Chapter IV.

Methodology and Model

The Vector Autoregression (VAR) methodology has a structural approach to equation modeling and uses econometrics to describe the relationships between several variables of interest.

The VAR methodology has a special characteristic that is appropriate to the economic theory. In general, economic theory is not rich enough to provide a strong specification of the dynamic relationship among variables. This is due to the nature of the economic system that makes it hard to define relation and effect among variables. In other words, it is almost impossible to define which variable is the endogenous variable and which variable is the exogenous variable. Sometimes the variable that is called the exogenous variable has an influence on the variable that is known as endogenous variable and the endogenous variable then has a feedback influence over the exogenous again. So, it is difficult to tell which one is an endogenous or exogenous variable. The endogenous may appear on both sides of the equation. In VAR methodology, analysis does not need to know their relation in order to estimate or test the empirical relevance of the theory because the VAR methodology treats all variables symmetrically without making reference to the issue of dependence versus independence. This is known as the non-structural approach to modeling the relationship between several variables. The VAR is commonly used for forecasting systems of interrelated time series and for analyzing the dynamic impact of random disturbances on the system of variables.

Using the VAR methodology to study the FRU has an advantage over other methodology from its ability to forecast over multiple periods.

The test has to be performed under the two assumptions of Rational Expectation and Risk Neutrality.

1. Rational Expectation

Rational Expectation is that investors or agents are able to take best advantage of news or information that they receive. They will not hesitate to acquire any information related to the variable that they want to forecast, such as past information of that variable or other related variables. Therefore, forecast error will not be continuous. However, this does not mean that there is no forecast error. There might be forecast error but investors or agents will quickly learn from the error and will not apply this knowledge to their next forecast. Thus, the forecasts of interested variable will most of the time be correct on average.

In a case of foreign exchange market, investors or agents will be able to utilize all news and information to help them make a decision to buy or sell currencies without mistake.

2. Risk Neutrality

Risk Neutrality is that investors or agents do not desire for higher risk in order to gain much more return or profit or by definition the risk neutrality means the market agents required no risk premium to persuade them to undertake risky transactions. This means they are willing to speculate on their judgement for no reward, or at least to the point where their reward is negligible. On the other hand, a risk averse investor would expect to receive a higher return as compensation for investing in a currency of a high risk country.

The simple case of VAR methodology is the case of two variables. The structure of the VAR model is that the equation has to model every endogenous variable as a function of the lagged values of all of the endogenous variables in the system. Thus, The forward rate unbiasedness condition in the simple form of bivariate structural VAR can be written as following:

$$\Delta S_{t+1} = a_{11}\Delta S_t + a_{12}fp_t + w_{1t+1} \tag{1}$$

Where;

ΔS_{t+1}	is change in spot rate at period t+1
ΔS_t	is change in spot rate at period t
fp_t	is forward premium at period t where fpt = (f-s)t
w_{1t+1}	is white noise

The hypothesis under the risk neutrality and rational expectation is that the change in spot rate at time t+1 is equal to the forward premium. So,

$$H_0\text{: } a_{11} = 0 \text{ and } a_{12} = 1 \tag{2}$$

Under the above hypothesis, the equation (1) reduces to be written in a form of unbiasedness condition for the forward rate by taking expectation as following:

$$E_t\Delta S_{t+1} = (f - S)_t \tag{3}$$

The forecast error based on VAR can be calculated by subtracting $E_t\Delta S_{t+1}$ from both sides of the equation (1) to get the following equation.

$$\Delta S_{t+1} - E_t\Delta S_{t+1} = a_{11}\Delta S_t + (a_{12} - 1)(f - S)_t + w_{1t+1} \tag{4}$$

Under the null hypothesis above $a_{11} = 0$ and $a_{12} = 1$, and the forecast error does not depend on ΔS_t and fp_t. If the null hypothesis is substituted into the equation, the forecast error will be equal to w_{1t+1}.

