Chapter 3

Mathematical model

In this chapter, two-fluid (Eulerian) model of gas-solid turbulence pipe flow is formulated. A set of Overall balance equation, Reynold-averaged conservation equation for the mass and momentum of both phases, gas kinetic energy of turbulent and its dissipation can be derived as follows ;

3.1 Overall balance equation

3.1a) Overall continuity equation

Gas phase overall continuity equation

$$\bar{\gamma}_g \bar{V}_{gz} = \frac{G_g}{\rho_g} \tag{3.1}$$

Solid phase overall continuity equation

$$\bar{\gamma}_p \bar{V}_{pz} = \frac{G_p}{\rho_p} \tag{3.2}$$

3.1b) Overall momentum equation

The mixture momentum equation assuming constant gas properties and wall friction is

$$\rho_g \frac{d\bar{\gamma}_g \bar{V}_{gz}^2}{dz} + \rho_p \frac{d\bar{\gamma}_p \bar{V}_{pz}^2}{dz} = -\frac{d\bar{P}}{dz} - (\bar{\gamma}_g \rho_g + \bar{\gamma}_p \rho_p)g - F_w \tag{3.3}$$

Using equation (3.1) and (3.2), eqn.(3.3) can be rewritten to give the pressure gradient in terms of the inertial, gravitational and wall friction forces. Thus

$$\frac{d\bar{P}}{dz} = (G_g \frac{d\bar{V}_{gz}}{dz} + G_p \frac{d\bar{V}_{pz}}{dz}) + (\bar{\gamma}_g \rho_g + \bar{\gamma}_p \rho_p)g + Fw \tag{3.4}$$

Base on incompressible flow assumption, gas mean velocity gradient along z direction can be said to be zero. Finally

$$\frac{d\overline{P}}{dz} = G_p \frac{d\overline{V}_p}{dz} + (\overline{\gamma}_g \rho_g + \overline{\gamma}_p \rho_p) g + F_w \tag{3.5}$$

Whilst $F_w = F_g + F_p$

Particulate wall friction force , F_p, can be evaluated from literature correlation but these correlation differ so widely in results (Littman et al. [1993]). Louge et al. [1991] show that particulate wall friction force in Tsuji et al. [1984]'s experiment did not exceed 8% of gas phase wall friction force, F_g, which can be calculated from the expression for turbulent gas flow without particle.

3.2 General and Reynolds-Averaged form of phase conservation equation

The first step of mathematical model formulation is to state the assumptions invoked in deriving the equations. These are :

(i) Both phases behave macroscopically as a continuum, but only the carrier fluid behaves microscopically as a continuum.

(ii) The dispersed phase consists of particles or droplets spherical in shape and uniform in size.

(iii) Dispersed phase is dilute and the two phase is homogeneously coexist everywhere, therefore, no phase separation regime occurs in flow system.

The instantaneous, volume averages conservation equations in Cartesian tensor notations are thus

The continuity equation for carrier (lighter) phase

$$\frac{\partial \Gamma_1 \rho_1}{\partial t} + \frac{\partial \Gamma_1 \rho_1 V_i}{\partial x_i} = 0 \tag{3.6}$$

The continuity equation for dispersed phase

$$\frac{\partial \Gamma_2 \rho_2}{\partial t} + \frac{\partial \Gamma_2 \rho_2 V_{2i}}{\partial x_i} = 0 \tag{3.7}$$

The global continuity equation

$$\Gamma_1 + \Gamma_2 = 1 \tag{3.8}$$

The instantaneous, volume-averaged momentum equations of carrier phase are

$$\frac{\partial \Gamma_1 \rho_1 V_{1i}}{\partial t} + \frac{\partial \Gamma_1 \rho_1 V_{1j} V_{1i}}{\partial x_j} = -\left(1 - K\Gamma_2\right)\frac{\partial P}{\partial x_i} - KF\gamma_2\left(V_{1i} - V_{2i}\right) + \frac{\partial}{\partial x_j}\left[\mu_1 \Gamma_1\left(\frac{\partial V_{1i}}{\partial x_j} + \frac{\partial V_{1j}}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu_1 \Gamma_1 \frac{\partial V_{1l}}{\partial x_l}\right) \tag{3.9}$$

