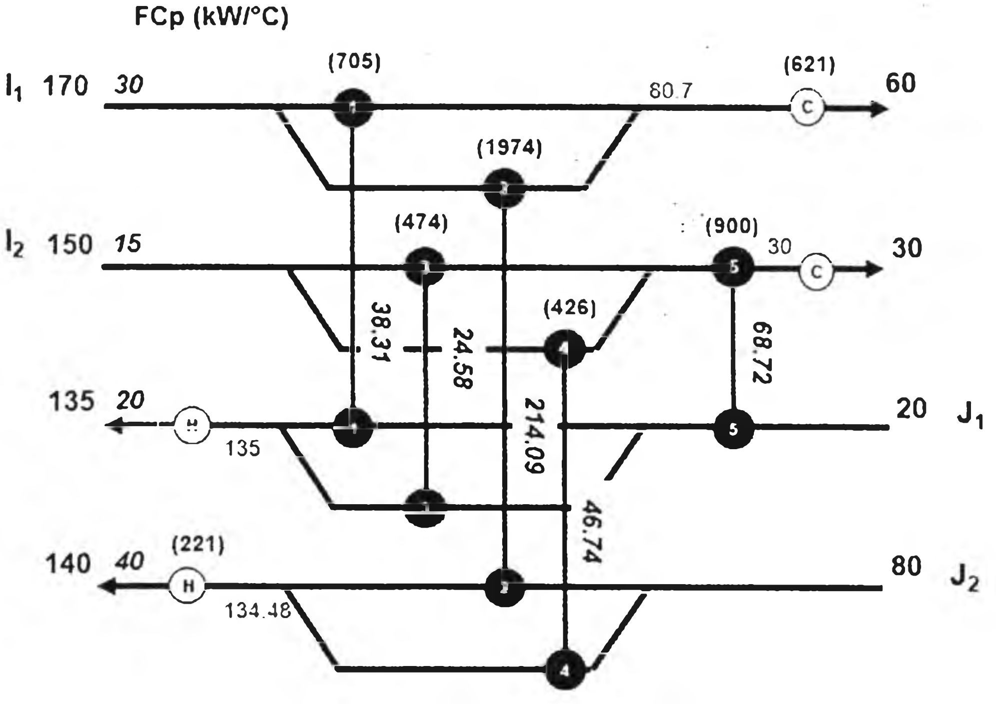

Figure 4.10 Retrofitted HEN from Liu *et al.* (2014).

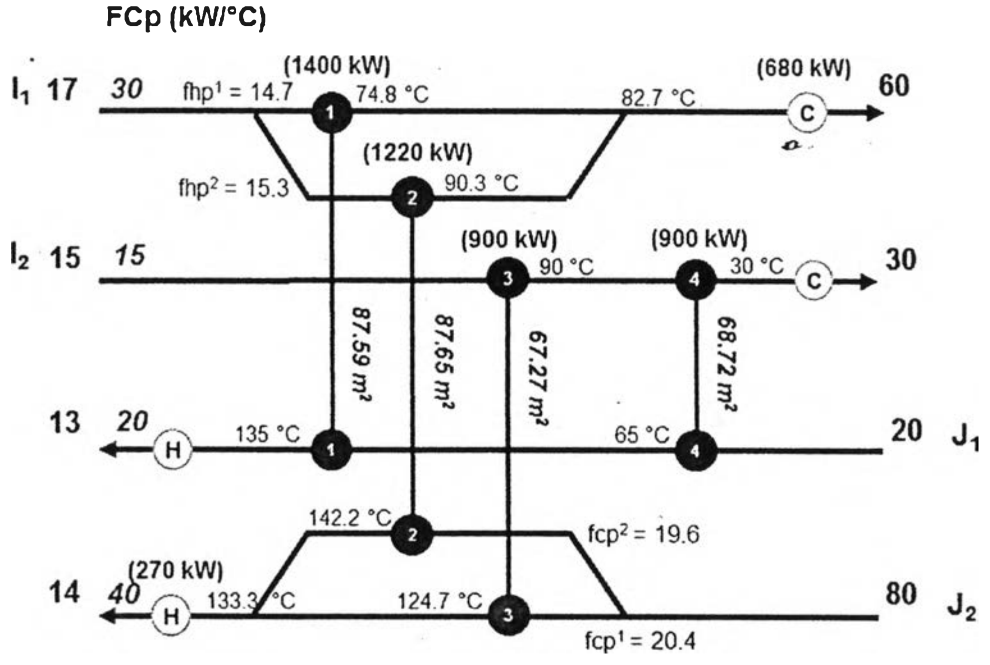

Figure 4.11 Retrofitted HEN for example 3.