For two-periods ahead, the unbiasedness condition for the forward rate is

$$E_t S_{t+2} - S_t = E_t \Delta_2 S_{t+2} = fp_t \tag{5}$$

The equation (3) is the unbiasedness condition of the forward rate one period ahead. Thus we can note that the identity to forecast two-periods ahead is

$$E_t \Delta_2 S_{t+2} = E_t \Delta S_{t+2} + E_t \Delta S_{t+1} \tag{6}$$

From the equation (1), we need to know a forecast of $E_t(fp_{t+1})$ in order to be able to forecast $E_t \Delta S_{t+2}$. Therefore, we require an equation to determine fp_t, which can be written by using the VAR approach as:

$$fp_{t+1} = a_{21} \Delta S_t + a_{22} fp_t + w_{2t+1} \tag{7}$$

The equation (1) and (7) are a simple bivariate Vector Autoregressive (VAR), S_t and f_t are stationary at first difference (I(1)). However, in our model S_t and f_t have a cointegrating parameter (1,-1). Hence, $fp_t = f_t - S_t$ is stationary at the level (I(0)). Thus we can conclude that all variables in VAR are stationary.

The VAR that we use to test for FRU imply a set of non-linear restrictions among the parameters (aij), which can be illustrated by using equation (1), (6) and (7).

$$E_t\Delta_2 S_{t+2} = E_t\Delta S_{t+2} + E_t\Delta S_{t+1}$$
$$= a_{11}(a_{11}\Delta S_t + a_{12}fp_t) + a_{12}(a_{21}\Delta S_t + a_{22}fp_t) + (a_{11}\Delta S_t + a_{12}fp_t)$$
$$= \theta_1\Delta S_t + \theta_2 Efp_t = fp_t \qquad (8)$$

Where;

$$\theta_1 \quad = a_{11}a_{11} + a_{12}a_{21} + a_{11} \qquad (8a)$$
$$\theta_2 \quad = a_{11}a_{12} + a_{12}a_{22} + a_{12} \qquad (8b)$$

The equation (8) will hold for all values of ΔS_t and fp_t if:

$$\theta_1 = 0 \text{ and } \theta_2 = 1 \qquad (8c)$$

The forecast error of equation (8) between time t and time t+2 is

$$\Delta_2 S_{t+2} - E_t\Delta_2 S_{t+2} \qquad (9)$$

This equation will combine equation (5) and (8) into

$$\theta_1\Delta S_t + \theta_2 fp_t - fp_t + \eta_{t+1} \qquad (10)$$

Where;

η_{t+1} is dependent on w_{it+1}.

In general, the expected value of the forecast error will depend on ΔS_t and fp_t, but if θ_1 and θ_2 are restricted under FRU conditions that assume $\theta_1 = 0$ and $\theta_2 = 1$ the expected forecast error will not depend on ΔS_t and fp_t and the restriction is

$$a_{21} = -a_{11}(1+a_{11})/a_{12} \quad (11)$$

$$a_{22} = (1-a_{12}(1+a_{11}))/a_{12} \quad (12)$$

Therefore, substitute a_{11} and a_{12} into equation (7) can write the VAR restriction as:

$$\Delta S_{t+1} = a_{11}\Delta S_t + a_{12}fp_t \quad + w_{it+1} \quad (13)$$

$$fp_{t+1} = (a_{11}(1+a_{11})/a_{12})\Delta S_t + (1-a_{12}(1+a_{11})/a_{12})fp_t + w_{2t+1} \quad (14)$$

Comparing the differential log-likelihood can compare the log-likelihood value from the restricted system of equation (13) and (14) with the unrestricted system of equation (1) and (7). If the different is large (small) then the restrictions are rejected (not rejected).

The estimation of a non-linear system is complicated when the estimation has a large number of lags such as in the multiperiod FRU. Hence, we can represent it in matrix form to demonstrate a computationally simpler procedure to estimate the unrestricted (linear in parameters) VAR.

The matrix form of the unrestricted VAR with log length (p=1) is

$$z_{t+1} = Az_t + w_{t+1} \quad (15)$$

Where;

$z_{t+1} = (\Delta S_{t+1}, fp_{t+1})$

$A = \{aij\} \quad (2\times2)$

Suppose that $e1' = (1,0)$ and $e2' = (0,1)$ then

$$\Delta S_{t+1} = e1'z_t \tag{16}$$

$$fp_{t+1} = e2'zt \tag{17}$$

These two assumptions made the expected value of equation (15) and (8) equal to

$$E_t z_{t+2} = E_t A z_{t+1} = A^2 z_t \tag{18}$$

$$E_t(\Delta_2 S_{t+1}) = E_t(\Delta S_{t+2} + \Delta S_{t+1}) = e1'(A^2 + A)z_t \tag{19}$$