The corresponding equations for the dispersed phase are

$$\frac{\partial \Gamma_2 \rho_2 V_{2i}}{\partial t} + \frac{\partial \Gamma_2 \rho_2 V_{2j} V_{2i}}{\partial x_j} = -\Gamma_2 \frac{\partial P}{\partial x_i} + \frac{\Gamma_2 \rho_2}{T}\left(V_{1i} - V_{2i}\right) + \frac{\partial}{\partial x_j}\left[\mu_2 \Gamma_2\left(\frac{\partial V_{2i}}{\partial x_j} + \frac{\partial V_{2j}}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu_2 \Gamma_2 \frac{\partial V_{2l}}{\partial x_l}\right) + g_i \Gamma_2\left(\rho_2 - \rho_1\right) + f \tag{3.10}$$

It should be noted that, K is the local effectiveness of momentum transfer from the dispersed phase to the fluid as discussed by Elgobashi & Abou-Arab [1983], Soo [1967]. K=1 for $|V_{1i}| > |V_{2i}|$, during acceleration of dispersed phase. When dispersed phase is decelerated, or $|V_{1i}| < |V_{2i}|$, $1 > K \geq 0$. This does not constitute a discontinuity in natural phenomena because K will change from 1 to 0 at $V_{1i} = V_{2i}$.

The instantaneous quantity V, Γ, P can be decomposed in to time mean quantities and its fluctuating component as follows

$$V = v + v' \quad ; \quad \Gamma = \gamma + \gamma' \quad ; \quad P = p + p' \tag{3.11a; b; c}$$

The above velocity represents for any phase in any direction.

Time mean quantities can be written as follows

$$v = \frac{1}{t}\int_{t=0}^{t} V dt \quad ; \quad \gamma = \frac{1}{t}\int_{t=0}^{t} \Gamma dt \quad ; \quad p = \frac{1}{t}\int_{t=0}^{t} P dt \tag{3.12a; b; c}$$

Time averaging value of fluctuating quantity must be zero;

$$\frac{1}{t}\int_{t=0}^{t} v' dt = 0 \quad ; \quad \frac{1}{t}\int_{t=0}^{t} \gamma' dt = 0 \quad ; \quad \frac{1}{t}\int_{t=0}^{t} p' dt = 0 \tag{3.13a; b; c}$$

Substitute right hand side expressions of 3.11a; b; c in instantaneous conservation equation (3.6) through (3.10) and apply time-averaging operator, the Reynolds-averaged conservation equations are obtained. The full form of equations up to triple correlation are presented in Elgobashi and Abou-Arab [1983]. In this study, Reynolds-averaged assumption are put forward as follows;

(i) γ_1 is independent of time so that $\gamma'_1 = 0$.

(ii) Triple correlations involving γ'_2 are negligible.

(iii) Time - averaged product between any components of dispersed phase volume fraction and fluctuating component of pressure is negligible.

Carrier phase mean continuity equation

$$\frac{\partial \gamma_1 \rho_1}{\partial t} + \frac{\partial \gamma_1 \rho_1 v_{1i}}{\partial x_i} + \frac{\partial \rho_1 \overline{\gamma'_1 v'_{1i}}}{\partial x_i} = 0 \tag{3.14}$$

Dispersed phase continuity equation

$$\frac{\partial \gamma_2 \rho_2}{\partial t} + \frac{\partial \gamma_2 \rho_2 v_{2i}}{\partial x_i} + \frac{\partial \rho_2 \overline{\gamma'_2 v'_{2i}}}{\partial x_i} = 0 \tag{3.15}$$

Carrier phase mean momentum equation

$$\frac{\partial\left(\gamma_1 \rho_1 v_{1i} + \rho_1 \overline{\gamma'_1 v'_{1i}}\right)}{\partial t} + \frac{\partial \rho_1 \gamma_1 v_{1j} v_{1i}}{\partial x_j} = -\left(1 - K\gamma_2\right)\frac{\partial p}{\partial x_i} - K\left[\frac{\gamma_2 \rho_2}{T}\left(v_{1i} - v_{2i}\right)\right]$$
$$+ \frac{\partial}{\partial x_j}\left[\mu \gamma_1 \left(\frac{\partial v_{1i}}{\partial x_j} + \frac{\partial v_{1j}}{\partial x_i}\right)\right] + \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu \gamma_1 \frac{\partial v_{1l}}{\partial x_l}\right) - \gamma_1 \rho_1 \overline{v'_{1i} v'_{1j}} \tag{3.16}$$