4.4.2 Retrofit of CDU Case Study

Amongst various types of applications of HEN, crude oil atmospheric distillation unit in the petroleum refineries case is one of the most challenging. Therefore, the objective of this work is retrofitting the existing HEN of Crude Distillation Units (CDU) to reduce the recent utility consumption and maximize Net Present Value (NPV). This work proposes the modified stagewise superstructure model by Yee and Grossmann (1990). The model is MINLP often involving highly non-linear and non-convex terms to account for the non-isothermal mixing and allow the multiple exchanger matches on each of branch stream. To provide good initial values for simultaneous retrofit of HENs, a two-step strategy is implied. The effective model formulations and effective strategy are applied to CDU case-study.

Figure 4.13 presents the original HEN for the CDU from Siemanond and Kosol (2012) consisting of 13 streams (10 hot and 3 cold process streams) and 18 exchangers (6 process exchangers, 3 hot utility exchangers and 9 cold utility exchangers). The original HEN uses two types of hot utility and three types of cold utility. Branch stream does not exist in the original HEN. Cost, film coefficient, supply and target temperature for process stream and utility are shown in Table 4.5 and 4.6. The project life is 5 years with 67,964 kW of hot utility and 75,051kW of cold utility consumption per year of original HEN. Modifications in the HEN account for new exchanger addition and area addition or reduction to existing exchangers. The limitation of additional and reduction area are 10% and 40% of existing exchanger area for all exchanger, respectively, except the two exchangers (HX5 and HX12). The limitation values of additional and removed area of H5-C1 match for both HX5 and HX12 are 20% and 30%, respectively. The maximum area per shell is 5,000 m^2 and the maximum number of shells per exchanger is 4. The fixed cost of branch streams is \$20,000 per branch. Equations (4.87), (4.88), (4.89) and (4.90) calculate the total cost of new exchanger, area reduction, area addition, and new shells made to existing exchangers, respectively.

$$\text{Exchanger cost (\$)} = 26{,}460 + [389\times\text{Area}\,(m^2)] \quad (4.87)$$

$$\text{Additional area cost (\$)} = 13{,}230 + [389\times\text{Area}^{add}(m^2)] \quad (4.88)$$

$$\text{Reduction area cost (\$)} = 13{,}230 + [0.5\times\text{Area}^{red}\,(m^2)] \quad (4.89)$$

$$\text{New shell cost (\$)} = 26{,}460 + [389\times\text{Areashell}\,(m^2)] \quad (4.90)$$

4.4.2.1 Solution Strategy for CDU Case Study

In retrofit design we can use the existing design as reference for evaluation. Comparing performance number of the new design and existing situation shows the improvement that is possible with the new design. The economy of a retrofit design is the trading-off achieved improvements and required investment. In energy saving retrofit the improvements are the operating cost savings which the utilities usage is the primary saving. The investment cost consists of the cost of new exchangers, additional area, repiping and splitting stream. Energy saving requires the investment in the HEN. From the economical point of view, the existing area must be reused as much as possible. The relocation is preferred over the installation of new exchanger. The systematic retrofit design methods tried to generate networks using synthesis design techniques, which are as close as possible to the existing design. The mathematical grassroots network generation method with some additional constraints is used to find new structures of HEN retrofit. The constraints are added to drive the generated network towards the existing structure. Energy saving retrofits require the addition of heat transfer area. This gives some flexibility in the arrangement of the existing area. It requires at least initialization stage prior to the solution of the actual optimization problem. To guideline the synthesis design, the number of exchanger is minimized to roughly estimate the required investment for HEN. During network improvement, only the tasks of the poorly performing exchangers are rearrangement by changing the splitting stream or the matched streams (relocation). To finalize the network, the area is added to or removed from to the existing exchanger and the new matches is assigned to the network to heat and cool process stream effectively.

