การบำรุงรักษาเชิงป้องกันที่เหมาะสมที่สุดสำหรับอุปกรณ์และเครื่องมือเช่าซื้อ

นางสาวจารุมนต์ จาตุรนต์นที

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรดุษฎีบัณฑิต
สาขาวิชาวิศวกรรมอุตสาหการ ภาควิชาวิศวกรรมอุตสาหการ
คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
ปีการศึกษา 2547

ISBN 974-17-4352-1

OPTIMAL PREVENTIVE MAINTENANCE OF LEASED EQUIPMENT

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A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of a Doctor of Philosophy in Industrial Engineering
Department of Industrial Engineering
Faculty of Engineering
Chulalongkorn University
Academic Year 2004
ISBN 974-17-4352-1

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จารุมนต์ จาตุรนต์นที : การบำรุงรักษาเชิงป้องกันที่เหมาะสมที่สุดสำหรับอุปกรณ์และเครื่องมือเช่าซื้อ. (OPTIMAL PREVENTIVE MAINTENANCE OF LEASED EQUIPMENT) อาจารย์ที่ปรึกษา : ผศ.ดร.เหรียญ บุญดีสกุลโชค, อาจารย์ที่ปรึกษาร่วม : Professor Murthy D.N.P. 116 หน้า. ISBN: 974-17-4352-1.

ในการวิจัยนี้ ได้ทำการพัฒนาตัวแบบการบำรุงรักษา (preventive maintenance และ minimal repair) เพื่อประกันความเชื่อมั่นของเครื่องมือที่มีอัตราเสียสูงขึ้น และได้นำมาประยุกต์ใช้สำหรับเครื่องมือเช่าในมุมมองการตัดสินใจของผู้ให้เช่า ในธุรกิจการเช่าเครื่องมือนั้น ผู้ให้เช่ามักทำการบำรุงรักษาเครื่องมือให้แก่ผู้เช่า ซึ่งความสามารถในการทำงานของเครื่องมือ และค่าชดเชยความเสียหาย เมื่อเครื่องมือไม่สามารถทำงานได้ตามมาตราฐานที่กำหนด มีผลกระทบอย่างมากต่อต้นทุนรวมของผู้ให้เช่า ดังนั้น นโยบายการซ่อมบำรุงรักษาเชิงป้องกัน ที่ประหยัดที่สุดจึงต้องมีการประเมินค่าใช้จ่ายระหว่างค่าซ่อม พร้อมทั้งค่าชดเชยกับค่าบำรุงรักษาป้องกัน เพื่อหาค่าตัวแปรการตัดสินใจที่เหมาะสม ผู้วิจัยได้พิจารณาเครื่องมือให้เช่าโดยแบ่งเป็น 2 ประเภท ได้แก่ เครื่องมือใหม่ และ เครื่องมือที่ผ่านการใช้งานแล้ว โดยนโยบายของเครื่องมือใหม่ขึ้นอยู่กับ 3 ตัวแปรการตัดสินใจ ดังนี้

1. จำนวนครั้งการปฏิบัติการบำรุงรักษาเชิงป้องกันในช่วงระยะเวลาการเช่า
2. กำหนดการการปฏิบัติการบำรุงรักษาเชิงป้องกัน
3. ระดับการบำรุงรักษา

ส่วนเครื่องมือที่ผ่านการใช้งานแล้ว ผู้ให้เช่ามีทางเลือกที่จะปรับปรุงค่าความเชื่อมั่นของเครื่องมือโดย การเพิ่มขีดความสามารถที่เหมาะสมก่อนให้เช่าเครื่องมือ

ภาควิชา	วิศวกรรมอุตสาหการ	ลายมือชื่อนิสิต	…...………………………..
สาขาวิชา	วิศวกรรมอุตสาหการ	ลายมือชื่ออาจารย์ที่ปรึกษา……………………..	
ปีการศึกษา	2547	ลายมือชื่ออาจารย์ที่ปรึกษาร่วม………………..	

##4371803721: MAJOR INDUSTRIAL ENGINEERING.
KEY WORD: PREVENTIVE MAINTENANCE / MINIMAL REPAIR / OPTIMAL STRATEGY / LEASE EQUIPMENT / LEASE CONTRACT

MS.JARUMON JATURONNATEE: (OPTIMAL PREVENTIVE MAINTENANCE OF LEASED EQUIPMENT). THESIS ADVISOR: ASSISTANT PROFESSOR REIN BOONDISKULCHOK, THESIS CO-ADVISOR: PROFESSOR MUTHY D.N.P., 116 pp. ISBN: 974-17-4352-1.

A preventive maintenance model has been developed to assure reliability for increasing failure rate equipment, based on the minimization of total cost. Failures are modeled at the system level and by utilizing the black-box approach. When equipment fails, failures are rectified through minimal repair according to intensity function. The proposed model is applied to leased equipment from the lessor's perspective. It is the first model that incorporates maintenance issue with lease equipment.

For lease equipment, the lessor carries out the maintenance of the equipment. Performance terms included in the lease contract and penalty for not meeting the performance standard have a significant impact on the total costs for the lessor. This implies that optimal preventive maintenance policy must take the penalty cost together with corrective maintenance cost into account and properly trade against the preventive maintenance cost. The thesis deals with a preventive maintenance policy for new and used equipment lease. With new equipment, the policy is characterized by three parameters; (i) the number of preventive maintenance actions to be carried out over the lease period, (ii) the time instants for such actions, and (iii) the level of actions. With used equipment, the lessor has an additional option to improve the reliability through an upgrade action before leasing.

Department: Industrial Engineering	Student's signature…………………………….
Field of study: Industrial Engineering	Advisor's signature…...……………………….
Academic year: 2004	Co-advisor's signature…………………………

Acknowledgements

I would like to express my deep gratitude and appreciation to my advisor Assistant Professor Rein Boondiskulchok for his advice, guidance and support throughout the course of this work.

The time spent doing research under Professor Murthy D.N.P. as a co-advisor from University of Queensland has helped in improving me as a researcher and a person. I am deeply grateful to Professor Murthy for his continuous guidance, support, encouragement, and inspiration.

My sincere thanks are extended to Associated Professor Chuvej Chansa-ngavej, Assistant Professor Manop Reodecha, Assistant Professor Jittra Rukijkanpanich, and Dr. Fujiwara Okitsugu for serving on my committee and giving my valuable suggestions.

I am also grateful to Thailand's grant of Commission on Higher Education and Thammasart University for allowing me the time to pursue my studying as well as Department of Industrial Engineering, Chulalongkorn University and Department of Mechanical Engineering, University of Queensland for offering me tremendous supports regarding facilities and resources.

Another notable mentioned is my family and Alex Pongpech, they are thoroughly supportive over the years. With their loves, I complete this work.

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NOTATIONS

$F(t)$	failure distribution function
$f(t)$	failure density function associated with $F(t)$
$r(t)$	failure rate [hazard] function associated with $F(t)$
β	shape parameter of Weibull distribution function
α	scale parameter of Weibull distribution function
$\lambda_0(t)$	failure intensity function with no preventive maintenance [$= r(t)$]
$\lambda(t)$	failure intensity function with preventive maintenance actions
$\Lambda(t)$	cumulative failure intensity function [$= \int_0^t \lambda(x)dx$]
$N(t)$	number of failures over $[0,t)$
Y	time to repair
$G(y)$	repair time distribution function
$g(y)$	repair time density function [$= dG(y)/dy$]
L	lease period
A	equipment age
x	upgrade level
k	number of preventive maintenance actions over the lease period
t_j	time instant for j^{th} preventive maintenance action
δ_j	reduction in intensity function due to j^{th} preventive maintenance action
θ	set of parameters $\{k, t_j, \delta_j, 1 \le j \le k\}$
$C_p(\delta)$	cost of preventive maintenance action resulting in a reduction δ in intensity function
TC_p	total cost of preventive maintenance actions
a	fixed parameter for preventive maintenance cost function
b	variable parameter for preventive maintenance cost function
C_f	average cost of corrective maintenance action to rectify failure
TC_f	total cost of corrective maintenance actions

$C_u(x)$	cost of upgrading resulting in a reduction x in equipment's virtual age
τ	repair time limit [parameter of lease contract]
η	limit on number of failures [parameter of lease contract]
C_t	Penalty cost per unit time if repair not completed within τ [Penalty-1]
C_n	Penalty cost per failure if $N(t) > \eta$ [Penalty-2]
ϕ_1	total cost due to Penalty-1
ϕ_2	total cost due to Penalty-2
ϕ_3	total cost due to Penalty-1 and Penalty-2
J	total expected cost to the lessor

CHAPTER I
INTRODUCTION

1.1 Introduction

1.1.1 Background

Businesses need different types of equipment to produce products and services—for examples, machines for production in factory, trucks for transportation of goods in company and, X-ray machines for diagnostic purpose in hospital. Every kind of equipment is unreliable in the sense that it degrades with age and/or usage, and ultimately fails. Equipment failures have a significant impact on the business performance. Therefore, maintenance actions are used to control equipment degradation and failures (called preventive maintenance) and to restore failed equipment back to operational status (called corrective maintenance).

The approach to maintenance has changed significantly over the last one hundred years. Prior to 1940, maintenance was viewed as an unavoidable cost and the approach was mainly to use corrective maintenance actions to restore failed equipment to operational state. Subsequent to the Second World War, the emphasis changed to preventive maintenance to avoid failures. Many different models were developed (and still continue to be developed) to achieve a proper trade-off between the extra cost involved in preventive maintenance and the reductions in the corrective maintenance costs. Since 1970, there were new maintenance approaches introduced. The emphasis was no longer solely on the maintenance costs, but also included the impact of failures on the overall business performance. Many different approaches, such as Reliability Centered Maintenance (RCM), Total Productive Maintenance (TPM) and several others, evolved and gained wide acceptance in industry. Also, advances in sensor technology, better understanding of the physics of failure, and better monitoring and data collection systems allowed one to use condition based maintenance where the maintenance actions were determined by the state of the equipment.

Prior to 1970, equipment was mostly owned and maintained in-house with very little outsourcing of maintenance. From 1970, this began to change with increased outsourcing of maintenance for a variety of reasons. Two main reasons for the change were addressed as follows. First, the complexity of equipment increased dramatically (due to advances in material, computer and other technologies). Second, the maintenance of such equipment required expensive tools and equipment in accordance with highly trained maintenance staff.

Maintenance outsourcing raised several new strategic and operational issues for both the equipment owners and the service agents who provide the maintenance under a service contract. The literature on outsourcing of maintenance is very limited. Murthy and Yeung (1995), Murthy and Asgarizadeh (1998 and 1999), Asgarizadeh and Murthy (2000), and Murthy (2000) deal with some of the issues and references to earlier literature can be found in Asgarizadeh (1997).

Since 1990, there is trend towards leasing of equipment as opposed to purchasing the equipment and outsourcing the maintenance. There are many reasons to lease equipment (Fishbein, McCarry, and Dillon, 2000). The considerations in financial terms are increasing of cash flow, convenience, flexibility, dollar value, and tax benefits. Buying equipment involves considerable capital investment. This is because the cost of equipment has been increasing due to greater equipment sophistication resulting from technological advances. On the other hand, the technical considerations entail the maintenance options and the opportunity to transfer the cost of equipment upgrades to the lessor. Equipment leasing provides protection against the risk of the rapid technological obsolescence.

Companies have begun to implement strategies aimed at choosing equipment leasing over other finance options (buy or loan). It is an alternative mean of acquiring equipment and plays important role in today business. In the United States, leasing is a widely used business strategy, with 80 percent of all companies leasing some or all of their equipment. Equipment Leasing Association (ELA) conducted the survey in 2002 and found that 73 percent of small business lease equipment. The leasing industry continues increasing until the last quarter of year 2001 which has the economic downturn impact from 9/11 (see Figure 1.1). However, the Department of Commerce estimated that 30 percent of all business equipment will be leased which is worth of $208 and $218 billion in year 2003 and 2004 respectively.

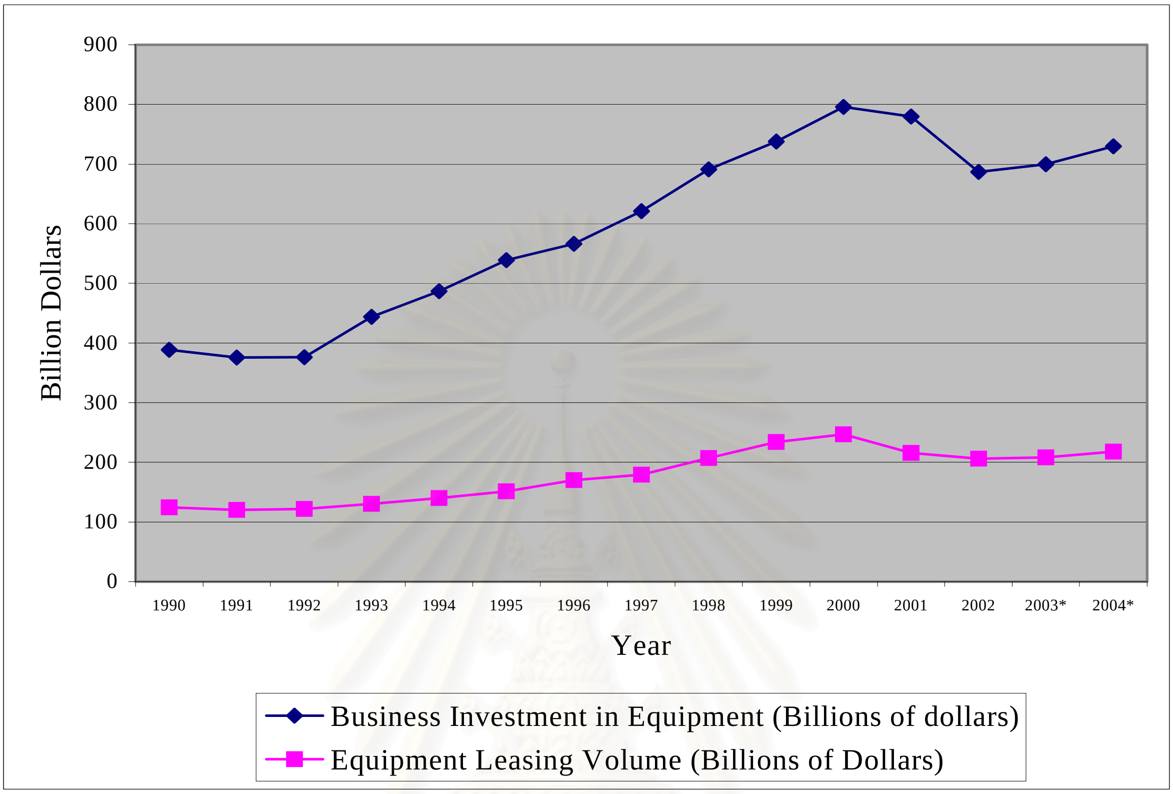

Figure 1.1 Trends and forecasts for equipment leasing in the U.S.

* Estimated volume

Source: Equipment leasing and financial foundation state of the industry report 2001-2003

Leased equipment is normally a complex and expensive product; hence maintenance becomes a major issue. For some systems, such as aircraft and automobiles, it is extremely important to avoid failure during actual operation because it is dangerous for human life and terrible lost in economy.

ELA Online Focus Groups Report (2002) also showed that 60 percent of leasing benefit comes from maintenance option. This is because lessees focus mostly on their core competencies. They are particularly interested in the leased equipment contract which also includes regular maintenance. For most commercial and industrial businesses, it is no longer economical to carry out the in house maintenance.

Additionally, as the equipment leasing continues to become commoditized, operational efficiency will become a competitive advantage. Lessor needs to focus on exploring opportunities to bundle the leasing equipment with other value-added service such as maintenance. It can be assumed that maintenance service for leased equipment is necessary to differentiate the competitors and to create customer royalty. As such, the equipment (a physical item) is bundled with maintenance (a service) and offered as a package to the lessee. Nevertheless, the lessor not only provides maintenance as a value-added service but do so in a very cost effective manner.

1.1.2 Problem Statement

In order to survive in today's market and to succeed in the future, the lessor must create cost effective and efficient leasing operation. One of significant problems of operational efficiency to manage the leased equipment during the economic downturn and the competitive era is to reduce cost of the lessor, particularly in the area maintenance. Since it helps business to be more productive and profitable which is critical to the business. For that reason, Lessor has to develop maintenance policies that could maintain lessee's requirement (equipment's reliability performance) as well as its own profitability. Thus, leased equipment needs suitable maintenance policies in order to improve requirement reliability, to prevent the occurrence of equipment failure, and to reduce maintenance costs.

The relationship between the lessor and the lessee is often characterized as one of competing interests. The lessor wishes to minimize the cost of maintenance while the lessee expects the excellent reliability performance of the leased equipment. Hence, in order to minimize the cost of the lessor in account of specified reliability performance, the lessor has to design the optimal maintenance policy for the leased equipment.

The optimal maintenance strategy is a lessor's important decision because it could bring about the lowest cost for the leased equipment. This decision has enormous strategic significant, as it influences the competitive position of the firm through cost and revenue implications for many years into the future. This is also even more considerable important when the impact of well-maintained equipment can increase lessee's satisfaction.

Maintenance policy is difficult to develop because of the complexity of assessing the effectiveness of maintenance as well as modeling the problem itself. This situation is further exacerbated by the fact that penalty included in the lease contract involves with the model. As the contract terms do not accomplish, the penalties occur to the lessor. Thus, the lessor has to consider not only maintenance actions but also impact of maintenance and penalty.

Technically, preventive maintenance involves additional costs and is worthwhile only if the benefits derived from it (such as lower corrective maintenance cost, greater customer satisfaction, higher yield etc) exceed the costs. Bulk of the literature dealing with optimal maintenance considers trade-off between corrective and preventive maintenance. A variety of objective functions have been proposed to obtain the optimal maintenance strategies. These include cost measures (such as expected cost over some interval, asymptotic cost per unit) and operational measures (such as availability, asymptotic availability) or combination of the two. Further details of the different models can be found in the review papers cited in Chapter 2.

The costs associated with failures of leased equipment are high for two reasons.

a) Since corrective maintenance actions are unplanned maintenance action, each of such action usually costs lot more than a planned preventive maintenance action.
b) The failures can result in penalty costs due to the reliability performance specified in the lease contract not being met.

This implies that the lessor's optimal preventive maintenance actions need to take both of rectification as well as penalty costs into account and properly trade against the cost of preventive maintenance actions to determine the optimal preventive maintenance strategy. Hence, the research questions are addressed as follows:

1. What is the optimal preventive maintenance strategy that minimises the total expected cost to the lessor?

2. How do penalty terms affect the total expected cost to the lessor and the optimal preventive maintenance strategy?

To the best of our knowledge, the study of maintenance of leased equipment has not received any attention from researchers in maintenance and there are no papers dealing with this topic. Hence, the problem of maintenance of leased equipment can serve as the first technical prototype for all problems connected with equipment support services in leasing industry.

1.2 Thesis objectives

The aim of the research is to develop model where the leased equipment is subjected to preventive maintenance actions to yield the optimal total expected cost over a lease period. The primary purpose of maintenance optimization model is to determine the optimum maintenance tasks that provide the most effective use of systems in order to secure the desired results at the lowest possible costs, taking all possible constraints into account. Therefore, the study was conducted to focus on two main objectives that are:

1. To investigate how preventive maintenance actions affect to the total expected cost of the lessor; and

2. To explore how penalties regard to equipment's reliability (repair time and failure number) affect to the total expected cost to the lessor and the optimal preventive maintenance strategy.

1.3 Thesis scope

We treat the maintenance of leased equipment on the lessor's perspective by minimizing the maintenance cost to the lessor. It is the most frequent used criteria for developing maintenance models. Specifically, we look at model in which the equipment has two states—working or failed.

The contract term includes two penalty terms regards to equipment's reliability. First penalty deals with a specified repair time completion for each failure.

Second penalty entails the guaranteed number of failures over a lease period. Both of penalties are crucial impact to the lessee’s business operation.

The model can be fitted with all leased equipment that has an increasing failure rate, therefore the scheduled preventive maintenance actions at anytime will improve reliability of the equipment.

We assume that minimal repair is carried out for failure and performed only at the time of failure. As a result, the failure rate of the equipment after rectification is the same as the rate just before failure. Preventive maintenance action is carried out over a lease period, and upgrading is considered as an option for used item.

1.4 Methodology

To achieve the objectives of this study, the method used for modeling preventive maintenance is to model the relationship among the preventive maintenance actions and two maintenance costs (corrective maintenance and preventive maintenance actions) including two penalty costs (repair time and failure numbers). The upgrade level and its cost also involve with the used equipment case.

The complexity of modeling preventive maintenance stems from the difficulty of quantifying the effect of performing preventive maintenance at different intervals. The effectiveness of preventive maintenance actions can be maximized by taking account of the time-to-failure distribution of the maintained items and of the failure rate function trend of the equipment. In order to optimize preventive maintenance actions, it is therefore necessary to know the followings:

- the time-to-failure distribution,
- the cost of failure,
- the cost of preventive maintenance action,
- the two penalty costs, and
- the upgrade cost.

Model analysis is undertaken to demonstrate that it is profitable to find the optimal number of preventive maintenance actions and its time instant in accordance with the optimal preventive maintenance effectiveness over a lease period for the new equipment. And, the optimal upgrade level is determined for the used equipment.

Model optimization is carried out to obtain the solution. Finally, numerical examples and sensitivity analysis are conducted to show how the parameters of the model affect to the solution.

1.5 Thesis outline

The outline of the thesis is as follows.

In Chapter 2, we first discuss an overview of equipment leasing. We highlight the different issues involved with the leased equipment and then focuses on the lease contract, the lessee's and the lessor's perspectives. Next, the basic concepts of reliability and maintenance are discussed in this chapter. We also carry out a review of literature on related works.

Through Chapter 3, it presents the knowledge needed for modeling and highlights issues of interest in later chapters. Furthermore, based on proposed maintenance policy, the general model is introduced here. We focus on the lease of new and used equipment.

Chapters 4 and 5 deal with the new and the used equipment lease respectively. We start with a discussion of the model formulation in detail. Following this, we carry out the model analysis to give a complete analytical characterization for the optimal strategy to the lessor. We also present the numerical examples for no penalty case as well as special cases. To investigate how significant parameters impact to the model, the sensitivity analysis is performed.

Finally, in Chapter 6, we give a brief summary of the work reported in the thesis and discuss extensions and topics for further study.

CHAPTER II
AN OVERVIEW OF EQUIPMENT LEASING AND MAINTENANCE SERVICE

2.1 Introduction

Business transactions in the product-service continuum can be classified into six varieties ranging from traditional to customized selling methods (Fishbein, McGarry, and Dillon, 2000). First transaction is sale of products. The traditional product sales mean that customers acquire equipment from the manufacturer in a one-time purchase. Purchaser owns equipment, and seller has no responsibility for service through the end-of-life. Second transaction refers to sale of products with/without service contract. This type of transaction offers service contract as an additional value-added service. Seller is paid to provide service for a specified period (often called “warranty”).

Third transaction deals with capital lease. Lessee pays lessor for the use of particular piece of equipment over a specified period. At lease end, lessee can assume ownership of equipment for a nominal price. Operating lease, also called “rental”, is the fourth transaction. Lessee/renter pays lessor for the use of a particular piece of equipment over a specified period. At lease end, lessor retains ownership of equipment, or on the other hand, lessee may purchase at fair market value. Lease with/without service contract is another business transaction. Any capital or operating lease during a lease period, lessor is responsible for equipment maintenance.

Next transaction is sale of functions. This means that the function of the equipment is being sold, not the equipment itself. Purchaser pays seller for use of equipment, repair and maintenance, supplies and staff training, and provides labor to operate equipment. Even though seller guarantees intended function of equipment but contract not tied to particular piece of equipment. Moreover, seller retains ownership at the end of contract.

Last transaction refers to sale of services. In this business transaction method purchaser pays seller based on the delivery of the desired end result of the service, e.g., clean floors. Purchase of services may also be called “outsourcing”. However, outsourcing is not restricted to the purchase of an end result, and can refer to any activity contracted to an outside party. Outsourcing can therefore perform at several places on the continuum. For example, under a lease with a service contract, the activity of cleaning floors would be carried out in-house, but the repair of the floor-cleaning machine would be outsourced to the lessor. Purchase of services refers strictly to the purchase of an end result. Also, seller retains control and ownership of equipment, supplies, etc., and provides labor to perform the service.

The relationships among different types of business transactions reveals that leasing falls in the midway between direct sales of equipment and sale of a service. In practice these distinctions are often fuzzy. Hence, a brief overview of equipment leasing as well as maintenance service are discussed in this chapter in order to make it more clear and be used as fundamental concepts for later chapters. In Section 2.2 we discuss the characterization of equipment leasing including leased equipment, contract, and the context of lessee's and lessor's perspectives. Following this, we give an introduction of equipment reliability and maintenance in Section 2.3. Literature review of related works is presented in Section 2.4.