Dispersed phase mean momentum equations

$$\frac{\partial \gamma_2 \rho_2 v_{2i}}{\partial t} + \frac{\partial \gamma_2 \rho_2 v_{2j} v_{2i}}{\partial x_j} = \gamma_2 \frac{\partial p}{\partial xi} + \rho_2 \gamma_2 \frac{1}{T}\left(v_{1i} - v_{2i}\right) +$$
$$+ \frac{\partial}{\partial x_j}\left[\mu_2 \gamma_2 \left(\frac{\partial v_{2i}}{\partial x_j} + \frac{\partial v_{2j}}{\partial x_i}\right)\right] - \frac{2}{3}\frac{\partial}{\partial x_i}\left(\mu_2 \gamma_2 \frac{\partial v_{2l}}{\partial x_l}\right) + \frac{\partial}{\partial xj}\gamma_2 \rho_2 \overline{v'_{2i} v'_{2j}}$$
$$+ g_i \gamma_2 \left(\rho_2 - \rho_1\right) + f \tag{3.17}$$

Turbulent kinetic energy equations for carrier phase

The first step in the derivation of the equations of carrier phase turbulent kinetic energy $\left(k \equiv \overline{v'_{1i} v'_{1i}} / 2\right)$ and its dissipation rate $\left(\varepsilon \equiv \nu \overline{\frac{\partial v'_{1i}}{\partial x_k} \frac{\partial v'_{1i}}{\partial x_k}}\right)$ is to obtain transport equation for v'_{1i} by subtracting equation (3.16) from equation (3.9). The k-equation is produced by multiplying the v'_{1i} equation throughout by v'_{1i} then taking the time average of all terms. For turbulence gas-particle flow, the resulting equation reduce to eleven terms (Louge et.al [1991]) :

$$\underbrace{\frac{\partial \gamma_1 \rho_1 k}{\partial t}}_{\text{transient}} + \underbrace{\frac{\partial \gamma_1 \rho_1 k}{\partial x_j}}_{\text{convection}} = \underbrace{-\overline{v'_{1i} v'_{1j}} \frac{\partial \gamma_1 \rho_1 v_{1i}}{\partial x_j}}_{\text{production}}$$

$$+ \underbrace{\left\{ -\frac{\partial \gamma_1 \overline{p' v'_{1i}}}{\partial x_i} - \overline{p' v'_{1i}} \frac{\partial \gamma_1}{\partial x_i} - \gamma_1 \rho_1 \overline{v'_{1i} v'_{1j} \frac{\partial v'_{1i}}{\partial x_j}} \right\}}_{\text{turbulent diffusion}} \underbrace{- \gamma_1 \rho_1 v_{1i} \overline{v'_{1i} \frac{\partial v'_{1j}}{\partial x_j}}}_{\text{extra production}}$$

$$+ \underbrace{\left\{ \mu \gamma_1 \overline{v'_{1i} \frac{\partial}{\partial x_j} \left(\frac{\partial v'_{1i}}{\partial x_j} + \frac{\partial v'_{1j}}{\partial x_i} \right)} + \mu \frac{\partial \gamma_1}{\partial x_j} \overline{v'_{1i} \frac{\partial}{\partial x_j} \left(\frac{\partial v'_{1i}}{\partial x_j} + \frac{\partial v'_{1j}}{\partial x_i} \right)} \right\}}_{\text{viscous diffusion}}$$

$$+ \underbrace{\left\{ -\frac{2}{3} \mu \gamma_1 \overline{v'_{1i} \left(\frac{\partial^2 v'_{1i}}{\partial x_i \partial x_j} \right)} - \frac{2}{3} \mu \frac{\partial \gamma_1}{\partial x_i} \overline{v'_{1i} \left(\frac{\partial^2 v'_{1i}}{\partial x_i \partial x_j} \right)} \right\}}_{\text{viscous dissipation}} + \underbrace{\frac{\gamma_2 \rho_2}{T} \overline{v'_{1i} \left(v'_{2i} - v'_{1i} \right)}}_{\text{extra dissipation}} \qquad (3.18)$$

The corresponding ε - equation is obtained by differentiating the v'_{1i} equation with respect to x_k multiplying throughout by $\nu \frac{\partial v'_{1i}}{\partial x_k}$ and finally time averaging.