The retrofit model is developed from the grass-roots model. The additional sets of constraints are added into the grassroots model to consider the network modifications that will allow a net reduction in the total annual cost. Therefore, the model consists of 2 sets of equations; the synthesis and retrofit equations. The objective of grassroots design is to minimize the total cost, which includes the utilities cost (i.e., operating cost) and the investment cost of the HEN. The goal for retrofit case is to maximize the heat integration among process streams or reduce utilities usage and therefore maximize the NPV calculated by the energy saving subtracted by the investment cost. Although the modified and extended Synheat model looks promising, the problem of the model occurs when it handles the effect of high non-convexities which prevent model from obtaining the feasible solution. This difficulty needs to be overcome by applying the effective initialization strategy. This strategy consists of two main steps; initialization and retrofitting steps as shown in Figure 4.10.The initialization steps are divided into two steps. The first initialization step is to find the minimum number of exchangers using the MILP solver. The initialized variables consist of continuous variables of heat load distribution, number of exchangers, calculated area and additional area. The second initialization step formulates the MINLP model to minimize total annual cost composing of capital and operational expenses. After initial HEN is provided, the retrofit step is done by using MINLP with the objective function of maximum NPV.

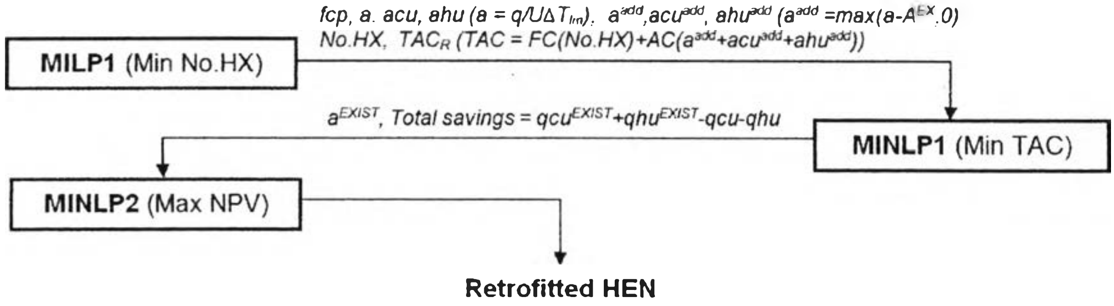

Figure 4.12 The HEN retrofit strategy.

From the work of Siemanond and Kosol (2012), they used pinch design method of Tjoe and Linnhoff (1987) in targeting step to optimize a HEN and followed by the n-stage model Grossmann and Zamora (1996) to design HEN at above and below pinch sections based on algorithm from Smith (1995). However, the retrofit constraints did not add to the stage model, and stream repiping did not occurred in the network. The model used in this work is MINLP and considers the modifications in the HEN including new exchanger addition, area addition or reduction to existing exchangers and relocation of heat exchangers. The objective of the MINLP model from this work is maximizing the net present value. The retrofit match and additional area for existing from Siemanond and Kosol (2012), displayed in Figure 4.13, uses 13 existing exchangers and requires 13 more new heat exchangers. The summarized result is shown in Table 4.7. The results of the retrofitted exchanger area compared to original exchanger area are summarized in Table 4.8. The retrofitted topology from this work is shown in Figure 4.14. The retrofitted HEN consists of 14 existing exchanger and 4 new exchangers added to the network (exchangers 19-22, highlighted by using a gray background). The total retrofitted area of process exchanger is 6,455.15 m^2. As the result of increased heat recovery by adding new exchangers or expanding existing areas that exchange heat between process streams, the usages of hot and cold utilities are decreased to 40,702.91 and 50,514.627kW from original case, respectively. The heat recovery improvement in the retrofitted network results in remarkable NPV: the hot and cold utilities usage are reduced by 40% and 33%, the energy savings is over $3.87 million per year, the NPV is $12,052,466. The NPV of Siemanond and Kosol (2012)'s work is $16,542,682; however, their work requires $2,180,230 more on total investment cost in exchanger area and new shell than this work. For practical reasons, such as limitation of investment cost or complex structure when installing new exchangers or repiping, their HEN, as depicted in Figure 4.14, requires 13 more new heat exchangers and more numbers of splitting. For this work, one splitting is introduced to the cold stream J1 and 4 new exchangers are used in our retrofitted HEN.