2.2 Equipment leasing

Since 1990's technology has been changed rapidly and this will be also certainly continue into the next century (Blischke and Murthy, 2000). As a result, equipment is becoming more and more complex according to the use of new materials and new manufacturing methods. This often makes it more complicated to fix the failed equipment. For traditional action, failure of equipment was rectified by the owner. When the equipment has become more complex, the maintenance was carried out by the external agent. Then, in the technology era equipment management is changing to the aspect of leasing combined with maintenance service.

Economic uncertainties and ongoing technological process stress the significance of properly performing equipment management subject to technological change. High-tech equipment is especially becomes obsolete rapidly due to the advances in technological research. The new alternative of equipment management in leasing is generally with more flexible, less risk of ownership, less capitalizing and more controllable of cost and budget, etc. Regarding to technology change, high efficiency is necessary for competitive business. As a result, the equipment leasing has received considerable attention in the past couple decades.

Equipment leasing involves the loan of equipment owned by the lessor to the lessee under a lease contract. It is a contractual agreement between a lessor[1] and a lessee[2] (Robins, 1999; Kieso, Weygandt, and Warfield, 2001). The lessor is the equipment owner and charges for the leased item and service. The lessee is a user of the equipment for a period of time that is specified in the contract and in turn, the lessee pays the rental fee to the lessor. Leasing is usually considered for automobiles and aircrafts, or high technology equipment such as specialized medical devices, office machines and computers (PCs and Works stations).

In this chapter, we discuss major elements of equipment leasing in general. These four elements interact as shown in Figure 2.1.

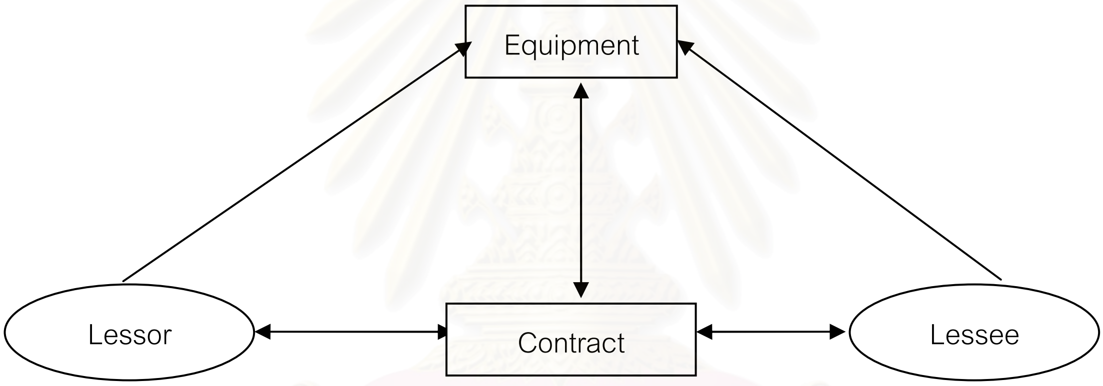

Figure 2.1 Conceptual model of equipment leasing

2.2.1 Leased equipment

2.2.1.1 Types of equipment

Equipment can be categorized into consumer, industrial and commercial equipments (Blischke and Murthy, 2000). Consumer equipments; for example, television sets, automobiles, PCs, are typified by a large number of society and commercial users. The technical complexity of the equipment varies from simple to very complex. Industrial and commercial equipments; for example, large-scale computers, medical equipment, office equipment, are characterized by a relatively

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[1] The term “lessor” may be used to refer to a manufacturer, an independent leasing company, or a manufacturer or a leasing subsidiary of manufacturer.

[2] The term “lessee” may be used to refer to an individual, a business or a government /an institution.

small number of consumers. The technological complexity and the mode of usage can vary noticeably.

2.2.1.2 Equipment characteristics

Leased equipment can be characterized as new and used items. For new item, its performance requirement is rather reliable. Nevertheless, the leasing price is more expensive than the used equipment lease. For used item, its reliability depends on the equipment age and maintenance history. The lessor may upgrade or overhaul the used equipment to improve its reliability. Accordingly, the leasing price is subjected to the upgrade level and equipment age.

2.2.1.3 Equipment degradation

As mentioned in Section 2.2, the complexity of the equipment has been increasing with technological advances. For more complex equipment, its reliability decreases as a result from deterioration and obsolescence. Most equipment is complex in the sense that they can be viewed as a system, which is comprised of several components. Equipment failures occur due to the failure of one or more of the components. Equipment fails due to deterioration resulting from age and/or usage and the failures occur in an uncertain manner. Failures mechanism can be classified into two categories (Dasgupta and Haslach, 1993). First is overstress failure due to brittle fracture, ductile fracture, yield, buckling, large elastic deformation and interfacial deadension. Second is wear-out failure due to wear, corrosion, dendritic growth, inter-diffusion, fatigue crack propagation, diffusion, radiation, fatigue crack initiation and creep. The deterioration can be controlled through preventive maintenance actions. When equipment fails, corrective maintenance actions are needed to restore the failed equipment to operational state. Many factors affect the failures. These include the decisions made during design and manufacturer as they affect the inherent reliability, usage mode and intensity and the operating environment.

2.2.2 Lease contract

There are three main issues in a lease contract. We start a discussion with contract terms and conditions followed by economic aspect of leasing and contract drafting so as to set the big scene of the topic of the thesis.

2.2.2.1 Contract terms and conditions

The first deals with the contract terms and conditions. These include the period of the lease, the performance requirements that the leased equipment should meet and the actions that each party (lessor and lessee) is obligated to take. The obligations of the parties should be explicitly and unambiguously stated. The performance requirements can involve several measures such as the upper limits on the number of failures over the lease period, the time between successive failures, the time to repair each failure and so on. When these are not met the lessor incurs a penalty as stated in the contract. In addition, it states the due care to be exercised by the lessee and the usage mode, which if violated results in lessee incurring penalty. In addition, to penalties the contract can also include incentives to ensure that both parties carry out their obligations.

2.2.2.2 Economic aspect of leasing

The second deals with the economic aspect of leasing. These include the amount that the lessee must pay the lessor (or also called the price) for the lease on equipment and the terms of payment. They can vary from a single payment to multiple payments over the lease period. The price is related to the performance requirements stated in the contract with higher price for more stringent performance requirement.

2.2.2.3 Contract drafting

Finally, the third deals with the drafting of the contract. The three different situations are as follows.

a) Drafted by the lessor (typical example being the lease of consumer durables). This corresponds to the case where there are several lessees and few lessors so that no one individual can influence a lessor in a significant manner. This type of contract is often referred to as a “standard contract”.

b) Drafted by the lessee (typical example being a business leasing a large number of similar equipment such as cars). This corresponds to the case where the lessee is the dominant player and can dictate the terms to the lessor. This type of contract is a “non-standard” contract.

c) Drafted jointly by both parties. This corresponds to the case where there are a few lessor and lessees and no one is dominant. As a result, each contract is drafted through a mutual agreement.

If the lessor drafts the contract, we define such contract as a standard contract. The content of the standard contract depends on the equipment type. On the other hand, if the lessee drafts the contract, we define such contract as a non-standard contract. The content of the non-standard contract is usually based on the needs of the lessee. For more complicated contract type, the terms and conditions are determined jointly by the both parties and they have to deal with contract negotiation.

It is important to note that the goals or objectives of the lessor and the lessee are different. As a result, one needs to understand the lessor and lessee perspectives.

2.2.3 Lessee

2.2.3.1 Lease contract classification

a) Standard Contract

For equipment leased under standard contract, the lessee decision is to choose the best lease arrangement from a set of alternate options offered by one or several lessors. The market can be either monopolistic (single lessor) or competitive (several lessors). In the latter case, the contract terms and conditions tend to be more competitive as long as there is no collusion between the different lessors. Figure 2.2 demonstrates the interactions between the four elements of equipment leasing under a standard contract.

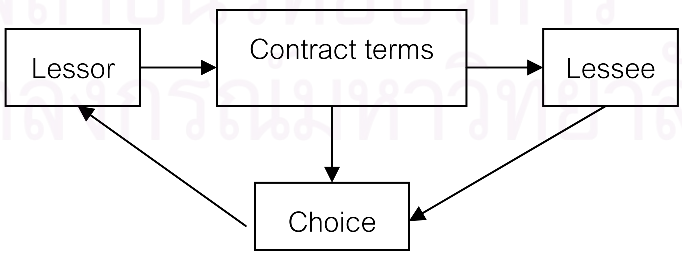

Figure 2.2 Elements of standard contract

Determining the best choice requires the lessee to formulate a suitable objective function for evaluation of different lease options. The objective function must take into account the impact of the contract terms and price on the lessee's business performance that can include variables such as cost, profit, sales, customer satisfaction etc.

b) Non-standard Contract

For equipment lease under a non-standard contract, the lessee states the performance requirements of the contract. The lessor decides on the contract price. As a result, the interactions between the main elements are as indicated in Figure 2.3.

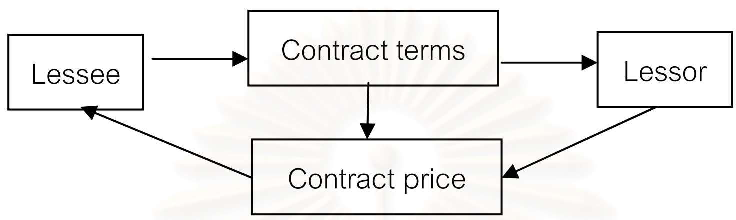

Figure 2.3 Elements of non-standard contract

Here the optimal decision of the lessee must take into account the price that can be viewed as a response function of the lessor to the terms stated by the lessee. As in the case of the standard contract, the objective function for making the final decision can include many other variables that are affected by the performance of the leased equipment.

2.2.3.2 Lease options

The lessor who can perform the business in the competitive world is required to provide varieties of service. In this thesis, we confine to bundle service between equipment leasing and maintenance. When a lease has ended, the lessee has several options as follows (Wijnands, 1986; Nisbet and Ward, 2001):

1. *Single lease option*—the lessee returns the equipment to the lessor with no further obligation.
2. *Replace option*—the lessee returns the equipment to the lessor and replace it with a newer one.
3. *Renew option*—the lessee renews the lease for an additional amount of time.
4. *Lease to purchase option*—the lessee buys the equipment outright as used equipment at its fair market value from the lessor.

2.2.3.3 Lessor selection

Vosicky (1992) suggested criteria for lessor selections as follows.

1. *Flexibility*—it is a criterion used to evaluate the lessor's ability to make changes in contract.
2. *Stability*—investigation of the lessor's financial status and years in business is an approach to ensure that it will remain in business for the duration of the lessee's contract term and beyond.
3. *Services*—the lessee should select the lessor that offers the types of value added services the lessee requires of such an important partner.
4. *Knowledge*—the lessee should ensure that the lessor will provide the competence service.

2.2.4 Lessor

The lessor is primarily interested in maximizing the expected profit through lease of equipment. For both the standard and non-standard contracts, this requires that the lessor take into account the lessee's response in deciding on the optimal strategy.

The lessor needs to make decisions at three dimensions—strategic, operational and other issues.

2.2.4.1 Strategic issues

At the strategic level the issues to be addressed are the following:

a) The numbers of lessee to serve

The lessor who sets only one service channel or outlet can provide service to one lessee or request at a time. When there is more than one lessee, they have to wait. This problem corresponds to the service time. As a result, the lessor deals with the scheduling problem. Waiting time, therefore, is a critical decision variable since if the lessees wait too long, they may switch to other leasing agents. On the other hand, the lessor may increase the number of service channels to achieve the lessee demand. Hence, the optimal number of service channels corresponds to service time and number of lessee to serve. There are several techniques used to set priority of lessee to gets service. For example, first come first serve, short service time first serve, critical request first serve, randomly serve, and etc.

b) The number and variety of equipment to stock for leasing

The number of equipment depends on demand of the lessee. Demand of service can be fixed or dynamic. The optimal number of equipment is one of strategic challenges. Such that, the service capacity is relevant to the lessor's capital fund.

The lessor may provide leasing equipment in identical or various types. The problem is simpler when the lessor provides in the same type of leasing equipment and is more complicated when the lessor provides in many choices of leasing equipment.

c) The infrastructure needed for leasing and servicing.

To provide maintenance service to very complex equipment, the sophisticated tools and equipment are crucial concerns for the service provider. This strategic issue associates with the number of lessee or requests as well as the variety of leased equipment.

d) The different contract options to offer in the case of standard contracts taking into account the varying requirements across the lessee population.

With the large number of lessee, their requirements vary so as to the lessor has to differentiate contract options. These might include performance requirement, payment option, period of contract, and, of course, contract price.

2.2.4.2 Operational issues

At the operational level, there are several issues to be addressed. These include the followings:

a) The pricing of the different lease options.

To determine the price of lease option, the lessor should clarify the total cost of service, which comprises of two main costs—leasing and maintenance costs. The leasing cost can be identified associated with the leasing fee—calculated from the fair market value of the subject equipment at the beginning of the lease term, the economic useful life and residual values at the end of the lease term. The anticipated maintenance cost comes from rectification costs of failures, preventive maintenance cost and penalty cost, if applicable. To obtain maintenance service cost, the lessor has to determine the lease term, the maintenance action plan and the expected number of failures. As part of appraisal analysis, the maintenance policy and the characteristics of leased equipment with its failures should be addressed. Moreover, the lessor needs to take into account factors such as competition, demand, etc.

b) The maintenance (corrective and preventive) strategies

The lessor can define many different types of maintenance strategies. Bulk of research had been proposed optimal maintenance policies. For further details, these can be found in Section 2.4.

c) The logistics needed for carrying out the maintenance.

These include spare part inventory management, scheduling of maintenance etc. The time to carry out corrective maintenance actions depends on the availability of repair crew and spare parts. This raises several issues such as the optimal inventory levels for spares, number of repair crews, etc. Large inventory and greater number of crews reduces the penalty cost but increase the inventory holding and operating costs. As a result, these must be selected optimally to achieve a proper trade-off.

2.2.4.3 Other issues

Both the strategic as well as the operational decisions must take into account the various forms of uncertainties. These include the usage intensity and environment varying across the lessee population, the varying level skills of operators using the equipment, demand for lease etc. Additional factors include issues such as moral hazard (for both lessor and lessee), adverse selection (for lessee due to limited information about the lessor and/or the equipment being leased) and competition (when there is more than one lessor).

A proper study requires a stochastic dynamic game-theoretic framework because of the uncertainties and the objectives of the lessor and lessee being different. In this research, we focus on the maintenance of leased item. This is an operational issue of great importance to the lessor as it not only affects the profits, but also the satisfaction of lessee and this in turn impacts on future business.

2.3 Equipment maintenance

Equipment is engineered and manufactured to perform in some specified manner when operated under normal operating conditions. However, it fails occasionally. When a failure occurs, it impacts the business operations. The occurrence of failure is uncertain. On the other hand, we can say that all equipment is unreliable in the sense that it will fail eventually. Therefore, maintenance is used to reduce the chance of occurrence o failures and also restore the failed item to a specified condition.

Hence, the maintenance becomes more important role in the new industrial and business environment as a key factor in organization efficiency and effectiveness. It also enhances the ability of organization to be competitive and meets its standard objectives (Jardine and Buzacott, 1985; Tomlingson, 1998). In this section we introduce the basic concepts of reliability and equipment failure. Furthermore, a general maintenance concept is also briefly discussed. Equipment reliability and maintenance serves as a foundation to suggest direction for the thesis problem.

2.3.1 Reliability

2.3.1.1 Reliability definition

According to theoretical meaning, reliability problem corresponds to the dependability concept, the successful performance, and the absence of failures. It requires a dynamic (changing with time) and probabilistic (stochastic) framework because equipment deteriorates with age and/or usage, and finally fails in an uncertain manner (Blischke and Murthy, 2000). Hence, the reliability of system (equipment) is defined as the probability that the system will perform its intended function for a specified time period when operating under normal (or stated) environmental conditions.

For practical meaning (Bentley, 1993), we can classify reliability definition into two cases. First case is non-repairable item. Failure of equipment corresponds with the replacement of the failed item by a new item. There are two criteria used to identify the equipment reliability. The mean time to fail is given by the total up time divided by the number of failures and the mean failure rate is given by the number of failures divided by the total up time. Second case is repairable item. The reliability definition associates with repair which depends on time. There are five criteria generally applied as the equipment reliability measures. The mean down time (MDT) is given by the total down time divided by the number of failures. The mean time between failures (MTBF), also called the mean up time, and the mean failure rate are similar to non-repairable item measures. Next, the availability is the fraction of the total up time to the total time (MTBF + MDT). Last, the unavailability is the compliment of the availability. It is given by the total down time divided by the total time. On the other hand, it is the proportion time that the item does not perform to specification.

2.3.1.2 Reliability measures

In this section we define various reliability measures. All the measures are defined based on the assumption that the time-to-failure (*TTF*) distribution of the system is known.

1. Failure Function

Failure function is a basic (logistic) reliability measure and defined as the probability that an item will fail before or at the moment of operating time *t* (see Figure 2.4). The time *t* can have units such as miles, hours, number of cycles, etc. The failure function is usually represented as $F(t)$.

$$\begin{aligned} F(t) &= P\{TTF \le t\} \\ &= \int_0^t f(u)\,du \end{aligned} \tag{2.1}$$

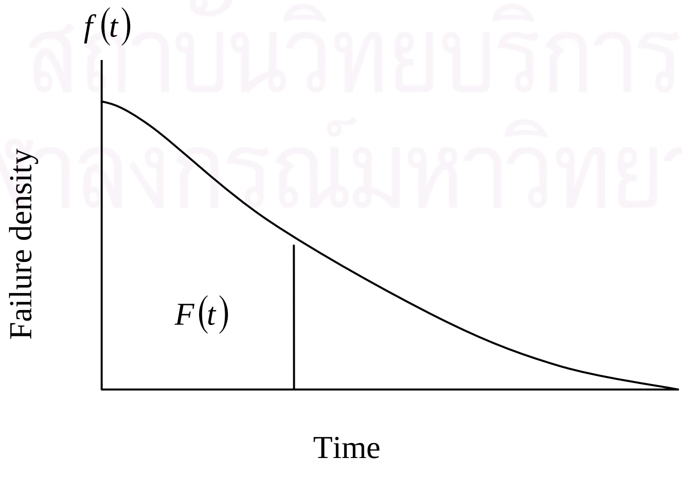

Figure 2.4 Failure function

Where $f(t)$ is the probability density function of the time-to-failure random variable *TTF*. Failure functions of popular theoretical distribution are listed in Table 2.1.

Table 2.1 Failure function

Distribution	**Failure Function,** $F(t)$
Exponential	$1-\exp(-\lambda t)$; $t>0$, $\lambda>0$
Normal	$\int_0^t \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]} dx$ or $\Phi\left(\frac{t-\mu}{\sigma}\right)$
Lognormal	$\int_0^t \frac{1}{\sigma_l x\sqrt{2\pi}} e^{-\left(\frac{1}{2}\left(\frac{\ln(x)-\mu_l}{\sigma_l}\right)^2\right)} dx$ or $\Phi\left(\frac{\ln(t)-\mu_l}{\sigma_l}\right)$
Weibull	$1-\exp\left(-\left(\frac{t-\gamma}{\eta}\right)^\beta\right)$; $\eta,\beta,\gamma>0, t\geq\gamma$
Gamma	$\frac{1}{\Gamma(\alpha)}\int_0^t \beta^\alpha x^{\alpha-1} e^{-\beta x} dx$

Characteristics of failure function:

- Failure function is an increasing function. That is, for $t_1<t_2$; $F(t_1)\leq F(t_2)$.
- For modeling purpose it is assumed that the failure function value at time t = 0, $F(0)=0$.

Applications of failure function:

- $F(t)$ is the probability that an individual item will fail by time t.
- $F(t)$ is the fraction of items that fail by time t.
- 1- $F(t)$ is the probability that an individual item will survive up to time t.

2. Reliability Function

Reliability is the ability of the item to maintain the required function for a specified period of time under given operating conditions (see Figure 2.5). $R(t)$ is given by

$$R(t)=1-F(t) \tag{2.2}$$

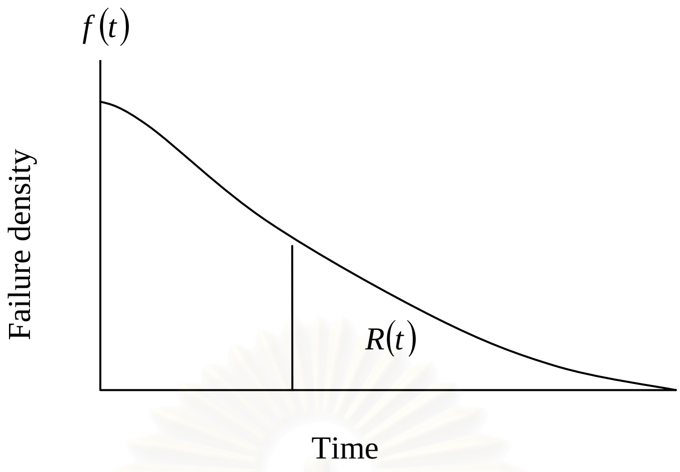

Figure 2.5 Reliability function

Reliability functions for some important life distribution are given in Table 2.2.

Table 2.2 Reliability function

Distribution	**Reliability Function,** $R(t)$
Exponential	$\exp(-\lambda t)$; $t>0$, $\lambda > 0$
Normal	$\Phi\left(\frac{\mu - t}{\sigma}\right) = 1 - \int_0^t \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]} dx$
Lognormal	$\Phi\left(\frac{\mu_l - \ln t}{\sigma_l}\right) = 1 - \int_0^t \frac{1}{\sigma_l x\sqrt{2\pi}} e^{-\left(\frac{1}{2}\left(\frac{\ln(x)-\mu_l}{\sigma_l}\right)^2\right)} dx$
Weibull	$\exp\left(-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right)$; $\eta, \beta, \gamma > 0, t \geq \gamma$
Gamma	$1 - \frac{1}{\Gamma(\alpha)}\int_0^t \beta^{\alpha} x^{\alpha-1} e^{-\beta x} dx$

Properties of reliability function:

- Reliability is a decreasing function with time t. That is, for $t_1<t_2$; $R(t_1) \geq R(t_2)$.

- It is usually assumed that $R(0) = 1$. As t becomes larger and larger $R(t)$ approaches zero, that is, $R(\infty)$.

Applications of reliability function:

- $R(t)$ is the probability that an individual item survives up to time t.
- $R(t)$ is the fraction of items in a population that survive up to time t.
- $R(t)$ is the basic function used for many reliability measures and system reliability prediction.

3. Hazard Function

Hazard function (or hazard rate) is used as a parameter for comparison of two different designs in reliability theory. Hazard function is the indicator of the effect of aging on the reliability of the system. It quantifies the risk of failure as the age of the system increases. Mathematically, it represents the conditional probability of failure in an interval t to $t+\Delta t$ given that the system survives up to t, divided by Δt, as Δt tends to zero, that is

$$h(t)=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\cdot\frac{F(t+\Delta t)-F(t)}{R(t)}=\lim_{\Delta t\to 0}\frac{R(t)-R(t+\Delta t)}{\Delta t R(t)} \tag{2.3}$$

Note that hazard function, $h(t)$, is not a probability, it is the limiting value of the probability. However, $h(t)\Delta t$ represents the probability that the item will fail between ages t and $t+\Delta t$ as $\Delta t \to 0$. Hence, we have

$$h(t)=\frac{f(t)}{R(t)} \tag{2.4}$$

Thus, the hazard function is the ratio of the probability density function to the reliability function. The hazard functions of some important theoretical distributions are given in Table 2.3.

Table 2.3 Hazard function

Distribution	**Hazard Function,** $h(t)$
Exponential	λ
Normal	$f(t)/\Phi\left(\frac{t-\mu}{\sigma}\right)$
Lognormal	$f_l(t)/\Phi\left(\frac{\mu_l - \ln t}{\sigma_l}\right)$
Weibull	$\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$
Gamma	$\left[\frac{\beta^{\sigma}}{\Gamma(\sigma)} t^{\sigma-1} e^{-\beta t}\right] / 1 - \frac{1}{\Gamma(\alpha)} \int_0^t \beta^{\alpha} x^{\alpha-1} e^{-\beta x} dx$

Characteristics of hazard function;

- $h(t)$ can be increasing, decreasing or constant.
- $h(t)$ is not a probability and hence can be greater than 1.

Applications of hazard function:

- $h(t)$ is loosely considered as failure rate at time *t* (time-dependent).
- $h(t)$ quantifies the amount of risk a system is under at time *t*.
- for $h(t) \le 1$, it is not recommended to carry out preventive maintenance.

a) Cumulative hazard function

Cumulative hazard function, *H*(*t*), represents the cumulative hazard or risk of the item during the interval [0, *t*]. *H*(*t*) is given by

$$H(t) = \int_0^t h(x)dx \qquad (2.5)$$

b) Typical forms of hazard function

In practice, hazard function can have different shapes. The hazard rate pattern usually has the general characteristics of a ‘bathtub’ such as shown in Figure 2.6.