The full expression of ε - equation contains a multitude of new fluctuation correlation, none of which explicitly appear in any other transport equation and most of which are practically impossible to experimentally quantify. According to Elgobashi and Abou-Arab [1983] , the significant form of ε - equation is reduced to

$$\frac{\partial\gamma_1\rho_1\varepsilon}{\partial t}+\left\{\frac{\partial\gamma_1\rho_1 v_1\varepsilon}{\partial x_l}+\varepsilon\frac{\partial\gamma_1\rho_1 v_1}{\partial x_l}\right\}=\left\{-2\gamma_1\rho_1\frac{\partial}{\partial x_l}\overline{\left(v'_{1i}\frac{\partial v'_1}{\partial x_k}\frac{\partial v'_{1i}}{\partial x_k}\right)}+\text{extra prod.}\right\}$$

transient convection total production

$$-2\nu\gamma_1\rho_1\overline{v'_1\frac{\partial v'_{1i}}{\partial x_k}\frac{\partial v'_{1i}}{\partial x_l\partial x_k}}+\left\{2\nu^2\rho_1\overline{\frac{\partial v'_{1i}}{\partial x_k}\frac{\partial}{\partial x_k x_l}\left(\frac{\partial v'_{1i}}{\partial x_l}+\frac{\partial v'_{1l}}{\partial x_i}\right)}+\text{extra dissipation}\right\}$$

turbulent diffusion total dissipation

(3.19)

3.3 Modeling of conservation equation for the flow considered

In this study, a steady, turbulent, developing, dilute flow system of gas-massive particles in cylindrical vertical pipe is considered. Two dimensional cylindrical coordinate (r-z) is employed following the assumption that the flow is axisymetric and no centrifugal or corriolis motion involve in the flow. It is further assumed that radial velocity component is very small compare with axial velocity component. Carrier phase subscripts, $_1$, are all changed to $_g$, as well as dispersed phase, $_2$, are changed to $_p$. Mathematical modeling of the flow is put forward by establishing the specific assumption for the flow considered as follows;

(i) The flow is characterized as high Reynolds number flow, viscous diffusion and dissipation can be neglected.

(ii) Particle density is much higher than gas density $\rho_p >> \rho_g$

(iii) Particle volume fraction is much lesser than gas volume fraction, therefore, $\gamma_p << 1$ (dilute flow assumption).

3.3.1 Gas phase conservation equation

3.3.1a Gas phase continuity equation

From (3.14), the transient term is cut off following steady flow assumption. The gas phase fluctuating volume fraction term γ'_1 can be neglected since γ_1 is considered independent of time in this study. Gas phase continuity equation is written in cylindrical coordinate as follows;

$$\rho_g\gamma_g\frac{\partial(r v_{gz})}{\partial z}+\rho_g\gamma_g\frac{\partial(r v_{gr})}{\partial r}=0 \qquad (3.20)$$

3.3.1b Gas phase momentum equation

Applying high Reynolds number assumptions to equation (3.16), resulting equation in z-direction is

$$\rho_g v_{gz}\frac{\partial \gamma_g v_{gz}}{\partial z} + \rho_g v_{gr}\frac{\partial \gamma_g v_{gz}}{\partial r} = -\frac{\partial \gamma_g P}{\partial z} - \frac{\rho_g}{r}\frac{\partial\left(r\gamma_g \overline{v'_{gz}v'_{gr}}\right)}{\partial r} + \rho_p\gamma_p\left(\frac{1}{T}\left(v_{pz}-v_{gz}\right)\right) + (\rho_g\gamma_g)g \tag{3.21}$$

Where $$-\overline{v'_{gz}v'_{gr}} = \upsilon_t\frac{\partial v_{gz}}{\partial r} \quad , \quad \upsilon_t = C_\mu\frac{k^2}{\varepsilon} \tag{3.21a}$$

In this study , C_μ=0.09

3.3.1c Gas phase kinetic energy equation and dissipation rate equation

According to equation (3.18) and (3.19), all terms are modeled using correlation procedure of Elgobashi and Abou-Arab [1983] except extra dissipation terms of both equations for which various expressions from the other authors are employed as follows;

Turbulence kinetic energy equation

$$\underbrace{\left\{\rho_g\gamma_g v_{gz}\frac{\partial k}{\partial z} + \rho_g\gamma_g v_{gr}\frac{\partial k}{\partial r}\right\}}_{\text{convection}} = \underbrace{\frac{\rho_g\gamma_g}{r}\frac{\partial}{\partial r}\left(r\frac{\upsilon_t}{\sigma_k}\frac{\partial k}{\partial r}\right)}_{\text{diffusion}} - \underbrace{\rho_g\gamma_g\upsilon_t\left(\frac{\partial v_{gz}}{\partial r}\right)^2}_{\text{production}} \underbrace{-\rho_g\gamma_g\varepsilon}_{\text{dissipation}} \underbrace{-P_k}_{\text{extra dissipation}} \tag{3.22}$$