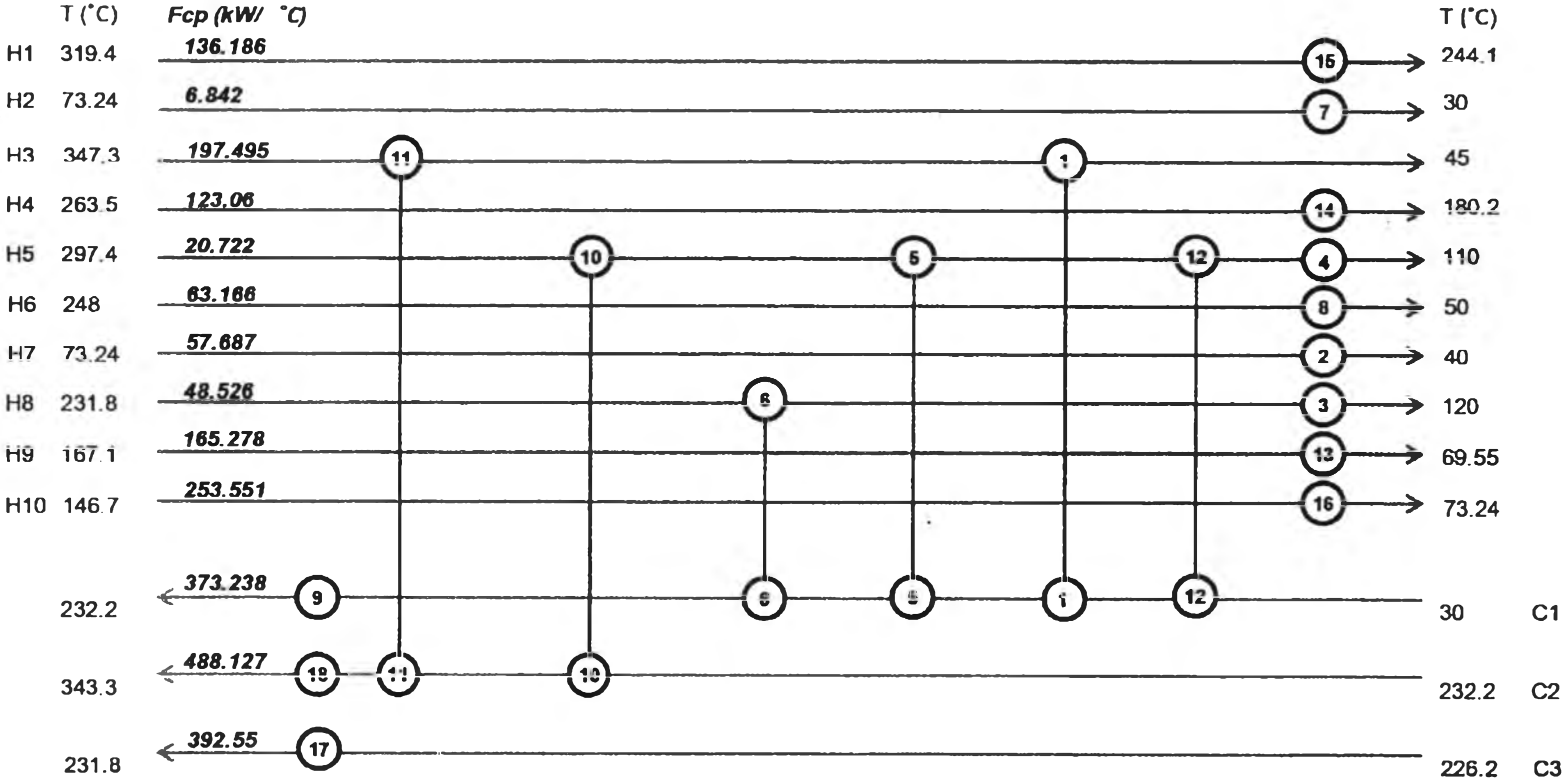

Figure 4.13 The grid diagram of the original HEN from Siemanond and Kosol (2012).

