การเปรียบเทียบแบบจำลองของอัตราดอกเบี้ยโดยประยุกต์ใช้กับการตีราคาหุ้นกู้อนุพันธ์ที่มีผลตอบแทนสะสมอ้างอิง

นาย ชวลิต กาจกำจรเดช

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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต
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ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

A COMPARISON OF INTEREST-RATE MODELS,
APPLICATION OF RANGE ACCRUAL NOTE PRICING

Mr. Chawalit Kajkumjorndej

ศูนย์วิทยทรัพยากร
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ชวลิต กาจกำจรเดช: การเปรียบเทียบแบบจำลองของอัตราดอกเบี้ยโดยประยุกต์ใช้กับการตีราคาหุ้นกู้อนุพันธ์ที่มีผลตอบแทนสะสมอ้างอิง (A COMPARISON OF INTEREST-RATE MODELS, APPLICATION OF RANGE ACCRUAL NOTE PRICING) อ. ที่ปรึกษาวิทยานิพนธ์หลัก: ดร.อนันต์ เจียรวงศ์, 47 หน้า.

จุดประสงค์ของวิทยานิพนธ์ฉบับนี้คือการศึกษาวิธีการหามูลค่าของหุ้นกู้อนุพันธ์ที่มีผลตอบแทนสะสมในทางปฏิบัติโดยใช้วิธีการของมอนติ คาร์โล นอกจากนี้ วิทยานิพนธ์ฉบับนี้ยังทดลองเปลี่ยนแปลงตัวแปรต่างๆในโมเดลเพื่อศึกษาคุณสมบัติของโมเดลที่นำมาใช้หามูลค่าของหุ้นกู้ชนิดนี้ โดยจะทดลองเปลี่ยนตัวแปรภายในของโมเดล, ช่วงของอัตราเบี้ยดอกเบี้ยที่จะจ่ายผลตอบแทนและอัตราดอกเบี้ยในวันแรกของการออกหุ้นกู้ ซึ่งหุ้นกู้ชนิดนี้จำเป็นที่จะต้องทำนายอัตราดอกเบี้ยไปในอนาคตเพราะผลตอบแทนของหุ้นกู้นี้จะได้รับตามจำนวนวันที่อัตราดอกเบี้ยอ้างอิงอยู่ในขอบเขตที่มีการกำหนดไว้ล่วงหน้า ในส่วนของผลการศึกษาสรุปได้ว่า วิธีในวิทยานิพนธ์ฉบับนี้ใช้ได้ดีกับการหามูลค่าของหุ้นกู้ดังกล่าว และตัวแปรของโมเดลก็สามารถหาได้จากข้อมูลในตลาดการเงิน นอกจากนั้นยังสรุปได้อีกว่าค่าของตัวแปรต่างๆของโมเดลมีผลต่อราคาของหุ้นกู้อย่างมาก รวมถึงการเปลี่ยนแปลงช่วงของอัตราดอกเบี้ยที่จะจ่ายผลตอบแทนและการเปลี่ยนแปลงของอัตราดอกเบี้ยในวันแรกของการออกหุ้นกู้ก็มีผลต่อราคาของหุ้นกู้เช่นกัน

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ภาควิชา การธนาคารและการเงิน ลงลายมือชื่อนิสิต..........ชวลิต กาจกำจรเดช..........

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CHAWALIT KAJKUMJORNDEJ: A COMPARISON OF INTEREST-RATE MODELS, APPLICATION OF RANGE ACCRUAL NOTE PRICING. ADVISOR: ANANT CHIARAWONGSE, Ph.D., 47 pp.

The objective of this thesis is to apply the Hull-White model and CIR++ model to price the range accrual note by using Monte Carlo simulation practically. This thesis also studies the behavior of the model by providing the sensitivity of the note's price, when changing the model's parameters. As a conclusion, the method used in this thesis is well applied to price the range accrual note by using Monte Carlo simulation. Models' parameters can be practically computed from the interpolated historical data which are daily 6M USD LIBOR and weekly U.S. swap curve. These models' prices are little affected by changing their mean reversion and speed of mean reversion in case of low volatility, but they are highly affected by changing them in case of high volatility. In addition, the changing of range of strike rates highly affects the prices of range accrual note. The narrow range provides the note's prices are lower than the wide range for all volatility cases. Moreover, range of strike rates which is near the movement of reference rate makes the note's prices more sensitive than other ranges. Therefore, the initial short rate highly affects the range accrual note's prices too. Lastly, the normal distribution function of Hull-White model makes the range accrual note's prices are higher than the chi-squares distribution function of CIR++ model.