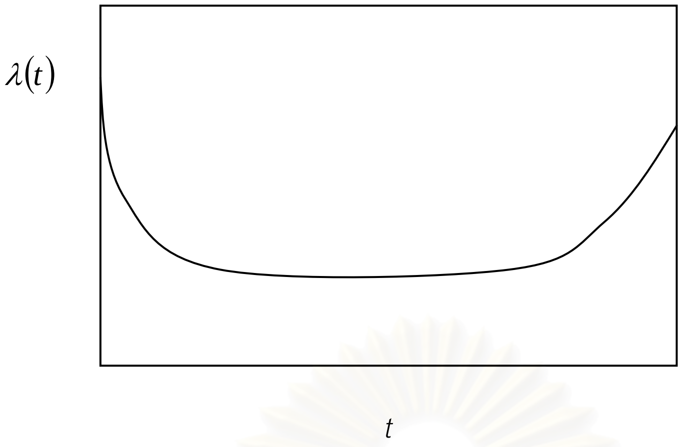

Figure 2.6 Bathtub curve

The bathtub curve consists of three distinct phases. The first phase is a region of decreasing hazard rates and referred as the period of infant mortality, or burn-in. It results from manufacturing defects. The next phase is a constant region or gradually increasing region and referred to as the useful life. The flat curve causes by random failures which result from unavoidable loads. Consequently, the probability that failure will occur in the next time increment is independent of the system's age. Then, it is followed by wear out region characterized by increasing hazard rate. The failures causes by deterioration of age and/or usage.

Figure 2.7 is a representative of much computer and other electronic hardware. This type of hazard function pattern is dominated by random failures with a long span of time over which the hazard rate is basically constant.

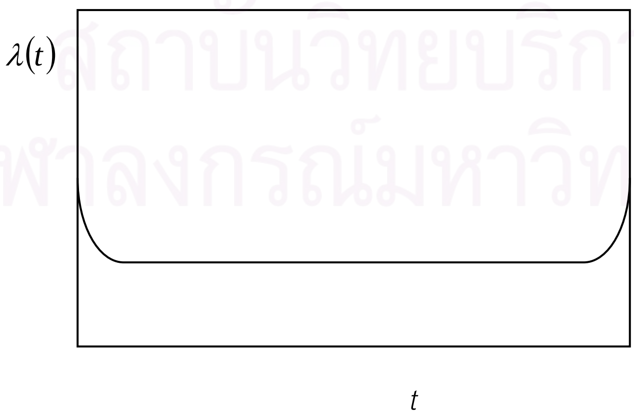

Figure 2.7 Hazard function pattern of electronic hardware

Figure 2.8 is a representative of typical mechanical equipment such as valves, pumps, etc. The initial wear in period is followed by a long span fo time with a monotonically increasing hazard rate.

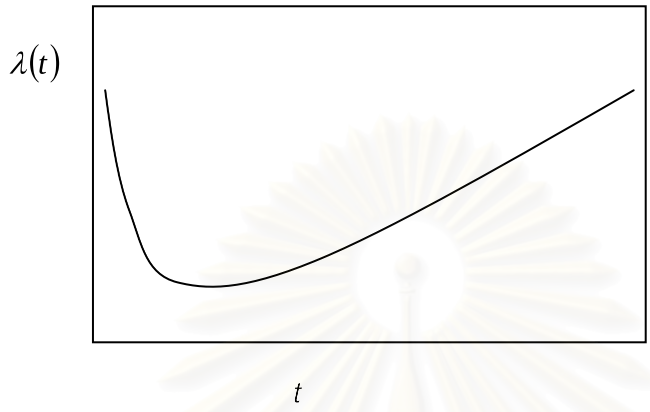

Figure 2.8 Hazard function pattern of mechanical equipment

c) Failure rate

Whenever the hazard function is constant, we call it as failure rate. That is, failure rate is a special case of hazard function (which is time dependent failure rate). Therefore, failure rate can be used only for a non-repairable system. Many defense standards such as MIL-HDBK-217 and British DEF-STAN00-40 recommend the following equation for estimating the failure rate.

$$\text{Failure rate} = \frac{\text{Total number of failures in a sample}}{\text{Cumulative operating time of the sample}} \tag{2.6}$$

Applications of failure rate:

- Failure rate represents the number of failures per unit time.
- If the failure rate is λ, then the expected number of items that fail in [0, t] is λt.

4. Mean Time To Failure (MTTF)

MTTF represents the expected value of a system's time to first failure. It is used as a measure of reliability for non-repairable items such as bulb, microchips and many electronic circuits. Mathematically, MTTF can be defined as

$$MTTF = \int_0^\infty t f(t) dt = \int_0^\infty R(t) dt \tag{2.7}$$

The MTTF of various failure distributions are listed in Table 2.4.

Table 2.4 Mean time to failure

Distribution	**MTTF**
Exponential	$1/\lambda$
Normal	μ
Lognormal	$\exp\left(\mu_l + \frac{\sigma_l}{2}\right)$
Weibull	$\eta \times \Gamma\left(1+\frac{1}{\beta}\right)$
Gamma	$\frac{\alpha}{\beta}$

Applications of MTTF:

- MTTF is the average life of a non-repairable system.
- For a repairable system, MTTF represents the average time before the first failure.

5. Mean Operating Time Between Failures (MTBF)

MTBF is used as a reliability measure for repairable system. Let $X_1, X_2, X_3, ..., X_n$ denote the operating time of the item before i^{th} failure (see Figure 2.9). MTBF can be predicted by taking the average of expected values of the random variables $X_1, X_2, X_3, ..., X_n$.

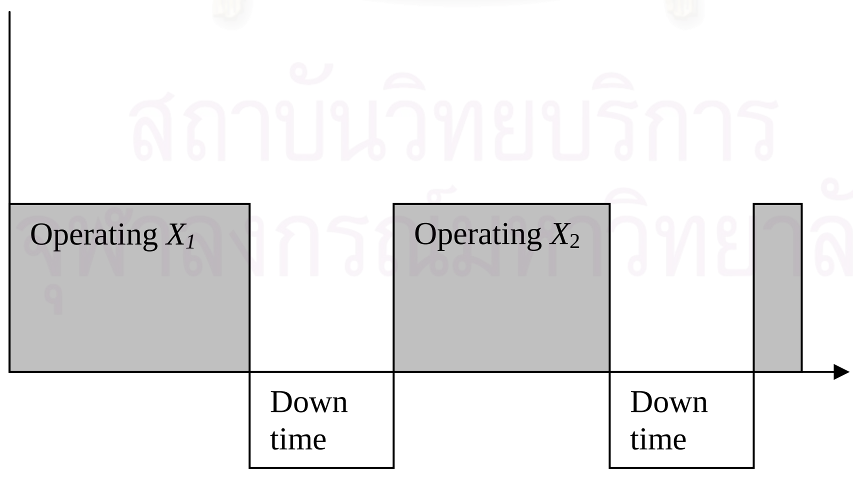

Figure 2.9 Operative profile of a generic item

2.3.2 Failure of unreliable equipment

2.3.2.1 Failure definition

Definition of failure is defined by many researchers as follows:

- "Failure is the event, or inoperable state, in which any item or part of an item does not, or would not, perform as previously." (MIL-STD-721C)

- "Failure is the termination of the ability of an item to perform a required function." (International Electronic Commission, IEC 50 (191), 1991)

- Equipment fails, if it is no longer able to carry out its intended function under the specified operational conditions for which it was designed." (Nieuwhof, 1984)

- "Failure is an event when machinery/equipment is not available to produce parts at specified conditions when scheduled or is not capable of producing parts or perform scheduled operations to specification. For every failure, an action is required." (Sae, No M-110, 1993)

- "Recent developments in products-liability law has given special emphasis to expectations for those who will ultimately come in direct contact with what we will do, make or say or be indirectly affected by it. Failure, then, is any missing of the mark or falling short of achieving these goals, meeting standards, satisfying specifications, fulfilling expectations and hitting the target." (Witherell, 1994)

When a failure occurs, it impacts to operations, behavior, or product applications that lead to dissatisfaction, or undesirable, unexpected side effects. For example, failure of car break results in personal injury, damage to property and a significant of economic loss.

2.3.2.2 Failure causes

Failures are influenced by several factors such as design, weakness, manufacturing, aging, usage, maintenance, operation, or mishandling. Failure is often a result of the effect of deterioration. Its mechanism may be divided into two categories (Dasgupta and Pecht, 1991). Firstly, overstress failures cause from brittle fracture, ductile fracture, yield, buckling, large elastic deformation and interfacial deadhesion. Secondly, wear-out failures cause from wear, corrosion, dendritic growth, interdiffusion, fatigue crack propagation, diffusion, radiation, fatigue crack initiation and creep.

2.3.3 Maintenance

2.3.3.1 Maintenance objectives

Maintenance can be defined as all actions which keep the system operating and ensure that it is maintained and restored to an acceptable standard in which it is able to operate at the required levels efficiently and effectively. Maintenance is becoming an important part in business operation as it is often called “asset management”. The objectives of maintenance are to:

1. Reduce the consequences of failure
2. Extend equipment lifetime
3. Ensure that the system is in operational state and safe to use
4. Ensure that the condition of the system meets all performance requirements such as reliability
5. Maintain the system’s availability
6. Minimize production and quality loss
7. Reduce overall maintenance costs and consequently minimize the life cycle cost

2.3.3.2 Maintenance approaches

Maintenance approaches can be classified into the following five categories:

a) Corrective Maintenance (CM)

Corrective maintenance is performed only at the time of failure for restoring the failed state back to its working state and referred to as an unscheduled or unplanned maintenance action. Two rectifications for failed equipment involve either repair or replacement. The choice between repair versus replacement is affected by cost and the performance of the system after the rectification. This may be the appropriate strategy in some cases, such as when the hazard rate is constant and/or when the failure has no serious cost or safety consequence or it is low on the priority list.

b) Preventive Maintenance (PM)

Preventive maintenance is performed on a schedule basis in order to reduce the likelihood of failure or to improve the reliability of the system and referred to as a scheduled or planned maintenance action. Common tasks for preventive maintenance involve inspection, cleaning, lubrication, adjustment and calibration, minor repair and major overhaul. Most of these actions can be performed at discrete time instant based on operating time (hours), distance (miles) or number of actions (landings) regardless of the actual condition of the item.

c) Condition-Based Maintenance (CBM)

Condition-based maintenance is to decide whether or not to maintain a system according to its state using condition monitoring techniques. Thus, it is used to avoid unnecessary maintenance and to perform maintenance activities only when they are needed to avoid failure. CBM relies on the following decisions. Firstly, one needs to select the parameters to be monitored which depend on several factors such as equipment type and technology available. Next, determining the inspection frequency is determined and, last, the warning limits that trigger appropriate maintenance action are established. The examples of condition monitoring techniques are oil analysis and vibration analysis.

d) Reliability Centered Maintenance (RCM)

Reliability centered maintenance is a systematic approach for selecting applicable and effective preventive maintenance actions for each item in a system taking into consideration failure consequences. It was derived from the approaches to structure airplane maintenance in the sixties. The primary objectives of RCM is to preserve system functions taking into account the objectives of maintenance such as minimising costs, meeting safety, environment and operational goals The RCM process begins with a failure mode and effects analysis (FMEA). Then, the examination of each failure mode is determined to obtain the optimum maintenance action to reduce or avoid failure. Such maintenance action is concerned with cost, safety, environmental and operational consequences. Other relevant parameters such as redundancy, spares costs, maintenance personnel costs, system aging and condition and repair time are also taken into account.

e) Total Productive Maintenance (TPM)

Total productive maintenance, an integrated approach to maintenance and production, was first introduced by (Nakajima, 1988), It is defined as a productive equipment maintenance strategy designed to improve overall equipment effectiveness (OEE) carried out by all employees participating through small group activities. TPM aims to eliminate the major six equipment losses including equipment failure, set-up and adjustment, idling and minor stoppages, reduced speed, process defects and reduced yield. The OEE is a function of the equipment availability, its performance efficiency and the corresponding quality rate taking into consideration the equipment loss.

2.3.3.3 Classification of preventive maintenance action

Preventive maintenance actions are divided into the following categories (Ben-Daya and Duffuaa, 2000; Blischke and Murthy, 2000; and Dohi, Kaio, and Osaki, 2000):

1. *Clock-based Maintenance*: preventive maintenance actions are carried out only when the item has been in use for a specified period of time.

2. *Age-based Maintenance*: preventive maintenance actions are based on the age of the item.

3. *Usage-based Maintenance*: preventive maintenance actions are based on usage of the equipment.

4. *Condition-based Maintenance*: preventive maintenance actions are based on the condition of the component being maintained. This involves monitoring of one or more variables characterizing the wear process.

5. *Opportunity-based Maintenance*: This maintenance is recommended for complex system, where maintenance actions (preventive and corrective maintenance) for a component provide an opportunity for carrying out preventive maintenance actions on one or more of the remaining components of the system.

6. *Design-out Maintenance*: This maintenance category deals with the modifications through re-design of the component. Accordingly, the new component has better reliability characteristics.

2.3.3.4 Maintenance costs

There are two fundamental costs associated with maintenance (Niebel, 1994; and Blischke and Murthy, 2000). First is a direct cost consisting of manpower cost, material and spares cost, tools (both hardware and software tools) and equipment needed for carrying out maintenance actions cost, and overhead cost. These costs are aggregated to be a repair cost. Second is failure costs including defective item cost, idle operation cost, late delivery cost, environment impacts cost, and human injured cost. These may refer to an indirect cost and be used as compensation or penalty costs in maintenance service contract when customers are not satisfy with the service or the contract terms can not be accomplished by the service provider.

2.3.3.5 Logistics

Logistic supports directly affect the maintenance effectiveness. Some significant issues; for instance, spare parts location and distribution, procurement, supply, storage condition, transportation, maintenance crews and facilities are discussed by Reiche (1993) and Blischke and Murthy (2000). A logistic time delay to maintain a system depends on its reliability/maintainability and the required availability. It is accounted for administrative delay which reflects to the performance of equipment maintenance. In the area of manpower and tools, Jardine and Buzacott (1985) stated that queueing theory and simulation can be used to assist maintenance decision makers. Furthermore, inventory control technique is used to forecast the demand for spare parts and to determine optimal purchasing and location strategies.

2.3.3.6 Maintenance management information system

Maintenance in the organization of complex system is sophisticated process and requires coordination among related functions (Blischke and Murthy, 2000). It is remarkable to see the evolution of computerized maintenance management systems (CMMS). A maintenance management information system is one of the means by which field data is converted into useful information. Such system provides technical data, maintenance resources information, maintenance history information, and performance report. As a result, maintenance can easily control its activities (Tomlingson, 1993). Such information is used for determining what task must be done, justifying the action that maintenance must carry out, assuring the validity of the actions, and measuring maintenance effectiveness.

2.4 Literature review

2.4.1 Equipment leasing

The literature related to equipment leasing is mainly qualitative. Most of leasing research has been an emphasis on strategic problems, particularly trade-off between buy or lease, service pattern, accounting, and regulations. For instance, Wijnands (1986), Trigeorgis (1990), Geoghegan (1994), Grenadier (1995), Sedimeier (1997), Akarakiri (1998), Desai and Purohit (1998), Robins (1999), and Kenyon and Tompaidis (2001).

Christer and Waller (1987) investigated the influence of variation of the input parameters on the replacement age and on a penalty measure for the leased commercial vehicle. The maintenance of leased equipment model, then, was first introduced. Kobbacy and Nicole (1994) extended the rent model by using statistical simulation. The sensitivity analysis presented effects of changes in parameters such as capital cost, maintenance cost, resale value, current discount factor as well as tax parameters on the optimal replacement age of the trucks.

Although practitioners have recognized leasing as one of major strategic challenges for their businesses, academics have spent little attention to the subject of bundling leasing with extension service like maintenance. This may be explained that equipment leasing and maintenance have been treated as a separated problem. In fact, when equipment is leased, lessees often believe that they are obtaining more than a physical item. They also have expectation about equipment maintenance service from the lessor over a lease period. Therefore, lessor must take maintenance service into account of equipment leasing operation in order to achieve lessee’s expectation and to be successful in the leasing business.

As such, in this thesis the optimal maintenance policy of leased equipment will be determined by minimizing the total expected cost to the lessor. There are several maintenance techniques can be used to accomplish the goals of the lessor. Traditionally, repair is carried out when equipment fails, while preventive maintenance action is performed to reduce failures. In next subsection, we give a brief review of repair and preventive maintenance policies literature as a fundamental concept used in later chapters.

2.4.2 Equipment maintenance

The literature on maintenance is very large. There are several books dealing with different aspects of maintenance – see, for example, Gertsbakh (1977), Mann (1983), Moubray (1997), and Niebel (1985). Several review papers have appeared over the last 30 years. Readers who are interested in maintenance introductions and frameworks could refer to Dekker (1995a and 1995b) and Aven and Dekker (1997).

Surveys/reviews on maintenance models can be found in McCall (1965), Pierskalla and Voelker (1976), Sherif and Smith (1981), Thomas (1986), Valdez-Flores and Feldman (1989), Cho and Parlar (1991), Pintelon and Gelders (1992), Dekker (1996), Vatn, Hokstad, and Bodsberg (1996), Scarf (1997), Vatn (1997), Dekker and Scarf (1998), and Wang (2002). A great deal of research has been done in the area of optimal maintenance modeling, involving the aspect of optimal preventive maintenance policies. This is due to increased complexity of systems, increased quality requirements, and rising costs of material and labor.

2.4.2.1 Types of repair

Basically, repair can be divided into three categories: perfect repair, minimal repair, and imperfect repair (Blischke and Murthy, 2000). The following relevant literature is briefly reviewed.

a) Perfect repair

When the failed component is replaced with a new one and the condition of repaired item is assumed to be as good as new, it is called “perfect repair”. Consequently, a system has the same lifetime distribution and failure rate function of the item after repair is identical to a new one. Perfect repair is suitable for a single unit system or electronic component. In real world, this is a rarely case. Examples are changing the light bulb and complete overhaul an engine of a car.

b) Minimal repair

The failure rate of the system after repair is the same as that before (as bad as old). So, it is often called “minimal repair”. This type of repair is suggested to be suitable for the large complex system; for example, changing a flat tire on a car. Minimal repair was first studied by Barlow and Hunter (1960). They have investigated systems subject to minimal repair and periodic renewal and established optimization models for determining the optimal replacement interval based on a non-homogeneous Poisson process. Since then, extensive research has been conducted in accordance with assuming minimal repair. The literature related to minimal repair can be found in the surveys and reviews stated above. Beichelt (1993) has conducted a study to present the mathematical background for analyzing maintenance policies with minimal repairs and summarized some standard policies.

c) Imperfect repair

The condition of failed item can be restored at some certain state which may be better or reverse than before. This repair type is called “imperfect repair”. The degree of repair effectiveness depends on the types of applications, repair costs, as well as reliability and safety requirements. Engine tune-up is an example of imperfect repair which can help the engine performance improve. There is an excellent review of imperfect maintenance conducted by Pham and Wang (1996).

2.4.2.2 Preventive maintenance policies

For deterioration system, preventive maintenance is an attractive option of maintenance optimization used to control the degradation process. The optimal preventive maintenance policy not only reduces the maintenance cost, but also improve the efficient and effectiveness of equipment. However, the failure behavior after preventive maintenance should be expected to be better than it was before preventive maintenance, but certainly not as good as new that of a new unit (as long as the unit is not replaced or completed overhauled). The concept of imperfect preventive maintenance of deteriorating system was introduced by Chaudhuri and Sahu (1977). Many extensions have been proposed for this concept as follows.

Malik (1979) proposed a preventive maintenance model with the concept that maintenance influences the reduction of failure rate by an age reduction factor which determined by expert judgments. Nakagawa (1980a) has also introduced improvement factor in hazard rate and age for preventive maintenance policy. Nakagawa (1980b) considered that due to imperfect preventive maintenance the effective age of the system is reduced by x time units after each preventive maintenance intervention. Optimal preventive maintenance policy using a function of maintenance cost and the age of the system was introduced by Lie and Chun (1986).

A different approach is proposed by Nakagawa and Yasui (1987) and Nakagawa (1988) who model the effect of preventive maintenance in accordance with a function of resource consumed and the system age. Chan and Shaw (1993) also suggested a treatment method which the failure rate corresponding on age and the number of preventive maintenance actions.

Other related papers in the literature include contributions from Malik (1985), Kijima, Morimura and Suzuki (1988), and Kijima (1989), Olorunniwo and Izuchuku (1991), Makis and Jardine (1993), and Love et al (2000) who have investigated the problem of improving a system’s reliability through optimal preventive maintenance using virtual age approach.

Chan and Downs (1978) modeled the effects of imperfect maintenance to determine which preventive maintenance is preferred under some specified condition. Murthy and Nguyen (1981) considered a more general approach by using a renewal process to derive the more directly and simply results. Nguyen and Murthy (1981a) proposed a different characterization of imperfect maintenance. Their proposed preventive maintenance policy can be considered as a semi-Markov process with four states: State 1—working after corrective maintenance, State 2—working after preventive maintenance, State 3—preventive maintenance, and State 4—corrective maintenance.

Other conventional replacement/preventive maintenance policies, such as Nguyen and Murthy (1981b), Boland (1982), Canfield (1986), Nakagawa (1979a, 1979b, and 1986), Abdel-Hameed (1987), Chun (1992), Jayabalan and Chaudhuri (1992c), Liu, Makis and Jardine (1991), Wang and Pham (1996a and 1996b), Zhang and Jardine (1998), and Lim and Lie (2000), are derived using non-decreasing hazard rate functions. Their models are extension of the imperfect effect model introduced by Brown and Proschan (1983) or the consideration with minimal repair proposed by Barlow and Hunter (1960). They hold generally periodic or block policies for replacement/preventive actions.

Park, Jung and Yum (2000) considered a repairable system subject to preventive maintenance periodically and minimal repair at each failure. The model assumes that each preventive maintenance relieve stress temporarily and then slows the degradation rate of the system. It further assumes that the hazard rate of the system increases monotonically. This model is extended by Park and Jung (2002) in aspects of preventive maintenance cost in which depends on the degree of effectiveness of the preventive maintenance action.

Most of these models formulate optimal policies for an infinite time horizon. Jayabalan and Chaudhuri (1990, 1992a and 1992b) considered the optimal preventive maintenance for a finite time period. They discussed the optimal maintenance policies of the system subject to failure and to be maintained at predetermined points to assure its reliability. The models assume that maintenance reduces the effective age of the system and hence the system ROCOF.

Usher, Kamal, and Syed (1998) expanded on Jayabalad and Chaudhuri's previous approaches by using three approaches—a random search, a genetic algorithm, and a branch-and-bound approach. Also, the periodic preventive maintenance policy for a system with mechanical components using genetic algorithms was developed by Tsai, Wang and Teng (2001).

Nakagawa (2000) has conducted surveys the imperfect maintenance models. He summarized the results in three policies. First, the unit after preventive maintenance has the same hazard rate as before preventive maintenance. Second, the age of the unit becomes x units of time younger at preventive maintenance. Third, the age reduces to at when it was t before preventive maintenance.

Although the preventive maintenance model of deteriorating systems has always been recognized, only a few works was conducted on maintenance issues, particularly the leased equipment. Most of the research is confined to traditional in-house or outsourcing maintenance approach. Nisbet and Ward (2001) have suggested that not only strategic considerations, but also technical considerations should be taken into account of equipment leasing. Those technical aspects are preventive maintenance service, leased equipment upgrading, and spare parts inventory management.

In this thesis, we consider maintenance problem for leased equipment, which is a complex system and subject to failure. Three kinds of maintenance action are taken into account; minimal repair, preventive maintenance, and upgrading, with different costs. To minimize the total expected cost, the preventive maintenance model will be developed regarding the lessor's perspective. The general concept of modeling is discussed in Chapter 3.

CHAPTER III

MAINTENANCE MODELING

3.1 Introduction

The likelihood of equipment failure increases as a result its reliability generally tends to decrease. However, it is expected that an effective maintenance policy can reduce the failure number. Maintenance, then, clearly affects equipment reliability.

For a complete evaluation of the effect of a maintenance policy, a mathematical model of the equipment deteriorating process is require to describe the effects of maintenance. Once the mathematical model is constructed, the process can be optimized with regard to changes in one or more of the variables. The optimization will result in the least cost of maintenance.

This chapter is organized as follows. Models of the failure process at component and system levels are introduced in Section 3.2. Following this, we discuss modeling of first and subsequent failures in Sections 3.3 and 3.4 respectively. In Section 3.5, modeling of minimal repair is examined. Section 3.6 deals with modeling of preventive maintenance, particularly preventive maintenance model concepts of new and leased equipment. Next, maintenance policy is proposed in Section 3.7. Regarding to the policy, a discussion of maintenance costs of leased equipment is given in Section 3.8. Last, optimal maintenance is considered in Section 3.9.