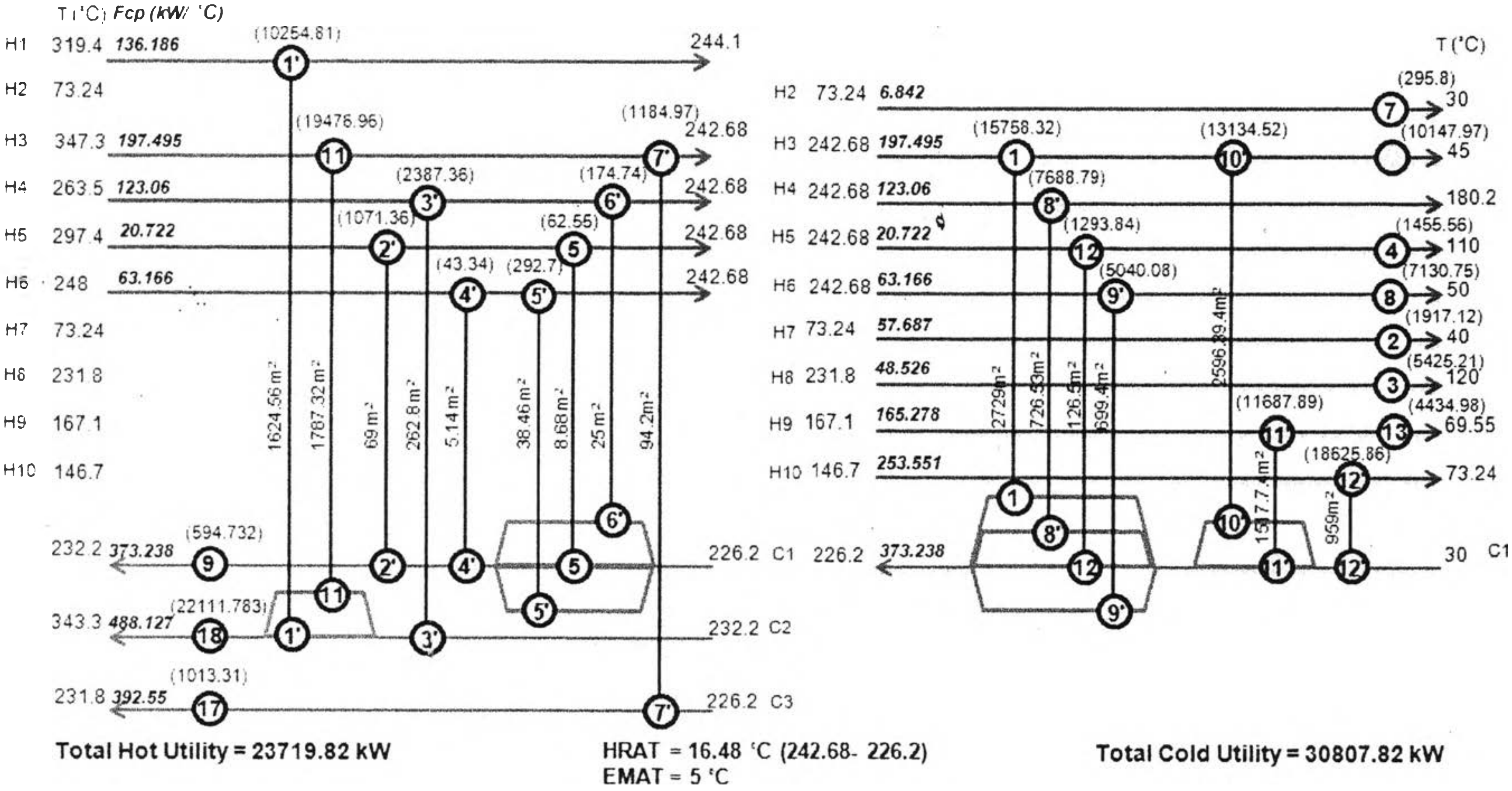

Figure 4.14 The grid diagram of retrofit case from Siemanond and Kosol (2012).