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CHAPTER I

INTRODUCTION

1.1 Background and Problem Review

In this thesis, I have two objectives. Firstly, I apply the Hull-White model and CIR++ model to price the range accrual note by using Monte Carlo simulation practically. Secondly, I study the behavior of the model by providing the sensitivity of the note's price, when changing the model's parameters. This study can present how the changing of models' parameters affects the note's prices and how the difference of the note's prices of both models.

To price the range accrual note, I apply the methods from the studies as follow. Brigo and Mercurio (2006) present the function which can easily compute the range accrual note's reference rate everyday and price this note by using Monte Carlo simulation. In addition, Shen, Huang, and Tao (2002) and Jagannathan, Kaplinb, and Sun (2003) also provide the observation-in-parameter method. The model's parameters can be observed from the historical data which are the instantaneous forward rate, swap curve, and discount curve.

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I choose the Hull-White model and CIR++ model in order to forecast a term structure of reference interest rates and price the range accrual notes by using Monte Carlo method practically. Both models are different in the random distribution function. The Hull-White model has the normal distribution function which is different from the chi-square distribution function of CIR++ model. So, I want to compare how this difference affects the range accrual note's price. In addition, I object to study the behavior of these models, so I provide the sensitivity analysis of models' parameters. It can be done by changing one of model' parameter and other parameters have to be fixed. The results show the sensitivity of range accrual note's prices from both models and the sensitivity of both models' parameters. I also provide how the changing of shapes of volatilities' term structures affects both models when pricing the note. I use the flat, linear, and historical market term structure of volatilities to be the models' volatility parameter. Moreover, I provide the sensitivity of the ranges of strike rates of range accrual note and the sensitivity of the initial short rate of both models.

Why is the valuation of Range Accrual Note (RAN) difficult? Because the payoffs depend on the reference rate, so it needs to forecast the reference rate everyday until maturity. The longer maturity this note has, the more complexity in the note's valuation. In addition, it's difficult to find the suitable model or suitable method to value this note, especially in practice, this method should be more quickly and easier to value than other methods. Moreover, the volatility is important when pricing this note because the high volatility can make the rate go outside the range. So, it leads to the change of the note's price. The important of volatility is proposed by Benhamou (2004) that "The range accrual note has the risk that investors may get nothing when the reference rate stays outside the range, so the volatility is very important when pricing it. A combination of increasing rates and increasing volatility levels may have significant impact on the price."

There is one suggestion for pricing range accrual note from Brigo and Mercurio (2006) that "The category of products that is usually considered for Monte Carlo pricing is the family of path-dependent payoffs. Their payoffs involve the history of underlying variable up to the final time. The Monte Carlo method works through forward propagation in time of the key variables, by simulating its transition density between dates where underlying-variable history matters to the final payoff. In

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conclusion, the range accrual note should be valued by Monte Carlo simulation." This raise to the question how to price the range accrual note by using Monte Carlo method practically? To answer whether the note's price is correct is difficult, because the note doesn't have the historical market price to compare. Therefore, the study of the model's behavior which can affect the note's price is very interesting. The sensitivity of the model's parameters when pricing the note should be considered. It leads to the question how does the changing of parameters affect the note's price?

1.2 Objective of the Study

This thesis applies the Hull-White model and CIR++ model to price the range accrual note by using Monte Carlo simulation practically. I also study the behavior of the models by providing the sensitivity of the note's price, when changing the model's parameters. This study can present how the changing of models' parameters affects the note's prices and how the difference of the note's prices of both models.

1.3 Scope of the Study

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Data for pricing Range Accrual Note are daily 6M USD LIBOR, weekly U.S. swap curves and Range Accrual Note details, terms and conditions. Daily 6M USD LIBOR data are used for determining the parameters of range accrual note. The historical data of daily 6M USD LIBOR span ten years from 18^{th} April 1997 – 18^{th} April 2007. The weekly U.S. swap curves are used for being the input of the short rate models to forecast the term structure of reference rate. The historical data of daily 6M USD LIBOR is obtained from Reuters 3000Xtra and the weekly U.S. swap curve is collected from DataStream. Range Accrual Note data are issued on 18^{th} April 2007 with maturity date 18th April 2017 (maturity period of ten years). A denomination of the note is USD 10,000.