Rate of energy dissipation

$$\left\{\rho_g\gamma_g v_{gz}\frac{\partial\varepsilon}{\partial z}+\rho_g\gamma_g\, v_{gr}\frac{\partial\varepsilon}{\partial r}\right\} = \frac{\rho_g\gamma_g}{r}\frac{\partial}{\partial r}\left(r\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial r}\right) - c_{\varepsilon1}\rho_g\gamma_g\frac{\varepsilon}{k}\upsilon_t\left(\frac{\partial v_{gz}}{\partial r}\right)^2$$

convection diffusion total production

$$- c_{\varepsilon2}\rho_g\gamma_g\frac{\varepsilon}{k} - P_\varepsilon$$

dissipation extra dissipation

(3.23)

Where δ_k=1.0, δ_ε=1.3, $c_{\varepsilon1}$=1.44, $c_{\varepsilon2}$=1.92

The term P_k, extra dissipation term in turbulence kinetic energy equation and P_ε , the extra dissipation term in dissipation rate equation is produced by time averaging process of slip velocity term as can be observed from equation (3.18). Many researcher in turbulence two phase flow performed their mathematical modeling for these two terms to account for the effect of gas turbulence promotion or modulation due to the presence of particles in gas stream. If both terms are set to zero, standard k-ε model for single phase flow is obtained. The standard k-ε model and the other two model of Chen and Wood [1986] and Mostafa and Mongia [1988] are given as:

Standard k-ε Model ; $P_k = 0$

$P_\varepsilon = 0$

. Chen-Wood 's Model ; $P_k = -\left(2\rho_p\gamma_p\gamma_g\frac{k}{T_p}\right)\left(1-e^{-\left(.0825\frac{T_p}{T_e}\right)}\right)$

$P_\varepsilon = -2\rho_p\gamma_p\gamma_g\frac{\varepsilon}{T_p}$

Where $T_p = \dfrac{\rho_p\gamma_p\left(v_{pz}-v_{gz}\right)}{\left|F_D\right|}$, $F_D = \rho_p\gamma_p\dfrac{1}{T}\left(v_{pz}-v_{gz}\right)$

$T_e = 0.165\dfrac{k}{\varepsilon}$

Mostafa-Mongia's Model ; $P_k = -2\rho_p\gamma_p\gamma_g\dfrac{k}{\left(T_p+T_e\right)}$

$P_\varepsilon = -2\rho_p\gamma_p\gamma_g\dfrac{\varepsilon}{\left(T_p+T_e\right)}$

Where $T_p = \dfrac{\rho_p \gamma_p \left(v_{pz} - v_{gz}\right)}{\left|F_D\right|}$, $F_D = \rho_p \gamma_p \dfrac{1}{T}\left(v_{pz} - v_{gz}\right)$

$$T_e = 0.35\frac{k}{\varepsilon}$$

3.3.1d Gas phase boundary conditions

Because the momentum equation expressed above is based on high Reynolds number assumptions, it is not valid for viscous and buffer layers adjacent to the wall according to "no-slip" condition. Close to the solid walls, and some other interfaces, there are inevitably regions where the local Reynolds number of turbulence is so small that viscous effects predominate over turbulence one. Launder and Spalding [1974] presented wall-boundary mathematical treatment for computational purpose. Their model, "wall-function-method" is developed as follows;

Dimensionless distance from the pipe wall is defined as;

$$y^+ = \frac{\rho_g (R - r) v_g^*}{\mu} \tag{3.24}$$

The shear velocity $v_g^* = \sqrt{\dfrac{\gamma_g \tau}{\rho_g}}$; τ is gas shear stress (3.25)

The shear velocity at wall is calculated from global momentum balance obtained by adding the momentum balances of the gas and particle phases and integrating between r=0 and r=R (Louge et al. [1991]) :

$$\frac{\partial \gamma_g p}{\partial z} = -\frac{2\rho_g \left(v_g^*\right)_w}{R} + \frac{2S_0}{R} - \overline{\gamma_p \rho_p g} \tag{3.26}$$

Where S_0 is particle shear stress evaluated at the wall.

The gas velocity at $30 < y^+ < 130$ is given by "logarithmic law of the wall" :

$$\frac{v_g}{v_g^*} = \frac{1}{\kappa}\ln\left(E y^+\right) \tag{3.27}$$

Where κ is Von Karman 's constant , κ= 0.4 in this study. E is a function of wall roughness, approximately equal to 9.0 for a smooth wall.