3.2 Modeling failures

In many applications, equipment failures can be divided into two categories; random failures and ones arising from a consequence of deterioration (aging). The deterioration process is represented by a sequence of stages of increasing wear, finally leading to equipment failure. Deterioration is of course a continuous process in time, and only for the purpose of easier modeling it is considered in discrete steps.

In the case of random failures, the constant failure-rate assumption leads to the result that maintenance cannot produce any improvement, because the chances of a failure occurring during any future time-interval are the same with or without maintenance. The situation is quite different for the deterioration process, where the time to failure is not exponentially distributed. In such a process the hazard function is increasing with time, and maintenance will bring about advantage in this case. Hence, it can be stated that maintenance can not improve random failures where the hazard function is constant, but has an important role when failures are the consequence of aging.

Equipment comprises of a number of components. For more complex equipment, the number of parts may be magnitude larger. The performance of equipment depends on the state of the system (working or failed) and in turn depends on the state (working or failed) of the various components. Therefore, equipment failures can be modeled at the component level or the system level.

3.2.1 Modeling failures at component level

Modeling at the component level can be either empirical or physical based. In the empirical case, the switch from working to failure treated as a random variable and modeled by a probability distribution. In the case of repairable component, the distribution can depend on the type and number of repairs carried out. In the physical case, failure is modeled in terms of the degradation mechanism and as such involves more complicated models which take into account the material properties and environmental factors affecting the degradation.

3.2.2 Modeling failures at system level

Modeling at the system level can be done by either treating the system as black-box or white-box in terms of its components (Blischke and Murthy, 2000). The component failures are modeled individually. The former approach is useful for modeling equipment performance (such as availability, number of failures over a specified time etc) whereas the latter approach is needed for logistical planning (such as need for spares etc).

The black-box approach is used where the state of the system is described either in terms of two states (working or failed). As failure of system is often caused by the failure of one or more components, the number of failed components is usually small relatively to the total number of components in the system. To make the system back to the operational state, replace or repair is the choice to take for the failed components.

The white-box approach is used where the state of the system is classified in terms of the states of the various components of the system. Failure is characterized through the physics of failure. For the overstress failure modeling, when the stress exceeds the strength and this results in item failure. For the wear-out failure modeling, failures occur when the effect of damage accumulates with time reaches some threshold level.

3.3 Modeling of first failure

In this thesis we assume system level modeling using the black-box approach. The time to first failure, *TTF*, is a random variable in the interval $[0,\infty)$ and modeled by Weibull distribution function $F(t)$. The distribution is characterized by three parameters. First is α referred to the scale parameter. Second is β referred to the shape parameter. When $\beta < 1$ and $\beta > 1$, $r(t)$ has a decreasing (DFR) and increasing (IFR) failure rate respectively. When $\beta = 1$, $r(t)$ is constant (CFR) which is reduced to exponential distribution function. Third is υ referred to the location parameter which is often set equal to zero. In this thesis, we confine our attention to the two-parameter Weibull distribution function. The shape parameter is utilized to model deterioration process, and affords considerable flexibility in capturing wide variety shapes of ROCOF, which may make it more realistic for many applications.

3.4 Modeling subsequent failures

Subsequent failures of the equipment depend on the type of corrective maintenance actions (to fix failures) and preventive maintenance actions (to avoid failures) which the equipment is subjected to. If the item is non-repairable, replacement with a new one is only a choice for the failed item. On the other hand, if the item is repairable, there are two choices of replacement or repair. With the repair case, there are several types of repair which are reviewed in Section 2.4.2.1. In this thesis, we focus on the repairable equipment. And, when failures occur, the failed item is minimally repaired.

3.5 Modeling of minimal repair

In real world situation, most complex mechanical systems deteriorated as time passes. A complex system may fail if one of its many components is out-of-function. The system is returned to the operating state when the failed component is replaced. As the majority of components have not been replaced, the remaining life distribution and failure rate of the system are not altered by the failure and repair.

To model deterioration, a Non-Homogeneous Poisson Process (NHPP) was first introduced by Barlow and Hunter (1960). The basic probabilistic assumptions of the NHPP are:

1. the repair times can be neglected if the time to restore the failed item to its operational state is very small relatively to the mean time between failures;

2. repairs take place instantaneously after failure; and

3. the failure rate of an item after a repair is the same as that just before failure.

Based on these assumptions, the failure process is time-dependent and failures are statistically independent. The intensity function of the failure process is given by $\lambda(t) = r(t)$, where t represents the age of the system. $\lambda(t)$ is an increasing function of *t*, reflecting the effective of age and associated with the failure distribution $F(t)$ (Nakagawa and Kowada, 1983; Murthy, 1991; and Coetzee, 1997). Let $N(t)$ denote the number of failures over $[0,t)$ and $\Lambda(t) = \int_0^t \lambda(x)dx$. Then

$$p_n(t) = P\{N(t) = n\} = \frac{e^{-\Lambda(t)}\{\Lambda(t)\}^n}{n!} \tag{3.1}$$

and the expected number of failures over $[0,t)$ is given by

$$E[N(t)] = \Lambda(t) \tag{3.2}$$

In this thesis, we commence two preventive maintenance strategies. One is discrete maintenance which is modeled as a regular maintenance action. Another one is upgrading which is modeled as a special maintenance action.

3.6 Modeling of preventive maintenance

3.6.1 Modeling of discrete maintenance

The effect of preventive maintenance is best modeled through its impact on the hazard function or ROCOF[1]. Since the equipment experiences an increasing of ROCOF, then, preventive maintenance action results in the reduction of ROCOF.

Let $t_j, j \geq 1$, denote the time instant of the j^{th} preventive maintenance action and it affects the ROCOF through a reduction in the intensity function. Let $\lambda(t)$ denote the intensity function with preventive maintenance and $\lambda_0(t)$ without preventive maintenance. We assume that the time for preventive maintenance action is small relative to the mean time between failures so that the effect of preventive maintenance is modeled by

$$\lambda(t_j^+) = \lambda(t_j^-) - \delta_j \qquad (3.3)$$

where δ_j is the reduction resulting from the preventive maintenance action at time t_j. δ_j is an increasing function of the level of preventive maintenance effort but is constrained by an upper limit. One such constraint is given by

$$0 \leq \delta_j \leq \lambda(t_j^-) - \lambda(0) \qquad (3.4)$$

This implies that preventive maintenance cannot make the equipment better than new. An alternate constraint is the following that we will use in Chapter 4 is shown as

$$0 \leq \delta_j \leq \lambda(t_j^-) - \lambda(t_j^+) \qquad (3.5)$$

This implies that the intensity function after preventive maintenance action cannot be smaller than that at the previous preventive maintenance action.

A feature of this type of preventive maintenance action is that each preventive maintenance action lowers the intensity function by a fixed amount. So, the intensity function with preventive maintenance action is given by

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[1]ROCOF refers to Rate of OCcurrence Of Failures, also termed "deterioration".

$$\lambda(t) = \lambda_0(t) - \sum_{i=0}^{j} \delta_i \qquad (3.6)$$

for $t_j < t < t_{j+1}, j \geq 0$, with $t_0 = 0$ and $\delta_0 = 0$. This implies that the reduction resulting from action at t_j lasts for all $t \geq t_j$ as shown in Figure 3.1.

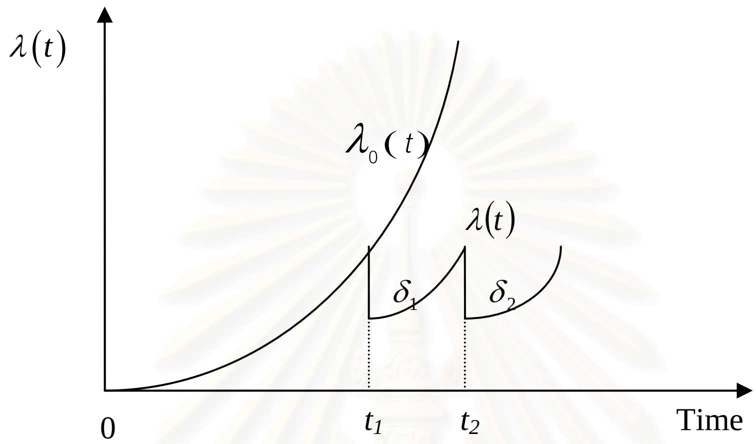

Figure 3.1 Intensity function with preventive maintenance actions for new equipment

3.6.2 Modeling of upgrade maintenance

Leasing allows the lessor retains ownership of leased equipment throughout the end-of-life and after the lessee use phase. As a result, leasing program has a considerable impact on how used equipment is handled. Hence, the lessor is more likely to develop the maintenance strategy model in a way that increases the reliability of used equipment.

The reliability of used equipment depends on its age and maintenance history. For leasing of used items, it might sometimes be more appropriate to subject the equipment to an upgrade (or overhaul) which improves its reliability. Since reliability decreases with age, one way of modeling the effect of upgrade is through a reduction in its age. Therefore, equipment of age A behaves like equipment of age $A - x$ after the overhaul. In other words, the equipment is rejuvenated so that its virtual age is $A - x$. Note that the magnitude of upgrading depends on the overhaul effort expended and must satisfy the constraint $x < A$. This implies that the intensity function after an overhaul at age A is given by $\lambda(t-x)$ for $t > A$ as shown in Figure 3.8.

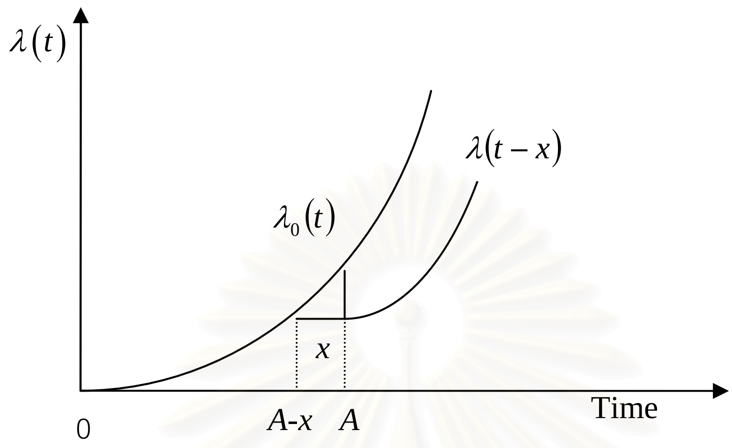

Figure 3.2 Intensity function with upgrading for used equipment

3.7 Maintenance policy

In this thesis we suggest one policy characterized by three or more parameters (depend on options) that need to be selected optimally. Here, the proposed preventive maintenance policy is as follows:

The equipment is subjected to k preventive maintenance actions over the lease period. The time instants at which these actions are carried out are given by $\{t_j, 1 \le j \le k\}$ with $t_i < t_j$ for $i < j$. The reduction in the intensity function during the preventive maintenance action is δ_j. Any failure over the lease period is rectified through minimal repair. The policy for new equipment lease is characterized by the set of parameters $\theta \equiv \{k, t_j, \delta_j, 1 \le j \le k\}$.

For used equipment lease, there is another one additional parameter dealing with the lessor's decision through a reduction in the equipment age, x. The upgrade is an option for the lessor to be carried out before the equipment is leased. Hence, the policy for used equipment lease is characterized by either the set of parameters $(x, \theta) \equiv \{x, k, t_j, \delta_j, 1 \le j \le k\}$ with the upgrade option or $\theta \equiv \{k, t_j, \delta_j, 1 \le j \le k\}$ with no upgrade option.

3.8 Maintenance costs of leased equipment

For leased equipment, the cost associated with maintenance comprises of corrective maintenance cost, preventive maintenance cost, upgrade cost, and penalty costs.

3.8.1 Corrective maintenance cost

When the lessor is responsible to maintenance service for leased equipment, the rectification cost is the inevitable cost to the lessor. Here, we assume that the all failures are rectified through minimal repair and that the cost of each corrective maintenance action is an average cost of C_f.

3.8.2 Preventive maintenance cost

Preventive maintenance cost is incurred when the lessor carries out the action at the optimal scheduled times. We assume that preventive maintenance actions result in a reduction in the intensity function. Let δ denote the reduction and the cost preventive maintenance action is given by a function of δ, $C_p(\delta)$. It increases with δ implying that the greater preventive maintenance effort is needed to achieve the larger reduction.

3.8.3 Upgrade cost

For used equipment, the lessor has an option to upgrade the leased item, and the upgrade cost has to be taken into account. This cost is considered as a function of the reduction in the age, *x*. It is given by $C_u(x)$ which is an increasing function in *x*, the reduction in the age, *A*.

3.8.4 Penalty costs

Let L denote the period lease, and $N(L)$ refers to the number of failures over the lease period. As mentioned earlier, the reliability performance requirements have a significant impact on the optimal preventive maintenance strategies. Two different reliability requirements and the associated penalties are as follows.

3.8.4.1 Penalty-1 cost

Each repair to be completed within a specified time τ. Failure to do so results in a penalty cost which is function of the repair time Y (a random variable from a distribution $G(y)$ called the repair time distribution) and τ. The penalty function is given by $C_t \max[0, Y-\tau]$ where $C_t > 0$. Let Y_i denote the time to rectify the i^{th} failure, $1 \le i \le N(L)$. Then, the total penalty incurred is given by

$$\phi_1(N(L), Y_i, \tau) = C_t\left\{\sum_{i=1}^{N(L)} \max[0, Y_i - \tau]\right\} \tag{3.7}$$

3.8.4.1 Penalty-2 cost

All failures occur over the lease period, $N(L)$, incur penalty to the lessor. Then, the total penalty incurred is given by the following function

$$\phi_2(N(L)) = C_n N(L) \tag{3.8}$$

If both of these are included in the contract, then the resulting penalty cost to the lessor is given by

$$\phi_3 = \phi_1 + \phi_2 = C_t\left\{\sum_{i=1}^{N(L)} \max[0, Y_i - \tau]\right\} + C_n\{\max[0, N(L)]\} \tag{3.9}$$

3.9 Optimal maintenance

Preventive Maintenance involves additional costs and is worthwhile only if the benefits derived from it (such as lower corrective maintenance cost, greater customer satisfaction, higher yield etc) exceed the costs (see Figure 3.3). Traditionally, the optimal maintenance considers trade-off between corrective and preventive maintenance. However, for the leased equipment, the contract usually includes penalties which are needed to be incorporated in the optimal preventive maintenance model.

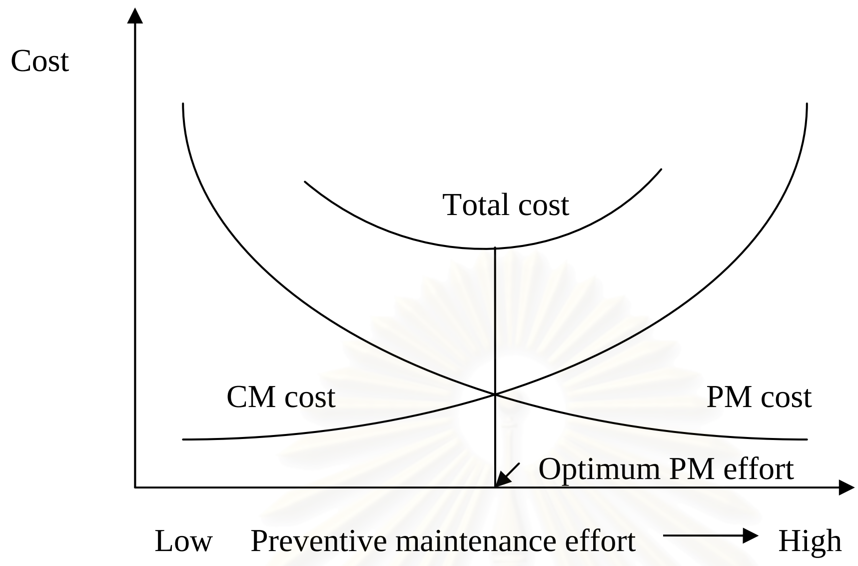

Figure 3.3 Optimum preventive maintenance effort

As such, from the preventive maintenance policy, a parameter set θ is selected optimally to minimize the total expected cost comprising of the penalty costs as well as the corrective and preventive maintenance costs. Let $J(\theta)$ denote the total expected cost for the case of new equipment lease and is the objective function for determining the optimal values for the parameters in the set θ.

3.9.1 New equipment lease

In this case, the objective function is given by

$$J(\theta) = C_f E[N(L)] + \sum_{j=1}^{k} C_p(\delta_j) + E[\phi_1(N(L), Y_i, \tau)] + E[\phi_2(N(L))] \qquad (3.10)$$

This includes the penalty for delay in not completing the rectification within the specified time limit and for the number of failures occurs over the lease period.

3.9.2 Used equipment lease

In this case, the lessor has the option to improve the reliability through upgrade which effectively reduces its age by *x*. As a result, the objective function is given by

$$J\left(x,\theta\right)=C_f E\left[N\left(L\right)\right]+\sum_{j=1}^{k}C_p\left(\delta_j\right)+E\left[\phi_1\left(N\left(L\right),Y_i,\tau\right)\right]+E\left[\phi_2\left(N\left(L\right)\right)\right]+C_u\left(x\right) \quad (3.11)$$

Thus we have both x and the parameters of the set θ which are selected optimally to minimize $J\left(x,\theta\right)$ is given by (3.11).

In the next chapter we will perform model formulation and model analysis to accommodate the following three special cases.

Special Case 1: No penalty

In this Special Case 1, $C_t = C_n = 0$ implies that no penalty so that (3.10) gets reduced to

$$J(\theta)=C_f E[N(L)]+\sum_{j=1}^{k}C_p\left(\delta_j\right) \quad (3.12)$$

Special Case 2: Penalty-1

In this Special Case 2, $C_n = 0$ implies that no Penalty-2 so that (3.10) gets reduced to

$$J(\theta)=C_f E[N(L)]+\sum_{j=1}^{k}C_p\left(\delta_j\right)+E[\phi_1(N(L),Y_i,\tau)]$$

(3.13)

Special Case 3: Penalty-2

In this Special Case 3, $C_t = 0$ implies that no Penalty-1 so that (3.10) gets reduced to

$$J(\theta)=C_f E[N(L)]+\sum_{j=1}^{k}C_p\left(\delta_j\right)+E[\phi_2(N(L))] \quad (3.14)$$

CHAPTER IV

ANALYSIS OF NEW EQUIPMENT LEASE

4.1 Introduction

In this chapter we carry out an analysis of optimal preventive maintenance actions for new equipment under lease. The outline of this chapter is as follows. We give the details of the model formulation in Section 4.2. The optimal preventive maintenance actions policy is characterized by a set of parameters $\theta = \{k, \underline{t}, \underline{\delta}\}$ where the last two are *k*-dimensional vectors. The parameters need to be selected optimally to minimise the total expected maintenance cost to the lessor. This topic is covered in Section 4.3 as the model analysis. In this section three special cases are also proposed. The numerical example of no penalty case is presented in Section 4.4 along with sensitivity analysis to indicate the influence of the shape parameter of failure distribution as well as the parameters of preventive maintenance cost on the optimal preventive maintenance strategy. Next, the numerical result of Penalty-1 case and its sensitivity study of the effect of the repair time limits on the optimal preventive maintenance actions are illustrated in Section 4.5. Penalty-2 numerical results and its sensitivity study of the effect of the failure costs on the optimal preventive maintenance strategy are demonstrated in Section 4.6. We give the numerical analysis of the general case which includes penalties 1 and 2 in the model in company with the sensitivity analysis of the effect of both penalties' parameters through Section 4.7. Last, a conclusion is given in Section 4.8.

4.2 Model formulation

The equipment is leased for a period L with penalty for failures over the lease period and/or for repairs not being completed within a time period τ. As a result, the penalty costs are random variables and are given by (3.7) and (3.8).

4.2.1 Failures and corrective maintenance

The lease equipment is repairable and all failures are rectified through minimal repairs. The average cost of a rectification is C_f. We further assume that the time needed to rectify a failed-equipment is small in relation to the mean time between failures and such that it can be ignored. In this case, equipment failures with no preventive maintenance action occur according to a non-homogeneous Poisson process with intensity function

$$\lambda_0(t) = r(t) \tag{4.1}$$

where $r(t)$ is the hazard function associated with the distribution function $F(t)$.

We assume that $\lambda_0(t)$ is given by the two-parameter Weibull intensity function $\lambda_0(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}$ with scale parameter $\alpha > 0$ and shape parameter $\beta > 1$ (indicating an increasing failure rate). Without loss of generality we assume $\alpha = 1$. If $\alpha \neq 1$, then a change in time scale given by $t' = t/\alpha$ results in failures occurring with an intensity function with scale parameter equal to 1. In this case, the lease period L changes to $L' = L/\alpha$. In the remainder of the thesis the intensity function is given by

$$\lambda_0(t) = \beta t^{\beta-1} \tag{4.2}$$

4.2.2 Preventive maintenance

The preventive maintenance actions under the proposed policy were discussed in Section 3.6. A preventive maintenance action results in a reduction, δ, in the intensity function. The cost of a preventive maintenance action is a function of δ and is given by

$$C_p(\delta) = a + b\delta \tag{4.3}$$

where $a > 0$ and $b \geq 0$. a is the fixed cost and b is the parameter of the variable cost of a preventive maintenance action.

4.2.3 Lessor's decision problem

The optimal preventive maintenance strategy is to select the optimal number of PM actions, k^*, the optimal preventive maintenance schedule, $\underline{t}^* = \{t_j^*\}$, and the optimal preventive maintenance levels, $\underline{\delta}^* = \{\delta_j^*\}$, for j = 1, 2, …, k^*, to minimise the total expected maintenance cost, $J(k^*, \underline{t}^*, \underline{\delta}^*)$, given by (3.10).

4.3 Model analysis

4.3.1 Expected costs

When no preventive maintenance is used, the failures over the lease period L occur according to an NHPP with intensity function given by (3.2). The expected number of failures over the lease period is given by

$$E[N(L)] = \Lambda_0(L) = \int_0^L \lambda_0(t)dt = L^\beta \tag{4.4}$$

The expected number of failures over the lease period with preventive maintenance actions according to the preventive maintenance policy is given by

$$\begin{aligned} E[N(L)] = \Lambda(L) &= \Lambda_0(L) - \sum_{j=1}^{k} \delta_j (L - t_j) \\ &= L^\beta - \sum_{j=1}^{k} \delta_j (L - t_j) \end{aligned} \tag{4.5}$$

Note that the second term in the right-hand side of (4.5) represents the reduction in the expected number of failures due to preventive maintenance actions under the preventive maintenance policy.

The total expected maintenance cost to the lessor is given by (3.10) and consists of four components. These are the costs resulting from

(i) corrective maintenance actions to repair failures, TC_f,

(ii) preventive maintenance actions, TC_p,

(iii) the penalty costs for repairs not completed within the specified time limit (henceforth referred to as Penalty-1), and

(iv) the penalty costs associated with the number of failures (henceforth referred to as Penalty-2).

The expected cost resulting from corrective maintenance actions is given by

$$E(TC_f) = C_f \Lambda(L) \tag{4.6}$$

with $\Lambda(L)$ given by (4.5).

The cost of preventive maintenance actions is given by

$$TC_p = \sum_{j=1}^{k} (a + b\delta_j) = ak + b\sum_{j=1}^{k} \delta_j \tag{4.7}$$

The expected Penalty-1 cost is given by (3.7) and can be rewritten as

$$E[\phi_1(N(L),Y,\tau)] = C_t\Lambda(L)\left\{\int_\tau^\infty (y-\tau)g(y)dy\right\}$$
$$= C_t\Lambda(L)\int_0^\tau [1-G(\tau)]dy \qquad (4.8)$$

Finally, the expected Penalty-2 cost is given by (3.8) and can be rewritten as

$$E[\phi_2(N(L))] = C_n\sum_{s=0}^{\infty} sP[N(L)=s] \qquad (4.9)$$

Since failures occur according to an NHPP, then we have

$$P[N(L)=s] = \frac{\exp^{-\Lambda(L)}[\Lambda(L)]^s}{s!} \qquad (4.10)$$

for s = 0, 1, 2,… with $\Lambda(L)$ given by (4.5).

Using (4.10) in (4.9) results in

$$E[\phi_2(N(L))] = C_n\sum_{s=0}^{\infty} s\frac{\exp^{-\Lambda(L)}[\Lambda(L)]^s}{s!} = C_n\Lambda(L) \qquad (4.11)$$

Combining all these costs yields

$$J(k,\underline{t},\underline{\delta}) = C_f\Lambda(L) + ak + b\sum_{j=1}^{k}\delta_j + C_t\Lambda(L)\int_0^\tau [1-G(\tau)]dy + C_n\Lambda(L) \qquad (4.12)$$

and the parameters of the preventive maintenance policy are selected to minimise this subject to the constraints $0 < t_t < t_2 < \ldots < t_k < L$ and

$$0 \le \delta_j \le \lambda_0(t_j) - \sum_{i=1}^{j-1}\delta_i, \qquad 1 \le j \le k \qquad (4.13)$$

4.3.2 Optimal preventive maintenance strategy

The optimal parameters for the policy are obtained using a four-stage process as indicated below:

<u>Stage 1:</u> Fix $k \in \{1,2,...,k_{\max}\}$, where $k_{\max}$ refers to the smallest integer that is not less than $C_f \Lambda_0(L)/a$.