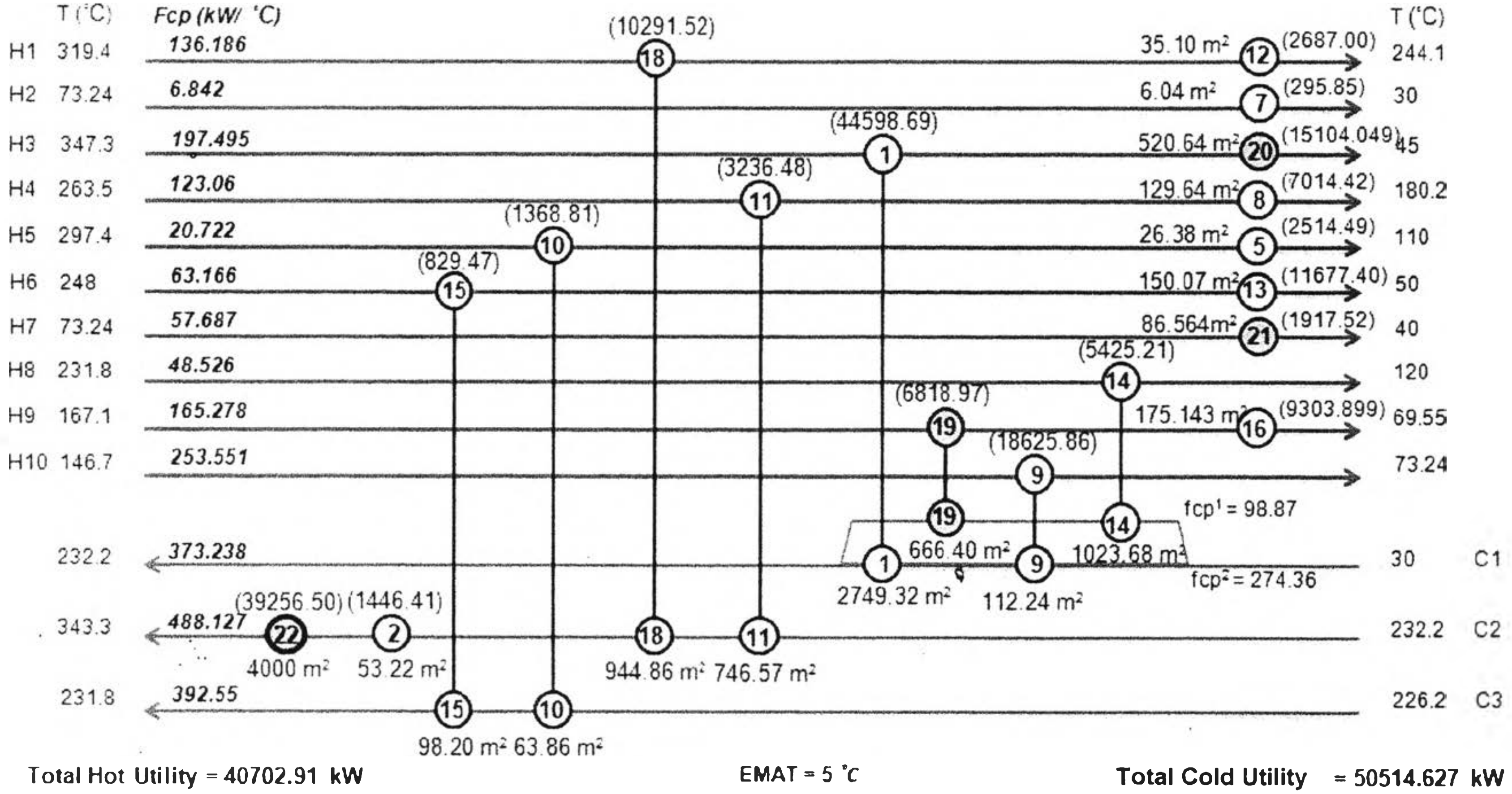

Figure 4.15 The grid diagram of our retrofitted HEN.

Table 4.6 Stream, cost and solution data for CDU case study

Stream	**Temperature**		**h** *(kW ℃$^{-1}$ m^{-2})*	**F** *(kW ℃$^{-1}$)*	**Cost** *(\$ kW^{-1} per year)*
	T_{in} (℃)	T_{out} (℃)			
Hot 1	319.4	244.1	1.293	136.186	-
Hot 2	73.24	30	5.063	6.842	-
Hot 3	347.3	45	0.7569	197.495	-
Hot 4	263.5	180.2	0.633	123.06	-
Hot 5	297.4	110	1.1995	20.722	-
Hot 6	248	50	1.2025	63.166	-
Hot 7	73.24	40	1.099	57.687	-
Hot 8	231.8	120	1.3714	48.526	-
Hot 9	167.1	69.55	1.3732	165.278	-
Hot 10	146.7	73.24	1.1729	253.551	-
Cold 1	30	373.238	0.5974	373.238	-
Cold 2	232.2	488.127	0.788	488.127	-
Cold 3	226.2	392.55	3.1902	392.55	-

Table 4.7 Stream, cost and solution data for CDU case study

Stream	**Temperature**		**h** *(kW ℃$^{-1}$ m^{-2})*	**F** *(kW ℃$^{-1}$)*	**Cost** *(\$ kW^{-1} per year)*
	T_{in} (℃)	T_{out} (℃)			
HU1	250	249	6	-	71.09
HU2	1000	500	0.1112	-	134
CU1	20	25	3.753	-	6.713
CU2	124	125	6	-	23.4
CU3	174	175	6	-	45.9

Table 4.8 Heat exchanger area from Siemanond and Kosol (2012)'s result.