Consider a computational point P, which locates in the distance (R-r) correspond to $30 < y^+ < 130$ and the second point W, locate on wall. The point P is selected for the reason that it is remote from point W enough so that the viscous effects are entirely overwhelmed by turbulent diffusivity effect. Assume that uniform shear stress prevails in the layer from W to P, and generation and dissipation of energy are in balance in this layer, then :

$$\gamma_g \frac{\tau}{\rho_g} = C_\mu^{1/2} k = \text{constant} \tag{3.28}$$

Substitute the relation (3.28) in (3.27) The fluxes of momentum to the wall are then supposed to obey the relations:

$$\frac{(v_g)_p}{(v_g^*)^2_w} C_\mu^{1/4} k_p^{1/2} = \frac{1}{\kappa} \ln\left[E \frac{y_p \rho_g (\gamma_g C_\mu^{1/2} k_p)^{1/2}}{\mu} \right] \tag{3.29}$$

The quantity k_P is supposed to be known. It should be calculated from the regular balance based on the assumption that diffusion of energy to the wall being equal to zero. When calculating k_p it is necessary to assign the average energy dissipation rate over control volume :

$$\int_0^{y_p} \varepsilon \, dy = C_\mu \frac{k_p^{3/2}}{\kappa} \ln\left[E y_p \rho_g \left(\sqrt{\gamma_g C_\mu^{1/2} k_p}\right)^{1/2} \right] \tag{3.30}$$

3.3.2 Particle phase equation

3.3.2a: Particle continuity equation

From equation 3.15 applying steady flow assumption

$$\rho_p \gamma_p \frac{\partial (r v_{pz})}{\partial z} + \frac{1}{r} \frac{\partial (r \gamma_p \rho_p v_{pr})}{\partial r} = -\frac{1}{r} \frac{\partial r \rho_p \overline{\gamma'_p v'_{pr}}}{\partial r} \tag{3.31}$$

Since $-\overline{\gamma'_p v'_{pr}} = D_t \frac{\partial \gamma_p}{\partial r}$, equation (3.31) then become;

$$\rho_p \gamma_p \frac{\partial (r v_{pz})}{\partial z} + \frac{\rho_p}{r} \frac{\partial (r \gamma_p v_{pr})}{\partial r} = \frac{\rho_p}{r} \frac{\partial}{\partial r}\left(\frac{D_t r \partial \gamma_p}{\partial r} \right) \tag{3.32}$$

Phase mass diffusivities D_t is defined as $D_t = \frac{\upsilon_t}{Sc_t}$ where the turbulent Schmidt number $Sc_t = 0.7$, based on turbulent mass transfer data.

3.3.2b Momentum equation

From equation 3.17 apply high Reynolds number assumption:

$$\rho_p v_{pz}\frac{\partial \gamma_p v_{pz}}{\partial z} + \rho_p v_{pr}\frac{\partial \gamma_p v_{pz}}{\partial r} = -\overline{\gamma_p' v_{pr}'}\frac{\partial v_{pz}}{\partial r} - \frac{\rho_p}{r}\frac{\partial\left(r\gamma_p \overline{v_{pz}' v_{pr}'}\right)}{\partial r} + \rho_p\gamma_p\left(\frac{1}{T}\left(v_{pz}-v_{gz}\right)\right) + (\rho_p\gamma_p)g \tag{3.33}$$

Where T is hydrodynamic relaxation time or time constant for momentum transfer from fluid to particle. T of a particle is defined in term of drag force on one particle by :

$$\frac{\rho_p}{T} = C_D\left|v_{gi} - v_{pi}\right|\frac{3\rho_g}{4d_p} \tag{3.34}$$

Empirical expression for Drag coefficient is

$$C_D = \frac{24}{Re_p}\left(1 + 0.15 Re_p^{0.687}\right) \quad ; \quad 0 < Re_p < 800 \tag{3.35}$$

Particle Reynolds number Re_p is define as

$$Re_p = \frac{\left|v_{gi} - v_{pi}\right|\rho_g d_p}{\mu} \tag{3.36}$$

The closure of particulate momentum equation (3.33) is obtained by the following correlation:

$$-\overline{v_{pz}' v_{pr}'} = \upsilon_p\frac{\partial v_{pz}}{\partial r} \quad , \quad \upsilon_p = \frac{\upsilon_t}{\sigma_p} \tag{3.37}$$