<u>Stage 2</u>: Fix $\underline{t}$. Solve $\min_{\underline{\delta}} J(k,\underline{t},\underline{\delta})$ globally for $\underline{\delta}^*(k,\underline{t}) = \{\delta_1^*, \delta_2^*, ..., \delta_k^*\}$.

<u>Stage 3</u>: Solve $\min_{\underline{t}} J(k,\underline{t},\underline{\delta}^*(k,\underline{t}))$ globally for $\underline{t}^*(k) = \{t_1^*, t_2^*, ..., t_k^*\}$.

<u>Stage 4</u>: Solve $\min_{k} J(k,\underline{t}^*(k),\underline{\delta}^*(k,\underline{t}^*(k)))$ globally for k^* by enumerating values of $k = 1,2,...$.

4.3.3 Special case 1: No penalty

The total expected cost to the lessor is given by (4.12) with $C_t = C_n = 0$. As a result, we have

$$J(k,\underline{t},\underline{\delta}) = C_f\left[\Lambda_0(L) - \sum_{j=1}^{k}\delta_j(L-t_j)\right] + ak + b\sum_{j=1}^{k}\delta_j \tag{4.14}$$

<u>Stage 1</u>: Fix $k \in \{1,2,...,k_{\max}\}$.

<u>Stage 2</u>: Fix $\underline{t}$. $J(k,\underline{t},\underline{\delta})$ can be rewritten as the function of $\underline{\delta}$ to be minimized as,

$$J_1(\underline{\delta}) = C_f\Lambda_0(L) + ak - C_f\sum_{j=1}^{k}\delta_j\left(L - t_j - \frac{b}{C_f}\right).$$

The constraints are

$$0 \le \delta_1 \le \lambda_0(t_1) - \lambda_0(t_0),$$
$$0 \le \delta_2 \le \lambda_0(t_2) - \lambda_0(t_1),$$
$$\vdots$$
$$0 \le \delta_k \le \lambda_0(t_k) - \lambda_0(t_{k-1}). \tag{4.15}$$

This is a separable program, thus it can be reduced to solving k one-dimensional optimization problems:

$$\min_{\delta_j} J_1(\delta_j) \equiv C_f\Lambda_0(L) + ak - C_f\left(L - t_j - \frac{b}{C_f}\right)\delta_j,$$

subjected to

$$0 \le \delta_j \le \lambda_0(t_j) - \lambda_0(t_{j-1}),$$

for $j = 1,2,...,k$.

Since $J_1(\delta_j)$ is linear in δ_j, the optimal value $\delta_j^*(k,\underline{t})$ is obtained as follows.

<u>Case 1</u>: $L - t_j - (b/C_f) \ge 0$ or $t_j \le L - (b/C_f)$. Then $\delta_j^*(k,\underline{t}) = \lambda_0(t_j) - \lambda_0(t_{j-1})$.

As a result, $J_1(\delta_j)$ has a global minimum at the maximum value of δ_j (see Figure 4.1).

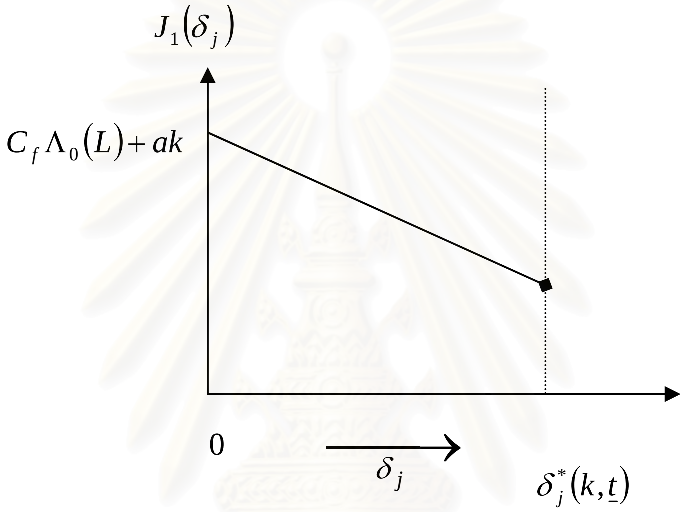

Figure 4.1 Global minimal $J_1(\delta_j)$ at maximum δ_j

<u>Case 2</u>: $L - t_j - (b/C_f) \le 0$ or $t_j \ge L - (b/C_f)$. Then $\delta_j^*(k,\underline{t}) = 0$.

As a result, $J_1(\delta_j)$ has a global minimum at the minimum value of δ_j (see Figure 4.2).

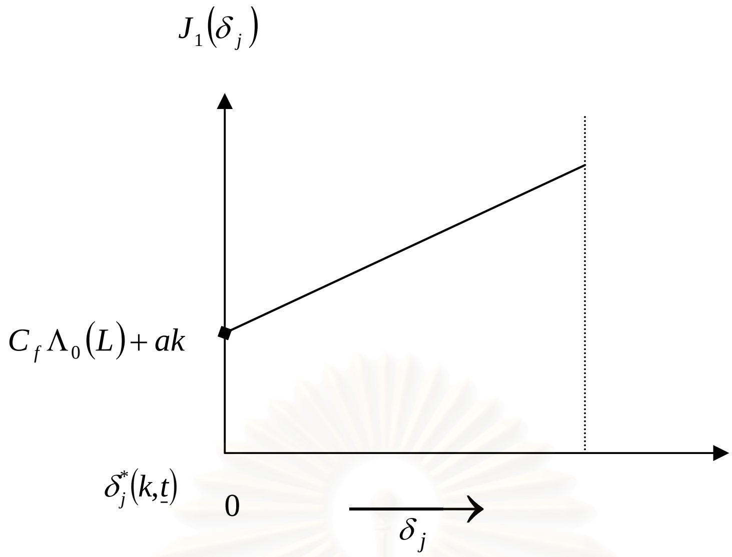

Figure 4.2 Global minimal $J_1(\delta_j)$ at minimum δ_j

Define

$$\tilde{L} = L - (b / C_f).$$

The optimal preventive maintenance action at t_j is, therefore, to reduce the failure intensity by the maximum amount if $t_j \le \tilde{L}$ and not to carry out any preventive maintenance action if $t_j \ge \tilde{L}$. Note that $\tilde{L}$ decreases as b increases and/or C_f decreases. Especially when $\tilde{L} \le 0$, then $t_j \ge \tilde{L}$ and hence no preventive maintenance action is needed for the optimal strategy. This situation arises when b is large (variable component of preventive maintenance cost) or C_f (corrective maintenance cost of each repair) is small so that it is cheaper to fix failure as opposed to carrying out preventive maintenance actions to reduce likelihood of failures.

<u>Stage 3</u>: Define $t_{k+1} = L$. As a result from Stage 2, $J(k, \underline{t}, \underline{\delta})$ to be minimised can be rewritten as

$$J\left(k, \underline{t}, \underline{\delta}^*(k, \underline{t})\right) = C_f \Lambda_0(L) + ak - C_f \sum_{j=1}^{k} \delta_j^* \left(\tilde{L} - t_j\right) \tag{4.16}$$

The constraints are $0 < t_1 < t_2 < ... < t_k < L$ for $j = 1, 2, ..., k$ and

$$\delta_j^* = \begin{cases} \lambda_0(t_j) - \lambda_0(t_{j-1}) & \text{if } t_j \le \tilde{L} \\ 0 & \text{if } t_j \ge \tilde{L} \end{cases} \tag{4.17}$$

We first focus on the analysis by ignoring the constraints $0 < t_1 < t_2 < ... < t_k < L$ and assume that $t_j \le \tilde{L}$ for all $1 \le j \le k$ are satisfied. Under these assumptions, for the fixed k, $J\left(k, \underline{t}, \underline{\delta}^*(k, \underline{t})\right)$ to be minimized can be rewritten as follows.

$$J_2(\underline{t}) = C_f \Lambda_0(L) + ak - C_f \left\{ \tilde{L}\lambda_0(t_k) - \sum_{j=1}^{k} t_j \left[\lambda_0(t_j) - \lambda_0(t_{j-1})\right] \right\} \qquad (4.18)$$

The problem is thus an unconstrained minimization problem $\min_{\underline{t}} J_2(\underline{t})$. The optimal values $\underline{t}^*(k)$ may be then obtained from the first order conditions (or differentiating $J_2(\underline{t})$ with respect to $t_j, 1 \le j \le k$, and setting the derivatives to zero). We will first demonstrate that there is only one solution satisfying the first order conditions. And, we will then show that these $\underline{t}^*(k)$ will indeed satisfy the all required constraints. Thus, if its Hessian matrix $\nabla^2 J_2\left(\underline{t}^*(k)\right)$ is positive definite, then $\underline{t}^*(k)$ obtained ignoring all constraints is indeed optimal at Stage 3.

Define

$$V_j = \frac{t_{j-1}^*}{t_j^*}, \qquad j = 1,2,...,k \qquad (4.19)$$

with $t_0^* = 0$. Then, after some analysis, we have

$$\frac{\partial J_2}{\partial t_j} = -C_f \left[t_{j+1}^* \frac{d\lambda_0(t_j)}{dt_j} - \left[\lambda_0(t_j^*) + t_j^* \frac{d\lambda_0(t_j)}{dt_j} \right] + \lambda_0(t_{j-1}^*) \right]$$

for $j \le k-1$ and equating it to zero, yields

$$\lambda_0(t_{j-1}^*) - \lambda_0(t_j^*) + (t_{j+1}^* - t_j^*)\frac{d\lambda_0(t_j)}{dt_j} = 0$$

$$\beta t_{j-1}^{*\,\beta-1} - \beta t_j^{*\,\beta-1} + t_{j+1}^* \beta(\beta-1) t_j^{*\,\beta-2} - t_j^* \beta(\beta-1) t_j^{*\,\beta-2} = 0$$

$$t_{j-1}^{*\,\beta-1} - \beta t_j^{*\,\beta-1} + (\beta-1) t_j^{*\,\beta-2} t_{j+1}^* = 0$$

$$V_j^{\beta-1} t_j^{*\,\beta-1} - \beta t_j^{*\,\beta-1} + (\beta-1) t_j^{*\,\beta-2} \frac{t_j^*}{V_{j+1}} = 0$$

$$V_j^{\beta-1} - \beta + \frac{\beta-1}{V_{j+1}} = 0$$

Then, the V_j 's are given by the following recursive relationship:

$$V_{j+1} = \frac{\beta - 1}{\beta - V_j^{\beta-1}} \text{ for } 1 \le j \le k-1. \tag{4.20}$$

We also have

$$\frac{\partial J_2}{\partial t_k} = -C_f \left[-\lambda_0\left(t_k^*\right) - t_k^* \frac{d\lambda_0\left(t_k\right)}{dt_k} + \tilde{L}\frac{d\lambda_0\left(t_k\right)}{dt_k} + \lambda_0\left(t_{k-1}^*\right) \right]$$

and equating it to zero, yields

$$\begin{aligned}
&\lambda_0\left(t_{k-1}^*\right) - \lambda_0\left(t_k^*\right) + \left(\tilde{L} - t_k^*\right)\frac{d\lambda_0\left(t_k\right)}{dt_k} &&= 0 \\
&\beta t_{k-1}^{*\,\beta-1} - \beta t_k^{*\,\beta-1} + \left(\tilde{L} - t_k^*\right)\beta(\beta-1)t_k^{*\,\beta-2} &&= 0 \\
&t_{k-1}^{*\,\beta-1} - t_k^{*\,\beta-1} + \left(\tilde{L} - t_k^*\right)(\beta-1)t_k^{*\,\beta-2} &&= 0 \\
&t_{k-1}^{*\,\beta-1} + \tilde{L}(\beta-1)t_k^{*\,\beta-2} - t_k^{*\,\beta-1} - (\beta-1)t_k^{*\,\beta-1} &&= 0 \\
&t_{k-1}^{*\,\beta-1} + \tilde{L}(\beta-1)t_k^{*\,\beta-2} - \beta t_k^{*\,\beta-1} &&= 0 \\
&V_k^{\beta-1}t_k^{*\,\beta-1} + \tilde{L}(\beta-1)t_k^{*\,\beta-2} - \beta t_k^{*\,\beta-1} &&= 0 \\
&V_k^{\beta-1} + \tilde{L}(\beta-1)\frac{1}{t_k^*} - \beta &&= 0
\end{aligned}$$

$$t_k^* = \left(\frac{\beta-1}{\beta - V_k^{\beta-1}}\right)\tilde{L} \tag{4.21}$$

As a result, $t_j^*, 1 \le j \le k$, are obtained recursively from (4.19) using (4.20) and (4.21), hence it is a unique solution of the first order conditions.

Next, we will show that $\underline{t}^*(k)$ indeed satisfies all the constraints. From (4.19) and $t_0^* = 0$, we have $V_1 = 0$. As a result, $V_2 = (\beta - 1)/\beta$ using (4.18). Noting $\beta > 1$, this implies that $0 < V_2 < 1$. On the other hand, V_2 is also given by (4.19) as $V_2 = t_1^* / t_2^*$. Thus, $0 < t_1^* / t_2^* < 1$ or this expression can be rewritten as $0 < t_1^* < t_2^*$. Next, by (4.20) $V_3 = (\beta - 1)/\left(\beta - V_2^{\beta-1}\right)$ where $V_2^{\beta-1} < 1$. As a result, $V_3 < 1$ and since $V_3 = t_2^* / t_3^*$ by (4.19), we have $t_2^* < t_3^*$. By repeatedly using (4.20) and (4.19), as to be expected, we can show that $V_j < 1$ for all j=1,2,…,k, and

$0 < t_1^* < t_2^* < ... < t_{j-1}^* < t_j^* < ... < t_{k-1}^* < t_k^*$. From (4.21) and $V_k^{\beta-1} < 1$, we have $t_k^* < \tilde{L}$. Hence, we can conclude that $\underline{t}^*(k)$ satisfies the constraints $0 < t_1^* < t_2^* < ... < t_{j-1}^* < t_j^* < ... < t_{k-1}^* < t_k^* < \tilde{L}$ as to be expected.

Although $\underline{t}^*$ can be obtained recursively, it cannot be analytically expressed. Therefore, it is very difficult to demonstrate $\nabla^2 J_2(\underline{t}^*(k))$ is positive definite. As such, we first consider this analysis for the case of k = 1. According to (4.17), we have

$$\delta_1^* = \lambda_0(t_1) = \beta t^{\beta-1}. \tag{4.22}$$

Then, from (4.21) we obtain

$$t_1^* = \left(\frac{\beta-1}{\beta}\right)\tilde{L} \tag{4.23}$$

where $V_1 = 0$.

To show that t_1^* is a global minimum, it is sufficient to show that the Hessian matrix of $\nabla^2 J_2(t_1^*)$ is positive. We have

$$\frac{\partial J_2(t_1)}{\partial t_1} = -C_f\beta\left[(\beta-1)\tilde{L}t_1^{\beta-2} - \beta t_1^{\beta-1}\right]$$

and

$$\begin{aligned}\frac{\partial^2 J_2(t_1)}{\partial t_1^2} &= -C_f\beta\left[(\beta-1)(\beta-2)\tilde{L}t_1^{\beta-3} - \beta(\beta-1)t_1^{\beta-2}\right] \\ &= -C_f\beta(\beta-1)t_1^{\beta-3}\left[(\beta-2)\tilde{L} - \beta t_1\right]\end{aligned}.$$

Replacing (4.23), then we obtain $\frac{\partial^2 J_2(t_1)}{\partial t_1^2} > 0$. This implies that $\nabla^2 J_2(t_1^*)$ is positive definite.

For the case k >1, we fail to prove it mathematically. Therefore, we only demonstrate through numerical experiment to verify the positive definiteness of the Hessian by randomly varying many parameters. As a result, we found that all the experiments show that the Hessian matrix of $J_2(\underline{t}^*(k))$ is positive definite (see more detail in Appendix A).

<u>Stage 4</u>: The k^* is obtained by minimising $J_3(k)$ given by

$$J_3(k) = J\left(k, \underline{t}^*(k), \underline{\delta}^*\left(k, \underline{t}^*(k)\right)\right) = C_f\left[\Lambda_0(L) - \sum_{j=1}^{k}\delta_j^*\left(L - t_j^*\right)\right] + ak + b\sum_{j=1}^{k}\delta_j^* \tag{4.24}$$

This is a complex discrete function of k and it is difficult to understand the nature of the function $J_3(k)$. Since $k_{\max}$ is not very large under a normal scenario, we adopt an enumerative search for k varying from 0 until $k_{\max}$ to obtain the global optimal k^*.

The total expected cost to the lessor under the optimal preventive maintenance policy is then given by (4.22) with k^*. As a result, we obtain k^*, $\underline{t}^*(k^*)$ and $\underline{\delta}^*(k^*, \underline{t}^*(k^*))$ that minimizes $J(k^*, \underline{t}^*, \underline{\delta}^*)$.

4.3.4 Special case 2: Penalty-1

The total expected cost to the lessor is given by (4.12) with $C_n = 0$. As a result, we have

$$
\begin{aligned}
J(k, \underline{t}, \underline{\delta}) &= C_f \Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j + C_t \Lambda(L)\int_0^{\tau} [1 - G(\tau)]dy \\
&= \left\{C_f + C_t \int_0^{\tau} [1 - G(\tau)]dy\right\}\Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j
\end{aligned}
\tag{4.25}
$$

Note that this is identical to Special Case 1 except that instead of C_f we have $\tilde{C}_f = C_f + C_t \int_0^{\tau} [1 - G(\tau)]dy$. Hence, the optimal parameters of the preventive maintenance policy for this case can be obtained as indicated earlier.

When $\tau \to \infty$, the case corresponds to no penalty associated with repair time. In this case, $\tilde{C}_f = C_f$ so that it reduces to the Special Case 1 as to be expected.

4.3.5 Special case 3 : Penalty-2

The total expected cost to the lessor is given by (4.12) with $C_t = 0$. As a result, we have

$$
J(k, \underline{t}, \underline{\delta}) = C_f \Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j + C_n \Lambda(L) \tag{4.26}
$$

We can rewritten (4.26) as

$$
J(k, \underline{t}, \underline{\delta}) = (C_f + C_n)\Lambda(L) + ak + b\sum_{j=1}^{k} \delta_j \tag{4.27}
$$

Note that this is identical to Special Case 1 except that instead of C_f we have $\tilde{C}_f = C_f + C_n$. Hence, the optimal parameters of the preventive maintenance policy for this case can be obtained as indicated earlier.

When there is no penalty for failures, $C_n = 0$, then $\tilde{C}_f = C_f$ so that it reduces to the Special Case 1 as to be expected.

4.4 Numerical example: No penalty

The nominal values for the model parameters are as follows:

- Lease period L = 5 (years)
- Corrective maintenance cost C_f = 100 (\$)
- Preventive maintenance cost parameters a = 100 (\$) and b = 50 (\$)
- Shape parameter for Weibull intensity function β = 2

4.4.1 Optimal preventive maintenance with no penalty

The optimal preventive maintenance strategy using the three-stage approach in Section 4.3 is shown in Table 4.1 for k varying from 1 to $k_{\max}$. Note that $k_{\max} = \lceil 100(5)^2/100 \rceil = 25$. With no preventive maintenance action, the expected cost is 2500.00 (\$). The optimal number of preventive maintenance actions that minimizes the total expected cost to the lessor is 4 as highlighted in the table and also see Figure 4.3. This implies that $k^* = 4$.

Table 4.1 Optimal preventive maintenance strategy for No Penalty case

k	$J(k)$
1	1587.50
2	1350.00
3	1281.25
4	**1280.00**
5	1312.50
6	1364.29
7	1428.13
8	1500.00
9	1577.50
10	1659.09
11	1743.75
12	1830.77
13	1919.64
14	2010.00
15	2101.56
16	2194.12
17	2287.50
18	2381.58
19	2476.25
20	2571.43
21	2667.05
22	2763.04
23	2859.37
24	2956.00
25	3052.88

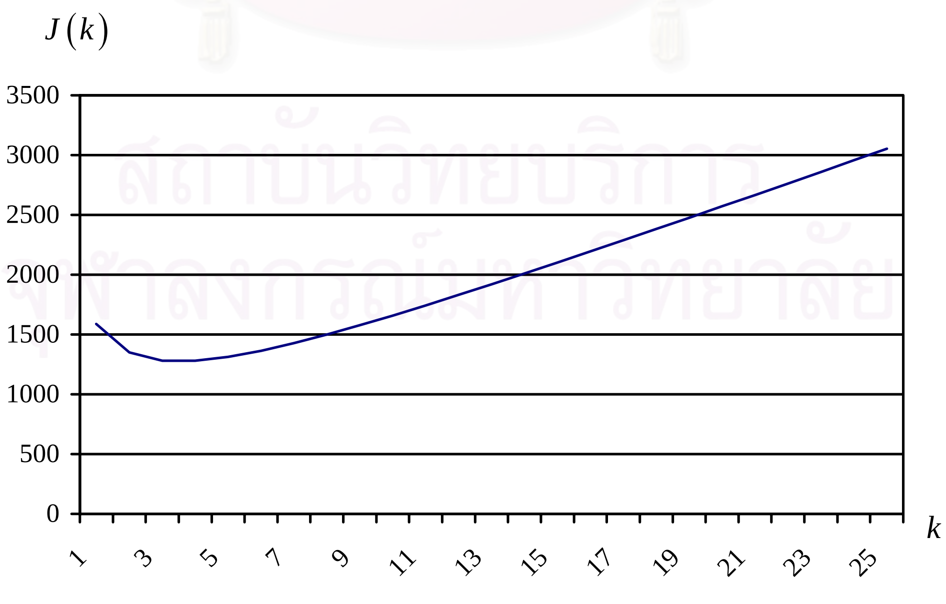

Figure 4.3 Total expected cost for No Penalty case

Note that the optimal $\{\underline{t}\}$ and $\{\underline{\delta}\}$ obtained from using Stages 1 and 2. Table 4.2 represents the optimal time instants for preventive maintenance action t_j^*, $1 \le j \le k^* + 1$ for $k^* = 4$. Note that $t_0^* = 0$ and $t_{k^*+1}^* = L$. The last preventive maintenance action t_4^* is less than $\widetilde{L} = 5 - 50/100 = 4.5$ years. We can see that the time intervals between optimal preventive maintenance actions are the same. And, we also have $\underline{\delta}^*$ = 1.8000 for all j as a result of β = 2. This is due to the fact that the failure intensity increases at the constant rate.

Table 4.2: Time intervals between optimal preventive maintenance actions

j	t_j^*
1	0.9000
2	1.8000
3	2.7000
4	3.6000
5	5.0000

Table 4.3 gives expected number of failures over $\left[t_{j-1}^*, t_j^*\right)$ for $1 \le j \le k^* + 1$ with no preventive maintenance and with optimal preventive maintenance.

Table 4.3 Expected number of failures in different intervals

j	t_j^*	No PM		Optimal PM	
		$E\left[N\left(t_j^*\right)\right]$	$E\left[N\left(t_j^*\right)\right] - E\left[N\left(t_{j-1}^*\right)\right]$	$E\left[N\left(t_j^*\right)\right]$	$E\left[N\left(t_j^*\right)\right] - E\left[N\left(t_{j-1}^*\right)\right]$
1	0.9000	0.8100	0.8100	0.8100	0.8100
2	1.8000	3.2400	2.4300	1.6200	0.8100
3	2.7000	7.2900	4.0500	2.4300	0.8100
4	3.6000	12.9600	5.6700	3.2400	0.8100
5	5.0000	25.0000	17.7100	5.2000	2.7700

Note that with no preventive maintenance action, the expected number of failures over time increases more rapidly than with preventive maintenance actions. This implies that the reduction of failures resulting from the effect of preventive maintenance actions. With preventive maintenance strategy, the expected number of failures between preventive maintenance actions is the same for all intervals except for the last.

4.4.2 Sensitivity analysis

We now carry out a sensitivity analysis to study the effect of changes to the model parameters on the optimal preventive maintenance actions. We vary one parameter at a time holding the remaining at their nominal values.

4.4.2.1 Effect of β variations

Table 4.4 illustrates the effect of the shape parameter on the optimal preventive maintenance actions. When β increases from 2 to 3, the optimal number of preventive maintenance actions k^* increases. In addition, the intervals between preventive maintenance actions become smaller while the reduction in the intensity function becomes larger. Note that for $\beta = 3$, δ_j^* increases with j. This is different from the case when β decreases from 2 to 1.5. In this case, k^* decreases, while the intervals between preventive maintenance actions become larger. Also, δ_j^* decreases with j. The optimal total expected cost to the lessor increases significantly with β increasing as to be expected.