From the equation (16), (17), (18) and (19) FRU hypothesis in equation (5) can be rewritten as

$$e1'(A^2 + A)z_t = e2'z_t \tag{20}$$

$$e2' - e1'(A^2 + A) = 0 \tag{21}$$

Where;

$$A^2 = \begin{bmatrix} a^2_{11} + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{21}a_{11} + a_{22}a_{21} & a_{21}a_{12} + a^2_{22} \end{bmatrix}$$

Forward prediction of ΔS_{t+m} for m period is given by

$$E_t \Delta_m S_{t+m} = \sum_{i=1}^{m} E_t \Delta s_{t+1} \tag{22}$$

This equation is similar to the equation (6) and that we can write the FRU for a m-period of forward rate $fp_t^{(m)}$

$$E_t \Delta_m S_{t+m} = \sum_{i=1}^{m} e1' A^i Z_t = fpt^{(m)} \tag{23}$$

Under the hypothesis

$$f(A) = e1' - e2' \sum_{i=1}^{m} A^{i} = 0 \tag{24}$$

The Uncovered Interest Parity (UIP) can be applied with the VAR in the same way as those in the case of FRU.

$$E_t \Delta_m S_{t+1} = E_t(S_{t+m} - S_t) = (r - r^*)_t \tag{25}$$

This equation is similar to equation (23), however, it should be obvious that the analysis for a VAR is ΔS_t and $(r_t - r_t^*)$ goes through in exactly the same fashion as for FRU. So, we can test the FRU and UIP simultaneously by consider the trivariate VAR where $Z'_{t+1} = (\Delta S_t, fp, r - r^*)_{t+1}$ under two set of restrictions.

The restriction of FRU

$$\sum_{i=1}^{m} e1' A^{i} - e2' = 0 \tag{26}$$

The restriction of UIP

$$\sum_{i=1}^{m} e1' A^{i} - e3' = 0 \tag{27}$$

Where the vectors eJ have unity in the J^{th} element and zeros elsewhere (J=1,2,3)

To illustrate 3X3 VAR system discuss above with lag length p=1, the VAR form equation can be written as:

$$\Delta S_t = a_{11}\Delta S_{t-1} + a_{12}fp_{t-1} + a_{13}d_{t-1} + w_{1t} \quad (28a)$$

$$fp_t = a_{21}\Delta S_{t-1} + a_{22}fp_{t-1} + a_{23}d_{t-1} + w_{2t} \quad (28b)$$

$$d_t = a_{31}\Delta S_{t-1} + a_{32}fp_{t-1} + a_{33}d_{t-1} + w_{2t} \quad (28c)$$

This thesis studies the parity among interest rate, exchange rate and forward premium before and after the change of the exchange rate regime on 2 July 1997 using the interbank rate and exchange rate of Baht per US dollar and the forward premium of Baht for dollar as representatives. For interest rate, we use the interbank rate as a representative because other interest rates are varied among bank policies, and it is less volatile when compare with other rates. For the exchange rate we use the exchange rate of Baht per US dollar due to the fact that the United State is the major trade partner of Thailand as represented in the weight of basket of currencies. Finally, for the forward premium we again use the forward rate of Baht per US dollar for the same reason. The forward premium has two types: export and import. In this study we average these two rates and test for one-period test, one month, and two-period test.

Data Source and Data Definition

Data Source

This study obtained data from:

- International financial statistic (IFS)
- Datastream
- Financial Institutions
- Bank of Thailand (BOT)
- Bangkok Bank Limited (BBL)

Data Definition

d_t	interest rate differential between domestic and foreign interest rate
f_t	the forward rate at period t delivery at t+1
fp_t	forward premium at period t where fpt = (f-s)t, calculated from forward rate minus spot rate delivery at t+1
r	domestic interest rate per month
r^*	foreign interest rate per month
$(r-r^*)_t$	interest rate differential per month
S	spot rate
S_t	spot rate at period t
ΔS_t	change in spot rate at period t
ΔS_{t+1}	change in spot rate at period t+1
E_tS_{t+1}	expected spot rate at period t+1
$E_t\Delta S_{t+1}$	expected value of spot rate at period t
$E(S_{t+m} - S_t)$	expected differential between spot rate at period t+m and t