The work of Louge et al. [1991] presented numerical simulation based on kinetic granular theory .They compared their simulation result with Tsuji et al. [1984] experimental result . They also showed the result of particle phase shear stress term inclusive and exclusive calculation. They concluded that particle shear stress play an important role for accuracy of simulation of this type of flow. Their particle shear stress term in cylindrical coordinate

$$\frac{1}{r}\frac{d(rS)}{dr} \tag{3.38}$$

Define particle phase shear stress on surfaces at constant radius ;

$$S = \frac{5\pi^{\frac{1}{2}}}{96}\rho_p d_p \Theta^{\frac{1}{2}} \frac{dv_{pz}}{dr} \tag{3.39}$$

where Θ is granular temperature which can be expressed in term of r.m.s particle velocity fluctuation ;

$$\frac{3}{2}\Theta = \frac{1}{2}v_p'^2 \tag{3.40}$$

In order to include particle phase shear stress into momentum equation solved by our CFD code, we treat Louge et al. [1991] particle shear stress term equal to particulate phase diffusivity term described in equation (3.33) (first term in the right hand side of equation (3.33) which involve particle volume fraction gradient is negligible in fully developed flow).

$$\frac{1}{r}\frac{d(rS)}{dr} = \frac{\rho_p}{r}\frac{\partial\left(r\gamma_p \overline{v'_{pz} v'_{pr}}\right)}{\partial r} \tag{3.41}$$

Substituting S expression defined in (3.39) into (3.41)

$$\frac{1}{r}\frac{d\left(r\frac{5\pi^{\frac{1}{2}}}{96}\rho_p d_p \Theta^{\frac{1}{2}}\frac{dv_{pz}}{dr}\right)}{dr} = \frac{\rho_p}{r}\frac{d\left(r\gamma_p \frac{\upsilon_t}{\sigma_p}\frac{dv_{pz}}{dr}\right)}{dr} \tag{3.42}$$

Therefore ;

$$\frac{5\pi^{\frac{1}{2}}}{96}\rho_p d_p \Theta^{\frac{1}{2}} = \gamma_p \frac{\upsilon_t}{\sigma_p} \tag{3.43}$$

Insert υ_t and Θ expressed in (3.21a) and (3.40) in (3.43)

$$\frac{5}{96}\pi^{\frac{1}{2}} d_p \sqrt{\frac{1}{3}}v_p' = \gamma_p \frac{k^2}{\varepsilon\sigma_p} \tag{3.44}$$

Consider dimensional similarity, we have

$$d_p v_p' = \gamma_p \frac{k^2}{\varepsilon} \tag{3.45}$$

Finally

$$\sigma_p = 1.697$$

3.3.2c Particle phase boundary conditions

For particle-wall boundary conditions, we assume slip boundary condition so that particle can slip along the pipe wall at the velocity solved by governing equation after gas phase equation at and nearby wall has been solved and substituted. The inelastic collisions between particles and wall is also assumed. These assumptions are put forward for the main reason that the particles considered in this study, the detergent powders, are relatively soft and brittle so that particle-wall restitution force is considered minor. The other reason is that even in glass bead or polystyrene particle flow, the negligible or assumed zero particle-wall interaction force is reasonable as discussed and reported by Littman et al. [1993]. However, it should be noted that this particle-wall boundary condition treatment is limited and applied only for the flow studied.

The boundary condition at inlet of particulate phase velocity is dependent and coupled with gas phase velocity, it can be calculated from the following mathematical treatment .

Let us assume that a particle enters into gas stream below pipe inlet in various direction, say, cross flow or free falling. It enters in main gas stream and is then accelerated by drag force toward pipe inlet. This is true for spouted bed feeding. In experiment set up by Tsuji et.al, vertical pipe inlet was abruptly reduced from 40 mm diameter pipe bend to 30.5 mm diameter, which we can assume well-mixing occur. Therefore the above spouted bed feeding assumptions are satisfied.

According to Spouted bed feeding assumptions, inlet particle velocity can be calculated using the correlation proposed by Littman et.al [1993] ;

$$F_D(0) = 0.923 \frac{dP(0)}{dZ} - 21.23 \tag{3.46}$$

Whereby (0) denoted pipe inlet condition.