Table 4.4 Effect of β

β	**1.5**			**2**			**3**		
$E\left[N(L)\right]$ No PM	11.18			25			125		
$E\left[N(L)\right]$ Optimal PM	3.60			5.20			22.70		
$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$615.31			\$1,280.00			\$5,437.03		
k^*	2			4			10		
j	t_j^*	$t_j^* - t_{j-1}^*$	δ_j^*	t_j^*	$t_j^* - t_{j-1}^*$	δ_j^*	t_j^*	$t_j^* - t_{j-1}^*$	δ_j^*
1	0.8129	0.8129	1.3524	0.9000	0.9000	1.8000	1.0374	1.0374	3.2287
2	2.4386	1.6257	0.9900	1.8000	0.9000	1.8000	1.5561	0.5187	4.0359
3				2.7000	0.9000	1.8000	1.9884	0.4323	4.5969
4				3.6000	0.9000	1.8000	2.3737	0.3863	5.0418
5							2.7277	0.3540	5.4178
6							3.0587	0.3310	5.7465
7							3.3718	0.3131	6.0406
8							3.6704	0.2986	6.3081
9							3.9569	0.2865	6.5542
10							4.2329	0.2760	6.7829

Table 4.5 represents the effects of preventive maintenance cost parameters *a* (fixed cost) and *b* (variable cost).

Table 4.5 Effect of *a* and *b*

Parameters			*b*		
			\$20	\$50	\$100
a	\$80	$J\left(k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$976.80	\$1200.00	\$1540.00
		k^*	4	4	3
	\$100	$J\left(k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$1056.80	\$1280.00	\$1600.00
		k^*	4	4	3
	\$150	$J\left(k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$1222.00	\$1431.25	\$173.33
		k^*	3	3	2

As can be seen from Table 4.5, we have k^* increases as *a* and *b* decrease. This is to be expected as lower fixed and variable preventive maintenance cost result in more often preventive maintenance actions.

4.5 Numerical example: Penalty-1

4.5.1 The optimal preventive maintenance with Penalty-1

In this section, we present the optimal preventive maintenance actions with τ = 2 days corresponding to $C_t = 300(\$)$ (see Table 4.6). Note that the repair time distribution is given by a two-parameter Weibull

$$G(y) = 1 - \exp\left[-\left(\frac{y}{\varphi}\right)^m\right],\ 0 \le y \le \infty \tag{4.28}$$

with the shape parameter *m* < 1 (implying decreasing repair rate). We assume that *m* = 0.5 and φ = 0.5 so that the mean time to repair is 1 day.

Table 4.6 Optimal preventive maintenance strategy for Penalty-1 case

k	$J(k)$
1	3116.89
2	2374.17
3	2052.81
4	1899.99
5	1831.45
6	**1811.06**
7	1820.77
8	1850.54

k	$J(k)$
9	1894.36
10	1948.39
11	2010.09
12	2077.68
13	2149.89
14	2225.82
15	2304.75
16	2386.16
17	2469.64
18	2554.85
19	2641.54
20	2729.51
21	2818.56
22	2908.57
23	2999.41
24	3090.98
25	3183.20
26	3276.00
27	3369.31
28	3463.08
29	3557.27
30	3651.84
31	3746.74
32	3841.95
33	3937.44
34	4033.19
35	4129.18
36	4225.39
37	4321.79
38	4418.38
39	4515.14
40	4612.05
41	4709.12
42	4806.32
43	4903.64
44	5001.09
45	5098.65
46	5196.31
47	5294.07
48	5391.92
49	5489.85
50	5587.87
51	5685.97
52	5784.13
53	5882.36
54	5980.66
55	6079.02

The optimal k that minimizes the total expected cost at 1811.06 ($) is $k^* = 6$ where $k_{\max} = \left\lceil 221.80(5)^2/100 \right\rceil = 55$ see Figure 4.4.

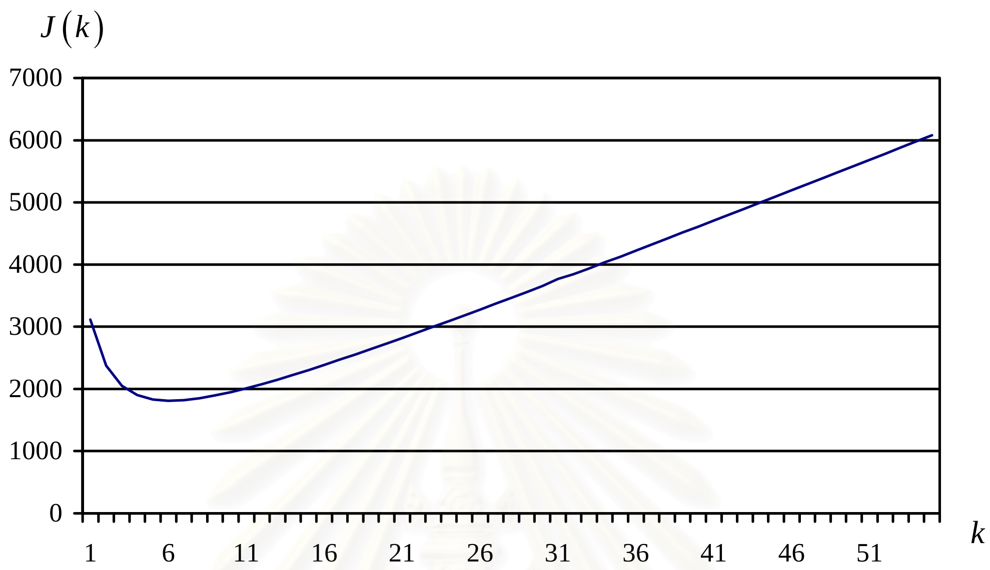

Figure 4.4 Total expected cost for Penalty-1 case

Table 4.7shows the optimal time instants for preventive maintenance action t_j^*, $1 \le j \le k^* + 1$ for $k^* = 6$. The last preventive maintenance action t_6^* is less than $\tilde{L} = 5 - 50/221.80 = 4.7746$ years. The optimal preventive maintenance levels $\underline{\delta}^*$ is 1.3642 for all j as to be expected due to β = 2.

Table 4.7 Optimal parameters for Penalty-1 case

j	t_j^*
1	0.6821
2	1.3642
3	2.0462
4	2.7283
5	3.4104
6	4.0925
7	5.0000

4.5.2 Sensitivity analysis

4.5.2.1 Effect of τ variations

Table 4.8 gives the optimal preventive maintenance actions for τ varying from 1 to 3 days. Note that $\tau \to \infty$ corresponds to No Penalty case.

Table 4.8 Effect of τ

Parameters	τ (Days)			
	1	2	3	∞
$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1992.33	\$1811.06	\$1693.49	\$1280.00
k^*	7	6	6	4

As can be seen, k^* increases as τ decreases. This can be explained as follows. As τ decreases, the expected Penalty-1 cost increases and consequently preventive maintenance action is needed to reduce the likelihood of failures. This implies more frequent preventive maintenance actions or increasing in k^*. The total expected cost to the lessor increases as τ decreases as to be expected.

4.6 Numerical example: Penalty-2

4.6.1 The optimal preventive maintenance with Penalty-2

The optimal preventive maintenance actions with Penalty-2 for C_n = 200 (\$) corresponding to all failures cause to penalty to the lessor (see Table 4.9).

Table 4.9 Optimal preventive maintenance strategy for Penalty-2 case

k	$J(k)$
1	4095.83
2	3027.78
3	2543.75
4	2293.33
5	2159.72
6	2092.86
7	**2067.71**
8	2070.37
9	2092.50
10	2128.79
11	2175.69
12	2230.77
13	2292.26
14	2358.89
15	2429.69
16	2503.92
17	2581.02
18	2660.53
19	2742.08
20	2825.40

21	2910.23
k	$J(k)$
22	2996.38
23	3083.68
24	3172.00
25	3261.22
26	3351.23
27	3441.96
28	3533.33
29	3625.28
30	3717.74
31	3810.68
32	3904.04
33	3997.79
34	4091.90
35	4186.34
36	4281.08
37	4376.10
38	4471.37
39	4566.88
40	4662.60
41	4758.53
42	4854.65
43	4950.95
44	5047.41
45	5144.02
46	5240.78
47	5337.67
48	5434.69
49	5531.83
50	5629.08
51	5726.44
52	5823.90
53	5921.45
54	6019.09
55	6116.82
56	6214.62
57	6312.50
58	6410.45
59	6508.47
60	6606.56
61	6704.70
62	6802.91
63	6901.17
64	6999.49
65	7097.85
66	7196.27
67	7294.73

68	7393.24
k	$J(k)$
69	7491.79
70	7590.38
71	7689.00
72	7787.67
73	7886.37
74	7985.11
75	8083.88

The optimal k that minimizes the total expected cost at 2067.71 (\$) is k^* = 7 where $k_{\max} = \lceil 300(5)^2/100 \rceil = 75$ see Figure 4.5.

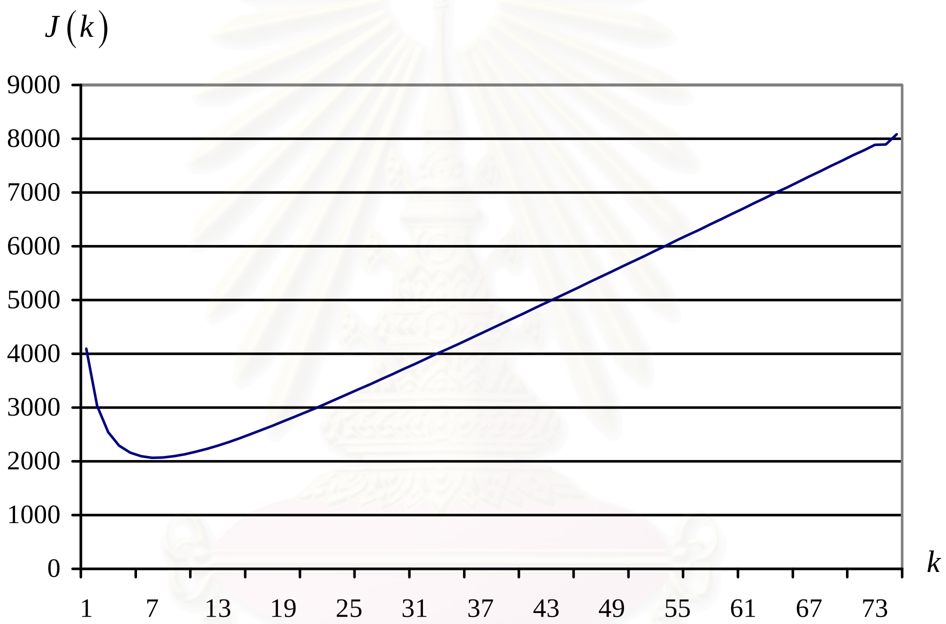

Figure 4.5 Total expected cost for Penalty-2 case

Table 4.10 demonstrates the optimal time instants for preventive maintenance action t_j^* , $1 \le j \le k^* + 1$ for $k^* = 7$. The last preventive maintenance action t_7^* is less than $\tilde{L} = 5 - 50/300 = 4.8333$ years. The optimal preventive maintenance levels $\underline{\delta}^*$ is 1.2083 for all j as to be expected due to β = 2.

Table 4.10 Optimal parameters for Penalty-2 case

j	t_j^*
1	0.6042
2	1.2083
3	1.8125
4	2.4167
5	3.0208
6	3.6250
7	4.2292
8	5.0000

4.6.2 Sensitivity analysis

4.6.2.1 Effect of C_n variations

Table 4.11 shows the results with C_n changing from zero. Note that, when $C_n = 0$ corresponds to No Penalty Case.

Table 4.11 Effect of C_n

Parameters	C_n			
	\$0	\$100	\$200	\$300
$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1280.00	\$1732.14	\$2067.71	\$2344.38
k^*	4	6	7	9

From the result of Table 4.11, k^* increases as C_n increases. An explanation is as follows. As C_n increases, the expected Penalty-2 cost increases and one way of reducing this is through more frequent preventive maintenance actions and this in turn reduces the expected number of failures. The total expected cost to the lessor increases as C_n increases as to be expected.

4.7 Numerical example: Penalties 1 & 2

4.7.1 The optimal preventive maintenance with Penalties 1 & 2

The general case includes both penalties in the model. For τ = 2 days and C_n = 200 (\$) (see Table 4.12).

Table 4.12 Optimal preventive maintenance strategy for Penalties 1 & 2 case

k	$J(k)$
1	5619.56
2	4044.40
3	3306.82
4	2904.27
5	2669.23
6	2529.93
7	2450.44
8	2410.85
9	**2399.17**
10	2407.80
11	2431.65
12	2467.22
13	2512.00
14	2564.14
15	2622.26
16	2685.31
17	2752.46
18	2823.07
19	2896.62
20	2972.69
21	3050.94
22	3131.07
23	3212.86
24	3296.11
25	3380.65
26	3466.33
27	3553.04
28	3640.66
29	3729.11
30	3818.30
31	3908.17
32	3998.65
33	4089.69
34	4181.24
35	4273.27
36	4365.72
37	4458.57
38	4551.79
39	4645.35
40	4739.22
41	4833.38
42	4927.82
43	5022.50
44	5117.43

k	$J(k)$
45	5212.57
46	5307.92
47	5403.47
48	5499.19
49	5595.09
50	5691.15
51	5787.36
52	5883.71
53	5980.20
54	6076.82
55	6173.55
56	6270.41
57	6367.37
58	6464.43
59	6561.59
60	6658.84
61	6756.19
62	6853.61
63	6951.12
64	7048.70
65	7146.36
66	7244.09
67	7341.88
68	7439.74
69	7537.66
70	7635.64
71	7733.67
72	7831.76
73	7929.90
74	8028.09
75	8126.32
76	8224.61
77	8322.93
78	8421.30
79	8519.71
80	8618.16
81	8716.65
82	8815.17
83	8913.73
84	9012.32
85	9110.94
86	9209.60
87	9308.29
88	9407.01
89	9505.75
90	9604.52
91	9703.32

k	$J(k)$
92	9802.15
93	9901.00
94	9999.87
95	10098.77
96	10197.69
97	10296.63
98	10395.60
99	10494.58
100	10593.59
101	10692.61
102	10791.66
103	10890.72
104	10989.80
105	11088.89

The minimum total expected cost to the lessor becomes 2399.17 ($) corresponding to $k^* = 9$ where $k_{\max} = \lceil 421.80(5)^2/100 \rceil = 105$ see Figure 4.6.

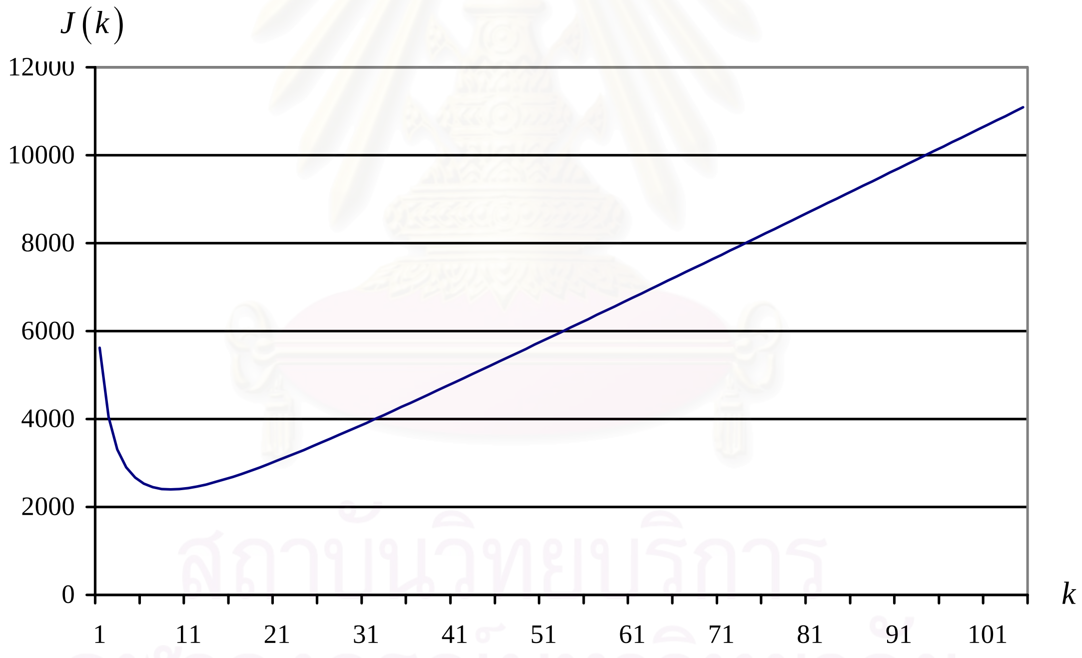

Figure 4.6 Total expected cost for Penalties 1 & 2 Case

The optimal time instants are shown in Table 4.13 for preventive maintenance action t_j^*, $1 \le j \le k^* + 1$ for $k^* = 9$. The last preventive maintenance action t_9^* is less than $\tilde{L} = 5 - 50/421.80 = 4.8814$ years. The optimal preventive maintenance levels $\underline{\delta}^*$ is 0.9763for all *j* as to be expected due to $\beta = 2$.

Table 4.13 Optimal parameters for Penalties 1 & 2 case

j	t_j^*
1	0.4881
2	0.9763
3	1.4644
4	1.9526
5	2.4407
6	2.9289
7	3.4170
8	3.9052
9	4.3933
10	5.0000

4.7.2 Sensitivity analysis

4.7.2.1 Effect of τ and C_n variations

$C_n = 0$ and $\tau \to \infty$ imply no penalty costs and the optimal preventive maintenance actions aim to achieve a trade-off between the two maintenance costs (preventive maintenance and corrective maintenance). When $C_n > 0$ and $\tau < \infty$ we have Penalty-1 and Penalty-2 costs and we now study the effect of these two parameters on the optimal preventive maintenance actions.

Table 4.14 Effect of τ and C_n

Parameters			C_n			
			\$0	\$100	\$200	\$300
τ (Days)	1	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1992.33	\$2283.20	\$2531.77	\$2754.52
		k^*	7	8	10	11
	2	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1811.06	\$2131.43	\$2399.17	\$2636.10
		k^*	6	8	9	10
	3	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1693.49	\$2034.15	\$2317.59	\$2562.05
		k^*	6	7	9	10
	∞	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$1280.00	\$1732.14	\$2067.71	\$2344.38
		k^*	4	6	7	9

Table 4.14 shows the effect of C_n and τ on the optimal preventive maintenance actions and the increase in the lessor's total expected cost. The number of preventive maintenance actions increases as τ decreases (for reasons discussed in

Section 4.6) and as C_n increases (for reasons discussed in Section 4.7). The number of preventive maintenance actions increases from 4 to 11 when the repair time limit $\tau = 1$ day and penalty for failures is $C_n = \$300$.

4.8 Comparison between many cases

In this thesis, we analyze many cases including No Penalty, Penalty-1, Penalty-2, and Penalties 1 & 2 (also called as general case). Table 4.15 show details of the results among these cases.

Table 4.15 Comparison between many cases

Cases	k^*	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$
No Penalty	4	\$1280.00
Penalty-1	6	\$1811.06
Penalty-2	7	\$2067.71
Penalties 1 & 2	9	\$2399.17

As can be seen, when the lessor does not include any penalty term in the contract, the total expected cost is smallest comparatively to other cases. However, the contract will not be attractive to the lessee. This can be explained that the penalty term can guarantee the equipment reliability performance for the lessee. With penalty included in the leasing contract, we can see that Penalty-1 has less influence than Penalty-2. If the lessor includes both penalties, he would incur to the largest amount of total expected cost of maintenance relatively to with only one penalty or without penalty term.

4.9 Conclusion

For the lease of new equipment, the lessor carries out the maintenance of the equipment and needs to take into account the penalty costs stated in the lease contract so as to determine the optimal preventive maintenance actions. The costs associated with failures are high as unplanned corrective maintenance actions are costly and the resulting penalties due to against to the lease contract terms. In this chapter we have developed a model where preventive maintenance actions result in a reduction in the failure intensity function. The optimal parameters are determined by minimizing a cost function that takes into account the corrective maintenance and preventive maintenance costs as well as the penalty costs. The decision variables of the policy are (i) the number of preventive maintenance action to be carried out over the lease period, (ii) the time instants for such actions, and (iii) the preventive maintenance action levels. We present a numerical example that represents the optimal values of preventive maintenance actions and highlights the effect of penalty terms on the optimal preventive maintenance strategy.

CHAPTER V

ANALYSIS OF USED EQUIPMENT LEASE

5.1 Introduction

In this chapter we extend the results to the lease of used equipment. Hence, an upgrade is considered as an additional option prior to it is being leased. The outline of this chapter is as follows. We give the details of the model formulations in Section 5.2. An extra decision variable that the lessor needs to select optimally to minimize the total expected cost of maintaining the equipment over the lease period is determined. Such analytical analysis is conducted in Section 5.3. Then, a numerical example for the optimal upgrade and PM actions is illustrated in Section 5.4. Besides, sensitivity analysis of all influenced parameters of the model and the effect of two penalties are also presented in this section. In Section 5.5, the special cases along with relative comparison are demonstrated. A conclusion is given briefly in Section 5.6.

5.2 Model formulation

The equipment age A is leased for a period L. The lease contract requires that the lessor is responsible for all rectifications cost when failures occur during the lease period at no additional cost to the lessee. The lease contract also includes penalties if the number of failures over the lease period and if a repair is not carried out within the specified time limit τ (> 0). $N(L)=0$ and $\tau \to \infty$ correspond to no penalty.

The lessor has an extra option of upgrading prior to lease the equipment and/or carry out preventive maintenance during the lease period to reduce the costs associated with corrective maintenance and penalty due to equipment failures. However, these upgrade action and preventive maintenance actions involve additional costs and are worthwhile only if the benefits exceed the additional costs. In other words, the upgrade and optimal preventive maintenance actions need to be selected in order to minimize the total expected maintenance cost over the lease period.

5.2.1 Equipment failures

We first study the case where the equipment is leased without any upgrade and no preventive maintenance actions used during the lease period. We assume that minimal repairs are carried out to rectify all failures with the time to repair relatively small in relation to mean time between failures (Barlow and Hunter, 1960). In this case, equipment failures over the lease period occur according to a Non-Homogeneous Poisson Process (NHPP) with intensity function $\lambda(t) = \lambda_0(A+t)$, $t \geq 0$. (This is also called as the rate of occurrence of failures – ROCOF – in the reliability literature). This is assumed to be an increasing function of t indicating that the number of failures per unit time (in a statistical sense) increases with age. We consider the two-parameter Weibull intensity function given by (4.2).

5.2.2 Upgrade and preventive maintenance actions

The effect of upgrade action is to rejuvenate the equipment so that its virtual age is $A - x$. This can be achieved through replacing worn out components. The level of upgrade depends on the number of components replaced (also called the upgrade effort). Replacing all the components implies restoring the equipment to as good as new which would costs to the lessor as buying new equipment. Therefore, the possible maximum magnitude of upgrading effort should not be equal or greater than equipment age. If the reduction in the age is x, then the constraint for upgrade will be $0 \leq x < A$. The failures over the lease period occur as a point process with intensity function given by

$$\lambda(t) = \lambda_0(A+t-x) \tag{5.1}$$

We confine our attention to the following preventive maintenance policy. The equipment is subjected to k preventive maintenance actions over the lease period and these occur at time instants $0 < t_1 < t_2 < .... < t_k < L$. Note that the failure rate never goes down below $\lambda(0) = \lambda(A-x)$. Thus, the effect of the j^{th} preventive maintenance action is to reduce the intensity function by δ_j subject to the constraint

$$0 \leq \delta_j \leq \lambda_0(A+t_j-x) - \sum_{i=1}^{j-1}\delta_i - \lambda_0(A-x), 1 \leq j \leq k \tag{5.2}$$

This implies that the rate of occurrence of failures decreases with each preventive maintenance action within the reduction limit.

The effect of preventive maintenance action is to reduce the failure intensity rate for failure occurrence so that it is given by

$$\lambda(t) = \lambda_0(A + t - x) - \sum_{i=0}^{j} \delta_i, t_j < t \le t_{j+1} \tag{5.3}$$

where $t_0 = 0$ and $\delta_0 = 0$.

5.2.3 Lessor's decision problem

The optimization problem is to select the optimal upgrade, x^*, the optimal number of preventive maintenance actions, k^*, the optimal preventive maintenance schedule, $\underline{t}^* = \{t_j^*\}$, and the optimal preventive maintenance levels, $\underline{\delta}^* = \{\delta_j^*\}$ to minimize the total expected cost given by (3.11).