HX No.	Match^{New} (Match$^{Exist,Base\ case}$)	A^{Exist} (m^2)	A^{new} (A$^{add/red}$) (m^2)
1	I3J1 (I3J1)	3,280	2,729 (-551)
2	I7CU1 (I7CU1)	62.6	62.6(0)
3	I8CU1 (I8CU1)	33.6	36.75 (+3.15)
4	I5CU1 (I5CU1)	4.08	14.3 (+10.22)
5	I5J1 (I5J1)	27.4	8.68 (-18.72)
6	*No match* (I8J1)	21.2	0 (-21.2)
7	I2CU1 (I2CU1)	5.63	5.65 (+0.02)
8	I6CU1 (I6CU1)	153	122.3 (-30.7)
9	J1HU (J1HU)	1,071	1,012 (-1,071)
10	*No match* (I5J2)	67.6	0 (-67.6)
11	I3J2 (I3J2)	688	1,787.317 (+1,099.317)
12	I1J1 (I5J1)	36	126.5 (+90.5)
13	I9CU1 (I9CU1)	182.57	74.4 (-108.17)
14	*No match* (I4CU2)	101.27	0 (-101.27)
15	*No match* (I1CU3)	93.8	0 (-93.8)
16	*No match* (I10CU1)	250.9	0 (-250.9)
17	J3HU1 (J3HU1)	51.7	26.24 (-25.46)
18	J2HU (J2HU)	942	588.25 (-353.75)
19 (New 1)	I1J2	-	1,624.56 (+1,624.56)
20 (New 2)	I5J1	-	69 (+69)
21 (New 3)	I4J2	-	262.8 (+262.8)
22 (New 4)	I6J1	-	5.14 (+5.14)
22 (New 5)	I6J1	-	38.46 (+38.46)
22 (New 6)	I4J1	-	25 (+25)
22 (New 7)	I3J1	-	94.2 (+94.2)
22 (New 8)	I4J1	-	726.53 (+726.53)
22 (New 9)	I6J1	-	699.4 (+699.4)
22 (New 10)	I3J1	-	2,596.39 (+2,596.39)
22 (New 11)	I9J1	-	1,517.7 (+1,517.7)
22 (New 12)	I10J1	-	959 (+959)
23 (New 13)	I3CU1	-	423.8 (+423.8)

Table 4.9 Heat exchangers from our retrofitted HEN result compared to base case

HX No.	$\text{Match}^{\text{New}}$ ($\text{Match}^{\text{Exist,Base case}}$)	A^{Exist} (m^2)	A^{new} ($A^{\text{add/red}}$) (m^2)
1	I3J1 (I3J1)	3,280	2,749.32 (-530.68)
2	J2HU2 (I7CU1)	62.6	53.22 (-9.38)
3	*No match* (I8CU1)	33.6	0 (-33.6)
4	*No match* (I5CU1)	4.08	0 (-4.08)
5	I5CU1 (I5J1)	27.4	26.38 (-1.02)
6	*No match* (I8J1)	21.2	0 (-21.2)
7	I2CU1 (I2CU1)	5.63	6.04 (+0.41)
8	I4CU1 (I6CU1)	153	129.64 (-23.36)
9	I10J1 (J1HU)	1,071	1,023.68 (-47.32)
10	I5J3 (I5J2)	67.6	63.86 (-3.74)
11	I4J2 (I3J2)	688	746.57 (+58.57)
12	I1CU1 (I5J1)	36	35.10 (-0.9)
13	I6CU1 (I9CU1)	182.57	150.07 (32.5)
14	I8J1 (I4CU2)	101.27	112.25 (+10.98)
15	I6J3 (I1CU3)	93.8	98.20 (+4.4)
16	I9CU1 (I10CU1)	250.9	175.14 (-75.76)
17	*No match* (J3HU1)	51.7	0 (-51.7)
18	I1J2 (J2HU)	942	944.87 (+2.87)
19 (New)	I9J1	-	666.40 (666.4)
20 (New)	I3CU1	-	520.64 (520.64)
21 (New)	I7CU1	-	86.56 (86.56)
22 (New)	J2HU1	-	4,000 (4,000)