By definition;

$$F_D = \left[C_D \frac{\pi}{8} d_p^2 \rho_g (\bar{V}_g - \bar{V}_p)^2\right]\left(\gamma_p \frac{6}{\pi d_p^3}\right) \quad (3.47)$$

Define Solid loading ratio ; $m = \dfrac{G_p}{G_g}$

From Overall material balance, assume $\gamma_g \cong 1$

Thus;
$$\gamma_p = m \frac{\rho_g \bar{V}_{gz}}{\rho_p \bar{V}_{pz}} \quad (3.48)$$

Within feeding and pipe inlet area, Particle Reynolds number fall within transition region, therefore, from standard drag curve equation (3.35);

$$C_D = \frac{24}{Re_p}\left(1 + 0.15 Re_p^{0.687}\right), \; 0 \le Re_p \le 800$$

Substitute γ_p and C_D into (3.47)

$$F_D(0) = \frac{16\mu}{d_p^2}\left[1 + 0.15\left(\frac{\rho_g d_p \bar{V}_{Slip}}{\mu}\right)^{0.687}\right]\bar{V}_{Slip} m \frac{\rho_g}{\rho_p}\frac{\bar{V}gz}{\bar{V}pz} \quad (3.49)$$

Where $\bar{V}_{Slip} = \left(\bar{V}_{gz} - \bar{V}_{pz}\right)$

Inlet mean particulate phase velocity can now be solved from (3.5), (3.46) and(3.49) numerically .

The above procedure can be calculated once axial pressure gradient at the point of inlet is known.

However, if axial pressure gradient is unknown, but the distance between the point that particle enter into main gas stream and the pipe inlet is known. Particle velocity at pipe inlet can be calculated using method described by Morsi and Alexander [1972] as follows;

When a cloud of particles is introduced into free boundary gas stream and it is assumed that there is no particle interaction and further that the presence of the particles does not change the flow patterns, the resulting equation of motion of single particle is

$$m_p \frac{dv_p}{dt} = C_D \frac{1}{2} \rho_g \left(v_g - v_p\right)^2 A_p \tag{3.50}$$

The mass of spherical particle is given by:

$$m_p = \frac{1}{6} \pi d_p^3 \rho_p \tag{3.51}$$

The drag coefficient can be approximated by an equation of the form

$$C_D = \frac{K_1}{\mathrm{Re}_p} + \frac{K_2}{\mathrm{Re}_p^2} + K_3 \tag{3.52}$$

K_1=29.1667 , K_2= -3.889 , K_3=1.222 for $1.0<Re_p<10.0$

K_1=46.50 , K_2= -116.67, K_3=0.6167 for $10.0<Re_p<100.0$

K_1=98.33 , K2= -2778 , K_3=0.3644 for $100.0<Re_p<1000.0$

For constant v_g, equation (3.50) can be written in the form.

$$\frac{dv_p}{\left(v_p - \eta_1\right)\left(v_p - \eta_2\right)} = A_1\,dt \tag{3.53}$$

The root of equation η_1,η_2 are given by

$$\eta_{1,2} = \frac{\phi_2}{2A_1} \pm \left[\left(\frac{\phi_2}{2A_1}\right)^2 - \frac{\phi_1}{A_1}\right]^{\frac{1}{2}}$$

Where:

$$\phi_1 = \frac{3\mu K_1 v_g}{4\rho_p d_p^2} + \frac{3\mu^2 K_2}{4\rho_g \rho_p d_p^3} + \frac{3\rho_g K_3 v_g^2}{4\rho_p d_p}$$

$$\phi_2 = \frac{3\mu K_1}{4\rho_p d_p^2} + \frac{3\rho_g K_3 v_g}{2\rho_p d_p}$$

$$A_1 = \frac{3\rho_g K_3}{4\rho_p d_p}$$

Equation (3.53) can be written in term of traveling distance x, in the following form:

$$v_p \frac{dv_p}{dx} = \phi_1 - \phi_2 v_p + A_1 v_p^2 \tag{3.54}$$

The solution of equation is

$$x = x_0 + \frac{1}{A_1}\left\{ \ln\left(\frac{v_p - \eta_1}{v_{p0} - \eta_1}\right) + \frac{\eta_1 (v_{p0} - v_p)}{(v_p - \eta_1)(v_{p0} - \eta_1)} \right\} \tag{3.55a}$$

for $\eta_1 = \eta_2$

$$x = x_0 + \frac{1}{A_1}\left\{ \frac{\eta_2}{\eta_2 - \eta_1} \ln\left(\frac{v_p - \eta_2}{v_{p0} - \eta_2}\right) + \frac{\eta_1}{\eta_2 - \eta_1} \ln\left(\frac{v_p - \eta_1}{v_{p0} - \eta_1}\right) \right\} \tag{3.55b}$$

for $\eta_1 \neq \eta_2$

If traveling distance from the point where particle entering in gas stream to pipe inlet (x-x_0) is known and v_{p0} at entering point is set to zero (this is reasonable for free stream or spouted bed feeding as particle enters in gas stream in almost radial and downward direction), v_p, inlet particle velocity can then be calculated from equation (3.53).