5.3 Model analysis

5.3.1 Expected costs

Define $\Lambda(L) = \int_0^L \lambda(t)dt$. Then, from (5.3) we have

$$\Lambda(t) = \Lambda_0(A + L - x) - \Lambda_0(A - x) - \sum_{j=1}^{k} \delta_j (L - t_j) \tag{5.4}$$

where $\Lambda_0(t) = t^\beta$. Since failures over the lease period occur according to a NHPP with intensity function given by (5.3) we have the result as being illustrated in (4.10).

$$E[N(L)] = \Lambda(L) = (A + L - x)^\beta - (A - x)^\beta - \sum_{j=1}^{k} \delta_j (L - t_j) \tag{5.5}$$

The total expected maintenance cost to the lessor is given by (3.11) and consists of five components. These are as follows.

(i) The expected cost resulting from corrective maintenance actions is given by (4.6).

(ii) The cost of preventive maintenance actions is given by (4.7).

(iii) The penalty costs for repairs not completed within the specified time limit, τ, is given by (4.8).

(iv) The penalty costs associated with the number of failures is given by (4.9).

(v) The upgrade cost, $C_u(x)$, is an increasing function of *x* and is given by

$$C_u(x) = \frac{wx}{1-e^{-\varphi(A-x)}} \tag{5.6}$$

where the parameters *w* > 0 and $\varphi > 0$. Note that when *x* = 0 we have no upgrade and as a result $C_u(0)=0$. As $x \to A, C_u(x) \to \infty$ implying that it is not possible to upgrade used equipment to as good as new.

Using all of those five costs in (3.11) results in

$$\begin{aligned} J(x,k,\underline{t},\underline{\delta}) = &\left\{C_f + C_t\int_{\tau}^{\infty}\left[1-G(\tau)\right]dy + C_n\right\}\left[(A+L-x)^{\beta} - (A-x)^{\beta} - \sum_{j=1}^{k}\delta_j(L-t_j)\right] \\ &+ak + b\sum_{j=1}^{k}\delta_j + C_u(x) \end{aligned} \tag{5.7}$$

and the parameters of the PM policy are selected to minimize this subject to the following constraints:

$$0 \le x < A,\ 0 < t_1 < t_2 < \ldots < t_k < L$$

and

$$0 \le \delta_j \le \lambda_0(A + t_j - x) - \sum_{i=1}^{j-1}\delta_i - \lambda_0(A-x), 1 \le j \le k$$

(5.8)

5.3.2 Optimal preventive maintenance strategy

The optimization is a four-parameter problem with two parameters of $\{\underline{t}\}$ and $\{\underline{\delta}\}$ being *k*-dimensional. We solve it using a four-stage approach where at each stage we solve a one-parameter optimization and the other parameters are fixed.

Stage 1: Compute $\underline{\delta}^*(x,k,\underline{t})$, the value of $\underline{\delta}$ that minimizes $J(x,k,\underline{t},\underline{\delta})$ globally.

Stage 2: Compute $\underline{t}^*(x,k)$, the value of $\underline{t}$ that minimizes $J(x,k,\underline{t},\underline{\delta}^*(x,k,\underline{t}))$ globally.

Stage 3: Compute $k^*(x)$, the value of k that minimizes $J\left(x,k,\underline{t}^*(x,k),\underline{\delta}^*\left(x,k,\underline{t}^*(x,k)\right)\right)$ globally.

Stage 4: Compute x^*, the value of x that minimizes

$J\left(x, k^*(x), \underline{t}^*\left(x, k^*(x)\right), \underline{\delta}^*\left(x, k^*(x), \underline{t}^*\left(x, k^*(x)\right)\right)\right)$ globally.

Once x^* is obtained, then proceeding backwards yields $k^* = k^*(x^*)$, $\underline{t}^* = \underline{t}^*(x^*, k^*)$ and $\delta^* = \underline{\delta}^*(x^*, k^*, \underline{t}^*)$, the optimal values that minimize $J(x, k, \underline{t}, \underline{\delta})$.

Define

$$\tilde{C}_f = C_f + C_t \int_{\tau}^{\infty} \left[1 - G(\tau)\right] dy + C_n,$$

$t_0 = 0$ and $t_{k+1} = L$.

We give the results of the analysis for the four stages as follows.

Stage 1: First x and k are fixed. Note that $x \in \{0.0, 0.1A, 0.2A, ..., 0.9A\}$ and $k \in \{1, 2, ..., k_{\max}\}$, where $k_{\max}$ refers to the smallest integer that is not less than $\tilde{C}_f \Lambda_0(L)/a$. $\underline{\delta}^*(x, k, \underline{t})$ is obtained by minimizing

$$\begin{aligned} J_4(\underline{\delta}) &= \tilde{C}_f \left[\Lambda_0(A + L - x) - \Lambda_0(A - x) - \sum_{j=1}^{k} \delta_j (L - t_j)\right] + \sum_{j=1}^{k} (a + b\delta_j) + C_u(x) \\ &= \tilde{C}_f \left[\Lambda_0(A + L - x) - \Lambda_0(A - x)\right] + ak - \tilde{C}_f \sum_{j=1}^{k} \delta_j \left(L - t_j - \frac{b}{\tilde{C}_f}\right) + C_u(x) \end{aligned} \tag{5.9}$$

subject to the constraints given by

$$0 \le \delta_1 \le \lambda_0(A + t_1 - x) - \lambda_0(A - x)$$

$$0 \le \delta_2 \le \lambda_0(A + t_2 - x) - \lambda_0(A + t_1 - x)$$

$$\vdots$$

$$0 \le \delta_k \le \lambda_0(A + t_{1k} - x) - \lambda_0(A + t_{k-1} - x)$$

This is a separable program, thus it can be reduced to solving k one-dimensional optimization problems:

$$\min_{\delta_j} J_4(\delta_j) \equiv \tilde{C}_f \left[\Lambda_0(A + L - x) - \Lambda_0(A - x)\right] + ak + C_u(x) - \tilde{C}_f \left(L - t_j - \frac{b}{\tilde{C}_f}\right)\delta_j$$

subjected to

$$0 \le \delta_j \le \lambda_0(A + t_j - x) - \lambda_0(A + t_{j-1} - x)$$

for j = 1, 2, …, k.

Since $J_4\left(\delta_j\right)$ is linear in δ_j, the optimal value $\delta_j^*\left(x,k,\underline{t}\right)$ corresponds to the extreme points of the constraint intervals. This yields two cases as follows.

Case 1: $L-t_j-\left(b/\tilde{C}_f\right)\geq 0$ or $t_j\leq L-\left(b/\tilde{C}_f\right)$ for $1\leq j\leq k$. Then, $\delta_j^*\left(x,k,\underline{t}\right)=\lambda_0\left(A+t_j-x\right)-\lambda_0\left(A+t_{j-1}-x\right)$. As a result, $J_4\left(\delta_j\right)$ has a global minimum at the maximum value of δ_j.

Case 2: $L-t_j-\left(b/\tilde{C}_f\right)\leq 0$ or $t_j\geq L-\left(b/\tilde{C}_f\right)$ for $j>k$. Then, $\delta_j^*\left(x,k,\underline{t}\right)=0$. As a result, $J_4\left(\delta_j\right)$ has a global minimum at the minimum value of δ_j.

Define $\tilde{L}=L-\left(b/\tilde{C}_f\right)$.

This implies that the optimal preventive maintenance action at t_j is to reduce the failure intensity by the maximum amount permissible if $t_j\leq\tilde{L}$ and not worthwhile to carry out preventive maintenance action if $t_j\geq\tilde{L}$.

Stage 2: Let $\tilde{k}$ (which can depend on x) be the largest j for $t_j\leq\tilde{L}$. We have

$$
\begin{aligned}
J_5(\underline{t}) &= \tilde{C}_f\left[\Lambda_0(A+L-x)-\Lambda_0(A-x)\right]+a\tilde{k}-b\lambda_0(A-x)+C_u(x) \\
&-\tilde{C}_f\left\{\sum_{j=1}^{\tilde{k}-1}\left[(A+t_{j+1}-x)\beta(A+t_j-x)^{(\beta-1)}-(A+t_j-x)\beta(A+t_j-x)^{(\beta-1)}\right.\right. \\
&\left.+(A+t_j-x)\beta(A+t_{j-1}-x)^{(\beta-1)}-(A+t_{j-1}-x)\beta(A+t_{j-1}-x)^{(\beta-1)}\right] \\
&+\left(A+L-x-\frac{b}{\tilde{C}_f}\right)\beta(A+t_{\tilde{k}}-x)^{(\beta-1)}-(A+t_{\tilde{k}}-x)\beta(A+t_{\tilde{k}}-x)^{(\beta-1)} \\
&\left.-(A+L-x)\beta(A+t_{\tilde{k}-1}-x)^{(\beta-1)}+(A+t_{\tilde{k}}-x)\beta(A+t_{\tilde{k}-1}-x)^{(\beta-1)}\right\}
\end{aligned}
\tag{5.12}
$$

The constraints are

$0<t_1<t_2<\ldots<t_k<L$ for j = 1, 2, …, k and

$$
\delta_j^*=\begin{cases}\lambda_0\left(A+t_j-x\right)-\lambda_0\left(A+t_{j-1}-x\right) & \text{if } t_j\leq\tilde{L} \\ 0 & \text{if } t_j\geq\tilde{L}\end{cases}.
$$

Define

$$
V_j=\frac{A+t_{j-1}^*-x}{A+t_j^*-x},\ j=1,2,\ldots,\tilde{k}
\tag{5.13}
$$

and $V_1=0$.

This can be rewritten as

$$A+t_{j+1}^{*}-x=\frac{A+t_{j}^{*}-x}{V_{j+1}},\ j=1,2,\ldots,\tilde{k}-1 \tag{5.14}$$

Note that

$$\frac{\partial J_5}{\partial t_j}=-C_f\left[\beta(\beta-1)(A+t_{j+1}^{*}-x)(A+t_j-x)^{\beta-2}-\beta^2(A+t_j-x)^{\beta-1}+\beta(A+t_{j-1}-x)^{\beta-1}\right]$$

and equating it to zero, yields

$$(\beta-1)(A+t_{j+1}^{*}-x)(A+t_j-x)^{\beta-2}-\beta(A+t_j-x)^{\beta-1}+(A+t_{j-1}-x)^{\beta-1}=0$$

$$\frac{(\beta-1)}{V_{j+1}}(A+t_j-x)^{\beta-1}-\beta(A+t_j-x)^{\beta-1}+V_j^{\beta-1}(A+t_j-x)^{\beta-1}=0$$

$$\frac{\beta-1}{V_{j+1}}-\beta+V_j^{\beta-1}=0$$

As a result, we have the V_j's given by the following recursive relationship:

$$V_{j+1}=\frac{\beta-1}{\beta-V_j^{\beta-1}}\ \text{ for } 1\le j\le \tilde{k}-1 \tag{5.15}$$

Also we have

$$\frac{\partial J_5}{\partial t_{\tilde{k}}}=-C_f\left[-\beta(\beta-1)\left(A+L-x-\frac{b}{\tilde{C}_f}\right)(A+t_{\tilde{k}}^{*}-x)^{\beta-2}-\beta^2(A+t_{\tilde{k}}^{*}-x)^{\beta-1}+\beta(A+t_{\tilde{k}-1}^{*}-x)^{\beta-1}\right]$$

and equating it to zero, yields

$$(\beta-1)\left(A+L-x-\frac{b}{\tilde{C}_f}\right)(A+t_{\tilde{k}}^{*}-x)^{\beta-2}-\beta(A+t_{\tilde{k}}^{*}-x)^{\beta-1}+V_{\tilde{k}}^{\beta-1}\beta(A+t_{\tilde{k}}^{*}-x)^{\beta-1}=0$$

$$\frac{(\beta-1)}{(A+t_{\tilde{k}}^{*}-x)}\left(A+L-x-\frac{b}{\tilde{C}_f}\right)-\beta+V_{\tilde{k}}^{\beta-1}\beta=0$$

$$(A+t_{\tilde{k}}^{*}-x)=\left(\frac{\beta-1}{\beta-V_{\tilde{k}}^{\beta-1}}\right)\left(A+L-x-\frac{b}{\tilde{C}_f}\right) \tag{5.16}$$

Note that the analysis for the optimal value $t_j^{*}(x,k)$ is the same as the new equipment demonstrated in Section 4.3.3.

From (5.14), (5.15), and (5.16), $t_j^{*}(x,k)$ for $1\le j\le k$ are obtained recursively. Hence, it is a unique solution of the first order conditions.

The optimal value $t_j^*(x), 1 \le j \le k$, are obtained by using iterative approach as shown below:

Step 1: Guess an initial value (t_1') and set $t_1 = t_1'$.

Step 2: Compute V_1 using (5.13) noting that $t_0 = 0$.

Step 3: Calculate $V_{j+1}, 1 \le j \le k-1$ using (5.15).

Step 4: Compute t_k and t_j, $j = 2, 3, \cdots, k$, using (51.4) and (5.16).

Step 5: Determine t_1'' from (5.14) using t_2 obtained from Step 4.

If $t_1'' = t_1'$, then $t_1^* = t_1$ and $t_{j+1}^* = t_{j+1}, 1 \le j \le \tilde{k} - 1$; else

if $t_1' < t_1''$, then increase t_1' and go to step 2; else

if $t_1' > t_1''$, then reduce t_1' and go to step 2.

<u>Stage 3</u>: $k^*(x)$, is obtained by minimizing

$$J_6(\underline{\delta}) = \tilde{C}_f \left[\Lambda_0 (A + L - x) - \Lambda_0 (A - x) - \sum_{j=1}^{k} \delta_j^* \left(L - t_j^* \right) \right] + ak + b \sum_{j=1}^{k} \delta_j^* + C_u(x) \quad (5.17)$$

This is a complex discrete function of k and $k_{\max}$ is not very large amount. So, we can adapt an enumerative search for k varying from 0 until $k_{\max}$ to obtain the global optimal k^* as indicated below:

Step 1: Set $k(x) = 0$ and compute $J\left(x, 0, \underline{t}^*(x, 0), \underline{\delta}^*\left(x, 0, \underline{t}^*(x, 0)\right)\right)$.

Step 2: $k(x) \leftarrow k(x) + 1$.

Step 3: Calculate $J\left(x, k(x), \underline{t}^*\left(x, k(x)\right), \underline{\delta}^*\left(x, k(x), \underline{t}^*\left(x, k(x)\right)\right)\right)$ using (5.17).

Step 4: If $J\left(x, k(x), \underline{t}^*\left(x, k(x)\right), \underline{\delta}^*\left(x, k(x), \underline{t}^*\left(x, k(x)\right)\right)\right)$ $< J\left(x, k(x)-1, \underline{t}^*\left(x, k(x)-1\right), \underline{\delta}^*\left(x, k(x)-1, \underline{t}^*\left(x, k(x)-1\right)\right)\right)$, then go to step 2; else stop and $k^*(x) = k(x) - 1$.

<u>Stage 4</u>: The optimal value x is obtained by minimizing $J\left(x, k^*(x), \underline{t}^*\left(x, k^*(x)\right), \underline{\delta}^*\left(x, k^*(x), \underline{t}^*\left(x, k^*(x)\right)\right)\right)$.

This is a one-dimensional optimization with x constrained to lie in the interval (0, A) with the step of 0.1A. One needs to use a computational method to obtain x^*.

The total expected cost to the lessor under the optimal preventive maintenance policy and upgrade is given by $J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$.

5.3.3 Special case 1: Upgrade only

For upgrade only case, there is no preventive maintenance action so that $k = 0$. As a result, we have the total expected cost to the lessor given by

$$J_7(x) = J(x,0,0,0) = \tilde{C}_f\left[\Lambda_0(A+L-x) - \Lambda_0(A-x)\right] + C_u(x) \tag{5.18}$$

From (5.18), we can obtain the optimal x by setting the first derivative to zero. The first derivative is given by

$$\frac{\partial J}{\partial x} = -\tilde{C}_f \beta\left[(A+L-x)^{\beta-1} - (A-x)^{\beta-1}\right] + \frac{dC_u(x)}{dx}$$

It is not possible to obtain an analytical expression for x^* and one needs to use a computational scheme.

If $\frac{dJ_7}{dx} > 0$ for $0 \le x < 1$, then $x^* = 0$. The total expected cost to the lessor is given by

$$J_7(0) = \tilde{C}_f\left[(A+L)^\beta - A^\beta\right] \tag{5.19}$$

5.3.4 Special case 2: preventive maintenance actions only

For preventive maintenance only case, there is no upgrade so that $x = 0$.The total expected cost to the lessor is given by

$$J_8(k,\underline{t},\underline{\delta}) = J(0,k,\underline{t},\underline{\delta}) = \tilde{C}_f\left[\Lambda_0(A+L) - \Lambda_0(A) - \sum_{j=1}^{k}\delta_j\left(L-t_j\right)\right] + ak + b\sum_{j=1}^{k}\delta_j \tag{5.20}$$

The analysis of this is similar to the general case and the optimal values are obtained using Stages 1 – 3 as discussed in Section 5.3.2.

(Note: When $A = 0$ this is the same as the problem studied in Chapter 4).

5.4 Numerical examples (Optimal upgrade and preventive maintenance actions)

We first consider the case with no penalty ($C_n = 0$ and $\tau \to \infty$) so as to study the relative impacts of preventive maintenance and upgrade actions on the total expected maintenance cost. We discuss the optimal upgrade and preventive maintenance actions, and the effect of model parameters on these optimal strategies of the lessor. Later, we look at the case where the two types of penalties are included and study their effects on the optimal maintenance strategies.

The nominal values for the model parameters are as follows:

- Lease period L = 5 (years)
- Equipment age A = 5 (years)
- Corrective maintenance cost C_f = 100 ($)
- Preventive maintenance cost parameters a = 100 ($) and b = 50 ($)
- Upgrade parameters $w = 10$ ($) and $\varphi = 0.01$
- Shape parameter for Weibull intensity function $\beta = 2$
- Penalty-1 cost when repair time exceeds a specified time $\tau = 2$ (days) $C_t = 300$ ($)
- Penalty-2 cost when a failure occurs $C_n = 200$ ($)

5.4.1 Optimal strategy with optimal upgrade and preventive maintenance actions

In this case following the four stage approach in that given x discussed in earlier section, we have the optimal number of preventive maintenance actions $k^* = 4$ and the optimal time instants for preventive maintenance actions $\left(\{t_j^*\}, 1 \le j \le 4\right)$ as given in Table 5.1. And, the optimal level of maintenance $\left(\{\delta_j^*\}, 1 \le j \le 4\right)$ is 1.8000. These optimal values give the minimum total expected cost to the lessor as 4,792.55 ($). We vary k from 1 to $k_{\max}$ using stages 1 and 2 while x is fixed. Here, we show the results only for k varying from 1 until 5. Then, the optimal upgrade $x^* = 2.5$ years is obtained using Stage 4 for x varying from 0 to 4.5 in step of 0.5. The results are shown in Table 5.2.

Table 5.1 Optimal preventive maintenance actions

j	t_j^*
1	0.9000
2	1.8000
3	2.7000
4	3.6000
5	5.0000

Table 5.2 Optimal upgrade

x	$J(x)$
0.0	\$6280.00
0.5	\$5893.63
1.0	\$5535.03
1.5	\$5216.12
2.0	\$4956.72
2.5	**\$4792.55**
3.0	\$4795.05
3.5	\$5130.88
4.0	\$6300.03
4.5	\$10802.52

Comment: The reason for the preventive maintenance actions being equi-spaced and the level of preventive maintenance actions being the same is due to the fact that $\beta = 2$. When this is not the case, this is no longer true.

5.4.2 Sensitivity analysis

The optimal upgrade and preventive maintenance actions depend on the values of the various model parameters. We carry out an investigation of the effect of model parameters on the optimal strategy by varying one parameter at a time and holding the remaining at their nominal values.

5.4.2.1 Effect of *A* variations

Table 5.3 shows the effect of equipment age on the optimal strategy. We consider *A* varying from 1 to 7 years.

Table 5.3 Effect of *A*

A	x^*	k^*	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$
0	0.0	4	\$1280.00
1	0.0	4	\$2280.00
2	0.6	4	\$3111.58
3	1.2	4	\$3752.68
4	2.0	4	\$4290.03
5	2.5	4	\$4792.55
6	3.6	4	\$5198.07
7	4.2	4	\$5601.10

As the equipment gets older, the expected number of failures increases. Therefore, x^* increases in order to reduce the likelihood of failures, but k^* does not change. This is because $\beta = 2$. The optimal time instants for preventive maintenance actions $\{t_j^*\}$ and the level of preventive maintenance actions $\{\delta_j^*\}$ are the same as that in Table 5.1 for all A again for the reason that $\beta = 2$. The upgrade is worthwhile when equipment age is greater than one year old.

5.4.2.2 Effect of β variations

Tables 5.4 indicates the effect of the shape parameter of the intensity function varying from 1.5, 2.0, and 3.0 for on the optimal maintenance strategy.

As to be expected β has significant impact to the optimal strategy. When $\beta = 1.5$, the rate of increasing in the intensity function decreases with age, and in this case no upgrade and only one preventive maintenance action is needed. When β = 3 the rate of increasing in the intensity function increases with age at a faster rate (compared to when $\beta = 2$), and as a result the intervals as well as the level of preventive maintenance actions.

Table 5.4 Effect of β

β	1.5		2		3	
$J\left(x^*,k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$2003.19		\$4792.55		\$13019.28	
x^*	0.0		2.5		4.0	
k^*	1		4		13	
j	t_j^*	δ_j^*	t_j^*	δ_j^*	t_j^*	δ_j^*
1	1.5000	0.4702	0.9000	1.8000	0.8880	7.6941
2			1.8000	1.8000	1.3321	5.6214
3			2.7000	1.8000	1.7021	5.5881
4			3.6000	1.8000	2.0319	5.6731
5					2.3349	5.7880
6					2.6183	5.9109
7					2.8863	6.0345
8					3.1419	6.1558
9					3.3871	6.2737
10					3.6234	6.3880
11					3.8520	6.4986
12					4.0739	6.6055
13					4.2896	6.7090

5.4.2.3 Effect of preventive maintenance cost $C_p(\delta)$

The preventive maintenance cost function comprises of two parameters. First one is an *a* as a fixed cost parameter of the preventive maintenance cost. Another one is a *b* as a variable cost parameter of the preventive maintenance cost. The effect of their variations on the optimal strategy is illustrated in Table 5.5.

Table 5.5 Effect of *a* and *b*

Parameters			*b*		
			\$20	\$50	\$100
a	\$80	$J\left(x^*,k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$4489.35	\$4712.55	\$5052.55
		x^*	2.4	2.5	2.5
		k^*	4	4	3
	\$100	$J\left(x^*,k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$4569.35	\$4792.55	\$5112.55
		x^*	2.5	2.5	2.5
		k^*	4	4	3
	\$150	$J\left(x^*,k^*,\underline{t}^*,\underline{\delta}^*\right)$	\$4734.55	\$4943.80	\$5245.89
		x^*	2.5	2.5	2.5
		k^*	3	3	2

As a and b increase, k^* decreases, but x^* remains the same for reasons discussed in Section 4.4.2.2.

5.4.2.4 Effect of upgrade cost $C_u(x)$

The effect of the upgrade cost parameter on the optimal strategy is shown in Table 5.6.

Table 5.6 Effect of w

w	5	10	15
$J(x^*, k^*, \underline{t}^*, \underline{\delta}^*)$	\$3,955.44	\$4,792.55	\$5,295.07
x^*	3.5	2.5	2.0
k^*	4	4	4

When the cost parameter of upgrade cost function increases, the upgrade level decreases as to be expected, while the total expected maintenance cost increases. However, there is no change to the optimal preventive maintenance actions. The optimal k is 4 for all values of w and $\{t_j^*\}$ and $\{\delta_j^*\}$ are as indicated in Table 5.1.

5.4.3 Effects of penalties

The optimal upgrade and preventive maintenance actions for three cases of penalty (Penalty-1, Penalty-2 and Penalties 1 & 2) are shown in Table 5.7.

Table 5.7 Effect of penalties

Case	No Penalty		Penalty-1		Penalty-2		Penalties 1 & 2	
$J(x^*, k^*, \underline{t}^*, \underline{\delta}^*)$	\$4792.55		\$7488.90		\$8918.59		\$10367.18	
x^*	2.5		3.5		3.5		4.0	
k^*	4		7		7		9	
j	t_j^*	δ_j^*	t_j^*	δ_j^*	t_j^*	δ_j^*	t_j^*	δ_j^*
1	0.9000	1.8000	0.6821	1.3642	0.6042	1.2083	0.4881	0.9763
2	1.8000	1.8000	1.3642	1.3642	1.2083	1.2083	0.9763	0.9763
3	2.7000	1.8000	2.0462	1.3642	1.8125	1.2083	1.4644	0.9763
4	3.6000	1.8000	2.7283	1.3642	2.4167	1.2083	1.9526	0.9763
5			3.4104	1.3642	3.0208	1.2083	2.4407	0.9763
6			4.0925	1.3642	3.6250	1.2083	2.9289	0.9763
7					4.2292	1.2083	3.4170	0.9763
8							3.9052	0.9763
9							4.3933	0.9763

As can be seen x^* and k^* increase to reduce the effect of penalty costs. The preventive maintenance actions are done more frequently and the level of preventive maintenance action is smaller compared to no penalty case. For each of the three cases, the preventive maintenance actions are equi-spaced and level of preventive maintenance actions constant as $\beta = 2$.

5.4.3.1 Effect of τ variations

The Penalty-1 deals with repair time limit. Table 5.11 demonstrates how τ influences to the optimal strategy.

Table 5.8 Effect of τ

τ	1 Day	2 Days	3 Days	∞ Days
$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$8484.35	\$7488.90	\$6884.50	\$4,792.55
x^*	3.5	3.5	3.0	2.5
k^*	7	7	6	4

As to be expected, as τ increases x^* and k^* decrease. This can be explained that as the repair time limit increases, the lessor has to carry out less frequency preventive maintenance actions in order to reduce the total expected cost. According to the fact that the less repair time promising to the lessee, then, the more attractive lease contract will be. However, the lessor has to consider the trade off between getting more contracts and the cost of the contract, which becomes a new interesting issue and will be discussed in Chapter 6.

5.4.3.2 Effect of C_n variations

The Penalty-2 deals with failure occurrence. Table 5.12 demonstrates how C_n affects to the optimal strategy.

Table 5.9 Effect of C_n

C_n	\$0	\$100	\$200	\$300
$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$4792.55	\$7083.02	\$8918.59	\$10364.41
x^*	2.5	3.5	3.5	4.0
k^*	4	6	7	9

The higher failure cost the more maintenance treats is needed to the leased item. This is because the more often preventive maintenance actions result in the reduction of the occurrence of failures. And also, the total expected cost is reduced.

5.4.3.3 Effect of τ and C_n variations

We now study the combination effects of Penalty-1 together with Penalty-2. We look at three different values for the two penalty parameters and the results are illustrated in Table 5.10.

As can be seen, when τ decreases and/or C_n increases, there is higher opportunity for the lessor to incur penalties. Therefore, the greater level of upgrade and the more frequent preventive maintenance actions are needed to reduce the likelihood of failures. Without any penalty ($\tau \to \infty$ and $C_n = 0$) the optimal strategy is given in the box in the southeast corner of Table 5.10 with bold font. With the most severe penalty ($C_n = 300$ and $\tau = 1$), the optimal strategy is given in the box in the northwest corner of Table 5.10 with bold font. As can be seen, the age reduction in upgrade jumps from 2.5 to 4.0, the number of preventive maintenance actions jump from 4 to 11 and the increase in the total expected maintenance cost jumps from $4,792.55 to $12,535.32.

Table 5.10 Effect of Penalties 1 & 2

τ	Parameters	C_n			
		$0	**$100**	**$200**	**$300**
1 Day	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	$8484.35	$10064.00	$11312.57	**$12535.32**
	x^*	3.5	4.0	4.0	**4.0**
	k^*	6	8	10	**11**
2 Days	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	$7488.90	$9309.28	$10637.18	$11874.11
	x^*	3.0	3.5	4.0	4.0
	k^*	5	8	9	10
3 Days	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	$6884.50	$8725.16	$10231.04	$11475.51
	x^*	3.0	3.5	4.0	4.0
	k^*	4	7	9	10
∞ **Days**	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	**$4792.55**	$7488.90	$8918.59	$10367.18
	x^*	**2.5**	3.5	3.5	4.0
	k^*	**4**	7	7	9

5.5 Numerical examples (Special cases)

The various special cases are considered in this section along with four different choices of penalty (No penalty, Penalty-1, Penalty-2 and Penalties 1 & 2). We begin with do nothing case, following by upgrade only then preventive maintenance only cases.

5.5.1 Corrective maintenance only

In this special case neither upgrade nor preventive maintenance action is performed for the leased item. Therefore, only the rectification cost is included in the contract cost to the lessor. The expected number of failures over the 5 years of lease period comes up with 75 failures resulting from $E\left[N\left(L\right)\right]=\left(5+5\right)^2-5^2=75$. Hence, with no penalty case it costs $7,500.00 to the lessor. With Penalty-1 case, the lessor incurs $221.80 per a failure resulting from $100.00 of rectification cost and $121.80 of Penalty-1 cost. Then, for 75 failures the total expected cost to the lessor with Penalty-1 case is $16,635.00. With Penalty-2 case, the lessor incurs $300.00 per one failure, which is obviously coming from $100.00 of rectification cost and $200.00 of Penalty-2 cost. As a result, $22,500.00 is the total expected cost to the lessor with Penalty-2 case. When two penalties included in the contract, one failure occur cost $421.80 to the lessor and the total expected cost is $31,635.00. Table 5.11 demonstrates the details of these results.

5.5.2 Upgrade only

Table 5.11 shows the optimal upgrade for four penalty cases. Penalty has strong influence to the optimal upgrade. x^* = 2.5 years, which is the same for the general case, but the total expected cost is $6,012.55. As can be seen, the Penalty-2 has more effect to the optimal strategy than the Penalty-1. That is x^* = 3.0 and 3.5 years for Penalty-1 and Penalty-2 cases respectively. When two penalties are considered in the same contract, x^* becomes 4.0 years.

5.5.3 Preventive maintenance only

The results of the study of preventive maintenance only option for different four penalty cases are also demonstrated in Tables 5.11. In this case the optimal number of preventive maintenance actions increases from k^* = 4 to 8 when there are penalty terms included in the contract. Furthermore, the total expected cost to the lessor increases.

5.4.4 Relative comparison

Table 5.11 summaries the optimal actions for all special cases as well as the total expected cost.

Table 5.11 Relative comparison of four options with and without penalties

Option	Parameters	No Penalty	Penalty-1	Penalty-2	Penalties 1&2
Optimal Upgrade and PM	$J\left(x^*, k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$4792.55	\$5814.75	\$8918.59	\$9610.07
	x^*	2.5	3.0	3.5	4.0
	k^*	4	5	7	8
Upgrade Only	$J\left(x^*\right)$	\$6012.55	\$7842.08	\$14350.88	\$15941.05
	x^*	2.5	3.0	3.5	4.0
PM Only	$J\left(k^*, \underline{t}^*, \underline{\delta}^*\right)$	\$6280.00	\$8517.71	\$17067,71	\$19214.06
	k^*	4	5	7	8
CM Only	J	\$7,500.00	\$16635.00	\$22,500.00	\$31635.00

As can be seen using either upgrade or preventive maintenance actions only, the total expected cost of maintenance is significantly lower than using only corrective maintenance actions for all penalty cases. When both upgrade and preventive maintenance actions are used, the total expected maintenance cost gets still smaller. In other words, we can say that the most economical option is to carry out the optimal upgrade and preventive maintenance actions. The results of penalty effect are similar to the new equipment lease in Chapter 4. Penalty-1 has less strong influence to the optimal strategy than Penalty-2.

5.5 Conclusion

For lease of used equipment, the lessor has an extra option of upgrade before lease in addition to preventive maintenance actions. The upgrade action makes the equipment younger and preventive maintenance actions lower the ROCOF while corrective maintenance actions are used to rectify the failure occurrence in order to restore the failed equipment back to working stage. In this chapter, we have studied the optimal upgrade and preventive maintenance actions taking into account two kinds of penalties – (i) repair time limit penalty and (ii) penalty for failures over the lease period. The model of which upgrade and preventive maintenance actions result in a reduction in the failure intensity function is developed to obtain the optimal preventive maintenance actions. The optimal parameters are determined by minimizing a cost function that takes into account the rectification, the preventive maintenance actions, the upgrade as well as the two penalties costs. The numerical analysis is presented to underline the effect of maintenance actions and penalty terms on the optimal strategy.

CHAPTER VI

CONCLUSION AND FURTHER RESEARCH

6.1 Introduction

This chapter is organized as follows. We provide the conclusions and discussion of the study in Section 6.2. Next, thesis contributions are stated in Section 6.3. Finally, future research is briefly discussed in Section 6.4. Topic of other preventive maintenance policies, the extensions of our models, and what relevant research issues need to be addressed of the leased equipment are demonstrated in the last section.

6.2 Conclusions and discussions

In this thesis, we have developed an optimal preventive maintenance policy of leased equipment with minimal repair, corresponding two types of equipment; new and used item. For leased equipment, the lessor carries out the maintenance of the equipment. Usually, the contract of lease specifies the penalty for repairs not being carried out within specified time limits and for equipment failures. Hence, the optimal preventive maintenance policy has taken these two penalties into account.

Through assuming that rectification time is negligible, NHPP was introduced into repair analysis. Furthermore, the preventive maintenance model has been suggested for leased equipment incorporated with increasing failure rate, and the upgrade has been offered as an option for the used item. Then, the structure of the decision model utilizes three or more variables (depends on the options) to obtain the optimal solution that minimizes the total expected cost to the lessor. Those decision variables include the number of preventive maintenance action to be carried out over a lease period, the time instants for such actions, the effectiveness of each preventive maintenance action, and the upgrade level.

In the model analysis and numerical experiments, we have addressed three special cases; no penalty case, Penalty-1 ($C_n = 0$) case, and Penalty-2 ($C_t = 0$) case. The numerical results show that the general case generates the highest cost to the lessor comparatively to those special cases. This is due to the fact that two penalties affect the increase in the lessor's total expected cost with optimal preventive maintenance actions. According to preventive maintenance action, it is used to lower ROCOF. This results in the reduction of the likelihood of failures which is corresponding to the less total expected cost to the lessor.

Having examined the shape parameter over a relatively wide range, it was found that the optimal solutions are significantly sensitive to this parameter. The increase of the shape parameter from 2 to 3 has produced very high total expected cost to the lessor. On the contrary, the decrease of the shape parameter from 2 to 1.5 has produced considerably reduction in the lessor's total expected cost. With the shape parameter = 2, the optimal preventive maintenance strategy associated with a periodic policy. This is not a surprising result in that the failure intensity increases at the constant rate.

In the sensitivity analysis, we also have investigated two parameters (fixed and variable) of preventive maintenance cost, which are typically of concern to the lessor. The study indicated that the parameters' variation has moderate influential towards any results.

The sensitivity analysis also found that increasing the cost of Penalty-2 caused the total expected cost to increase sharply, while increasing the promised repair time of Penalty-1 caused the total expected cost to drop fairly. Then, it is clear that Penalty-1 has more influence than Penalty-2.

With used equipment, the lessor has an option to improve the reliability through an upgrade action before leasing in addition to preventive maintenance to reduce the occurrence of failures over the lease period. Several options have been examined before deciding on the optimal strategy. We found that upgrade and preventive maintenance actions option is the most economical strategy comparatively to either upgrade or preventive maintenance actions only.

6.3 Contributions

Since more and more customers are becoming interested in leasing equipment and contracting out maintenance, the lessor takes great interest in dealing with current issues. To industrial engineering research area, we have proposed the first decision model to support the preventive maintenance problem of leased equipment. The model is obvious original and new contribution. Even though preventive maintenance policies have been investigated intensively in the literature, however, most of the work in this area focused on deriving optimal policies without considering the leased equipment. In practice, it is common to associate preventive maintenance with leased equipment when the maintenance of equipment requires special professional techniques as in cases with aircrafts, high-technology computers, or automobiles. For these items, a preventive maintenance policy within a lease period is usually specified in the lease contract. This is due to the fact that once equipment is leased and operating, the lessee needs to assure that the equipment will perform properly. Therefore, maintenance practices are noticeably important to ensure that the equipment is maintained at an acceptable level for the lessee as well as at an economical level for the lessor. Such complex situation, the proposed model has been thoroughly investigated to gain the knowledge of maintenance issues of leased equipment, particularly in the lessor perspective.

A major effort in modeling of leased equipment with maintenance practice (minimal repair and preventive maintenance action) is presented in this thesis. Furthermore, two penalties as mentioned earlier have been incorporated with the model, which makes it much more practical to the real world. We utilized the optimization technique based on the minimization total expected cost to the lessor in order to balance the cost of rectification combining with the two costs of penalty and the cost of preventive maintenance actions.

The impacts of the model on decision making have been revealed as the following. First of all, the optimal preventive maintenance policy can be evaluated with respect to cost effectiveness and reliability characteristics. Secondly, the model can assist in the timing aspect: how often to carry out preventive maintenance actions. Finally, the model can also be of help in determining effective and efficient schedules, taking all kind of constraints into account.

Also the model is expected to be valuable not only to maintenance management issues, but also to logistics and other management issues such as contracting, pricing, and etc. in leasing business. As a result, the effects of decision across maintenance and leasing area yield considerable benefits.

6.4 Further research

This section provides a brief concept of other related issues for the subsequent research. Three major topics are shown as follows.

6.4.1 Other preventive maintenance strategies

In the previous chapters, we discussed the analysis of an optimal preventive maintenance strategy. Then, other preventive maintenance strategies may be investigated as indicated here.

Strategy-A: The equipment is subjected to preventive maintenance action whenever the intensity function reaches a specified level ρ. Each preventive maintenance action reduces the intensity function by a fixed amount δ. Any failure over the lease period is rectified through minimal repair. The policy is characterized by the set of parameters $\theta \equiv \{\rho, \delta\}$.

Strategy-B: The equipment is subjected to preventive maintenance actions periodically so that the j^{th} preventive maintenance action is carried at time $t_j = j\mu$. After each preventive maintenance action the intensity function is reduced to a level ν. Any failure over the lease period is rectified through minimal repair. The policy is characterized by the set of parameters $\theta \equiv \{\mu, \nu\}$.

Strategy-C: Let $0 < \varsigma_1 < \varsigma_2 < L$. The equipment is subjected no preventive maintenance actions in over the interval $[0, \varsigma_1)$, periodic preventive maintenance actions with period 2Δ in the interval $[\varsigma_1, \varsigma_2)$ and period Δ in the interval $[\varsigma_2, L)$. Each preventive maintenance reduces the intensity function to a specified level ν. Any failure over the lease period is rectified through minimal repair. The policy is characterized by the set of parameters $\theta \equiv \{\varsigma_1, \varsigma_2, \nu\}$.

6.4.2 Some of model extensions

The proposed model can be extended in several ways and we discuss some of these:

a) Cost modeling

Maintenance cost structures involved in these models are rectification and preventive maintenance costs. We can look at different cost structures as follows.

1. *Rectification Cost*: The models are assumed C_f being constant. We may consider that C_f is uncertain and need to be estimated before an appropriated decision is made. There are two choices for rectification involving either repair or replacement of failed items. These choices depend on the relative costs.

2. *Preventive Maintenance Cost:* The models are assumed $C_p(\delta) = a + b\delta^m$ being a linear form, where m = 1. We may confine our attention to $C_p(\delta)$ as a non-linear function where m > 1.

b) Quality of repair

Repair type of the models is subjected to minimal repair. We may consider other types of corrective maintenance; for example imperfect repair. When failure is repaired, we might achieve some improvement at a certain amount of $\varepsilon\xi$, where ε is a random variable from [0,1]. If ε = 0, the quality of repair is equal to minimal repair while ε = 1, the quality of repair become perfect repair. We expect to achieve at amount of ξ. In case of $\varepsilon\xi$ can improve the failed item as new, it also can say that as a replacement.

c) Preventive maintenance policy

These models are subjected to the optimal schedule time to perform preventive maintenance and periodic preventive maintenance policy. Other types of preventive maintenance policy; for example, age policy, block policy, and etc. may be considered.

d) Penalty terms

In the contract, we can look at new penalty terms, which some of these are discussed as follows.

1. *Specified failure numbers is greater than one*: The specified failure number, which is greater than one is yet to be studied. The model will be much more complex.

2. *Multiple specified numbers of failures:* We may have multiple specified numbers of failures as η_1, η_2 and η_3 for subintervals L_1, L_2 and L_3 respectively for a new item.

3. *Times between subsequent failures*: We can look at time between subsequent failures that is specified to be greater than γ. If the time between subsequent failures is less than γ, then, the lessor incurs penalty. If failure occurs after fixing shortly within υ, it need to be fixed immediately. This means that the failure has not been fixed properly.

6.4.3 Other related issues

There are several interesting topics have been discussed. However, some other areas also need further investigation for the subsequent research. The different scientific disciplines such as management accounting, operation management, computer science, and sociotechnology all need to be involved at different stages. The following discussion highlights the other related issues.

a) Operational issues

1. *Different criterion*: Beside the minimal cost to the lessor, the maximum availability of the lease equipment is another criterion that one may look at.

2. *Logistics issues*: Some of logistics related issues are as follows.

2.1 <u>Maintenance capacity planning</u>: The maintenance capacity planning determines the optimal level of resources (workers, skills, spare parts, equipment and tools) required to meet the forecasted maintenance works. We need to determine all parameter values based on available sources. Effective planning can improve the utilization of maintenance resources. This also aids in spare part provisioning policy to minimise the cost of maintenance.

2.2 <u>The number of locations</u>: An important aspect of leasing service is the ability to allocate equipment to different locations of the lessee and to manage the maintenance work in an effective manner. The objective of this problem is to minimise cost that optimizes the allocation of equipment as well as maintenance resources.

b) Strategic issues

The important of service quality has become increasingly critical in choosing among different service providers (lessor). Lessees are interested in meeting their time and service response requirements in a cost effective manner. In this subsection, we indicate some of the strategic topics as follows.

1. *Contract*: The models deal with one contract. We have considered one lessee to serve at a time. We can look at different types of customer, such that different contracts are needed to be determined. When the number of lessor providing service is more than one, then one need to look at different scenarios—competition, collusion etc. Promotion and reputation are important aspects and needed to take into consideration of model formulation. The analysis of these cases are more complex in terms of contracting problems: Many lessor – One lessee, One lessor – Many lessee, Many lessor – Many lessee.

A lessor can offer a wide range of options all of which are based on financial constructs. When the lessor offers service to the lessee, the lessor has to decide on the optimal pricing strategy taking into account the reliability of the equipment and the response of the lessee. This is very important to ensure the economical leasing service price. In case of more than one option is feasible, the lessee needs to determine which options are most profitable, both for the lessor and the lessee.

2. *Leased equipment*: One of strategic issues is the determination of what capabilities the lessor must have. Since the demand is time varying and uncertain, it is difficult to forecast the demand properly. From the lessor point of view, to achieve the competitive situation, the number and variety of equipment to stock for leasing become significant concerns. In this case, we need to look at the optimality of the number and types of equipment according to the limited capital budget.

Other strategic issues which must be researched has to do are time to replace item based on age or cost of maintenance and acquisition policy, which might be buy or lease of new item.

c) Other issues

The contract requires a maximum of mutual trust and openness with respect to sharing information. If the lessee does note take care the equipment properly when they use it. This relates to moral hazard issue. If lessee lost the leased equipment, how lessor will charge to the lessee. In addition, if eventually a contract has been established, how can contracts be extended, or alternatively, how can they be terminated before ending period of contract time.

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APPENDIX

POSITIVE DEFINITE TEST

To proof that $\nabla^2 J_2\left(\underline{t}^*(k)\right)$ is positive definite so that we take the second order condition $\nabla^2 J_2(\underline{t}) = 0$ as shown here.

$$\frac{\partial J_2(\underline{t})}{\partial t_1} = -C_f \left\{ t_2 \frac{d\lambda_0(t_1)}{dt_1} - \left[\lambda_0(t_1) + t_1 \frac{d\lambda_0(t_1)}{dt_1} \right] + \lambda_0(t_0) \right\}$$
$$= -C_f \beta\left[(\beta-1)t_1^{\beta-2} t_2 - \beta t_1^{\beta-1}\right]$$

Setting $\frac{\partial J_2(\underline{t})}{\partial t_1}$ to zero and solving for t_1^*. Then, from the first order condition and after some analysis, we have

$$\frac{t_1^*}{t_2^*} = \frac{\beta-1}{\beta}. \tag{A.1}$$

For $2 \le i \le k-2$

$$\frac{\partial J_2(\underline{t})}{\partial t_i} = -C_f \left\{ t_{i+1} \frac{d\lambda_0(t_i)}{dt_i} - \left[\lambda_0(t_i) + t_i \frac{d\lambda_0(t_i)}{dt_i} \right] + \lambda_0(t_{i-1}) \right\}$$
$$= -C_f \beta\left[t_{i-1}^{\beta-1} - \beta t_i^{\beta-1}(\beta-1)t_i^{\beta-2} t_{i+1}\right]$$

Setting $\frac{\partial J_2(\underline{t})}{\partial t_i}$ to zero and solving for t_i^*.

Define

$$V_i = \frac{t_{i-1}^*}{t_i^*} \qquad \text{for } 2 \le i \le k-2 \tag{A.2}$$

Then, from the first order condition and after some analysis, we have

$$V_{i+1} = \frac{\beta - 1}{\beta - V_i^{\beta-1}} \qquad \text{for } 2 \le i \le k-2 . \tag{A.3}$$

$$\frac{\partial J_2(\underline{t})}{\partial t_{k-1}} = -C_f \left\{ t_k \frac{d\lambda_0(t_{k-1})}{dt_{k-1}} - \left[\lambda_0(t_{k-1}) + t_{k-1} \frac{d\lambda_0(t_{k-1})}{dt_{k-1}} \right] + \lambda_0(t_{k-2}) \right\}$$
$$= -C_f \beta \left[t_{k-2}^{\beta-1} - \beta t_{k-1}^{\beta-1} (\beta - 1) t_{k-1}^{\beta-2} t_k \right]$$

Setting $\frac{\partial J_2(\underline{t})}{\partial t_{k-1}}$ to zero and solving for t_{k-1}^*.

Define

$$V_{k-1} = \frac{t_{k-2}^*}{t_{k-1}^*}, \text{ and } V_k = \frac{t_{k-1}^*}{t_k^*} \tag{A.4}$$

Then, from the first order condition and after some analysis, we have

$$V_k = \frac{\beta - 1}{\beta - V_{k-1}^{\beta-1}} . \tag{A.5}$$

$t_j^*, 1 \le j \le k,$ are obtained recursively from (A.1) – (A.5).

Let $A(\underline{t}) = \nabla^2 J_2(\underline{t})$ which refers to a $(k-1)\times(k-1)$ matrix. The component $A_{im} = A(\underline{t})_{im}, 1 \le i, m \le k-1$, is given by $A_{im} = \frac{\partial^2 J_2(\underline{t})}{\partial t_i \partial t_m}$ and $A_{im} = A_{mi}$.

Note that for $|i - m| > 1$, we have $A_{im} = 0$. This implies that A is a tri-diagonal matrix.

The second order conditions are conducted and their expressions are as follows:

$$\frac{\partial^2 J_2(\underline{t})}{\partial t_1^2} = -C_f \beta \left[(\beta - 1)(\beta - 2) t_1^{\beta-3} t_2 - \beta(\beta - 1) t_1^{\beta-2} \right]$$

$$\frac{\partial^2 J_2(\underline{t})}{\partial t_1 \partial t_2} = -C_f \beta\left[(\beta-1)t_1^{\beta-2}\right]$$

For $2 \le i \le k-2$

$$\frac{\partial^2 J_2(\underline{t})}{\partial t_i^2} = -C_f \beta\left[(\beta-1)(\beta-2)t_i^{\beta-3}t_{i+1} - \beta(\beta-1)t_i^{\beta-2}\right]$$

$$\frac{\partial^2 J_2(\underline{t})}{\partial t_i \partial t_{i+1}} = -C_f \beta\left[(\beta-1)t_i^{\beta-2}\right]$$

$$\frac{\partial^2 J_2(\underline{t})}{\partial t_{k-1}^2} = -C_f \beta\left[(\beta-1)(\beta-2)t_{k-1}^{\beta-3}t_k - \beta(\beta-1)t_{k-1}^{\beta-2}\right]$$

It is not possible to derive any analytical results, as the expressions for $\underline{t}^*$ are complex and $A(\underline{t}^*)$ is analytical intractable. One can computationally test for the positive definiteness using standard packages such as MATLAB.

In this thesis, MATLAB is used to compute the eigenvalues. These are given by the solution of the equation $|A-\lambda \mathrm{I}|=0$ where $|\ |$ is the determinant. Note that this is a polynomial of order (k^*-1) as the matrix is of order $(k^*-1)\times(k^*-1)$.

The test for positive definiteness is also conducted by randomly generating each nominal value within its range. Note that more than 30 values are generated randomly for each parameter. The results show that all eigenvalues for all combinations are positive, then $A(\underline{t}^*)$ has to be a positive definite and resulting in $\underline{t}^*$ to be a local minimum.

In conclusion, the set of k^*, $\underline{t}^*(k^*)$ and $\underline{\delta}^*(k^*)$ is the global solution that minimizes *J*. Even though we select each parameter randomly, the solution is still global, since $\underline{\delta}^*(k^*)$ and $\underline{t}^*(k^*)$ are obtained globally from stages 2 and 3 respectively. Then we obtain k^* from searching for the minimum J within the bound of $k = 1$ till $k_{max} = \left\lceil \frac{C_f \Lambda(L)}{a} \right\rceil$.

VITA

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