1.4 Contribution

This thesis seeks to provide the valuation of range accrual note in practice. I focus on pricing the range accrual note by using Monte Carlo simulation. The model's parameters can be observed from the historical market data. Moreover, this study also provides the behavior of the model by providing the sensitivity of the note's price, when changing the model's parameters. I show the sensitivity analysis of the mean reversion parameter, the speed of mean reversion parameter, the volatility parameter, the initial short rate, and the range of strike rate of range accrual note.

1.5 Organization of the Study

The remaining of this paper is organized as follow. Chapter 2 discusses the literature reviews, the theoretical background of the study. It reviews the past several methods of range accrual note pricing. Chapter 3 describes data, methodology, and simulation model. It discusses the data collection, the method of range accrual note pricing, the way to apply the models of Hull-White model and CIR++ model to price the range accrual note. Chapter 4 provides the results and sensitivity analysis of range accrual note pricing. Finally, conclusion and recommendations are provided in Chapter 5.

CHAPTER II

LITERATURE REVIEW

There are many studies of range accrual note pricing; each study provides the different methods to price it or the improvement from others. Many kinds of interest-rate models are applied to value this note such as HJM (Heath-Jarrow-Morton) model, BGM (Brace, Gaterek, and Musiela) model, Hull-White model, and Black model etc. I review some of the papers that study about range accrual note pricing.

This section is described as follows; Section 2.1 describes the works of closed-form solution of range accrual note pricing. Section 2.2 describes the work of Monte Carlo simulation with BGM model of range accrual note pricing. Section 2.3 describes the studies of range accrual note pricing with trinomial trees of Hull-White model.

2.1 The Range Accrual Note Pricing with Closed-Form Solution

Firstly, Turnbull (1995) studies the valuation of interest rate structured products. This paper provides the closed-form solutions which are derived for European interest rate digital options, digital range options, and range notes. Using a one-factor Gaussian HJM model, this paper has priced explicitly each coupon of a floating range note as a portfolio of range-contingent payoff options plus an extra term, that only involves the univariate normal distribution function.

Secondly, Navatte and Quittard-Pinon (1999) aim to value interest rate structured products in a simpler and different way than Turnbull (1995). Considering some assumptions with respect to the evolution of the term structure of interest rates, the price of a European interest rate digital call option is given. Then using a one factor linear Gaussian model and the change of numeraire approach, a closed-form formula is found to value range notes which pay at the end of each defined period, a sum equal to a prespecified interest rate times the number of days the reference interest rate lies inside a corridor. Under the same framework as Turnbull (1995) but using the change of numerical technique, this paper have rewritten each coupon of a floating range note as a portfolio of digital options plus the same extra term, only involving the univariate normal distribution function.

Then, Nunes (2004) provides closed-form solutions in the context of multifactor Gaussian HJM framework which extended from a one-factor Gaussian HJM model in order to value the digital options, floating range notes and fixed range notes. It is remarkable that this closed-form formula involves the normal distribution function. This paper shows that, when moving toward a multifactor framework, the same structure as Turnbull (1995) and Navatte and Quittard-Pinon (1999) will be obtained for the price of each coupon, the only difference being that the extra term will have to be expressed as an integral over a bivariate normal density function.

Lastly, Eberlein and Kluge (2006) extends Nunes (2004) by providing explicit pricing solutions for digital options and range notes in the multivariate Lévy term-structure model introduced in Eberlein and Raible (1999) and pushed further in Eberlein and Ozkan (2003), Eberlein, Jacod, and Raible (2005), and Eberlein and Kluge (2006). This model generalizes the multifactor Gaussian Heath–Jarrow–Morton (HJM) model, by replacing the driving Brownian motion with a multivariate Lévy process. As a byproduct, they obtain a pricing formula for floating range notes in the special case of a multifactor Gaussian HJM model that is simpler than the one provided by Nunes (2004).

As a conclusion, all the papers of the range accrual note pricing with closed-form solution are suitable for digital options, range digital options, and range accrual notes. In their closed-form solutions, they try to estimate the expected term that determines the payoff each day. Next, they obtain the price from the summation of all expected terms. The cumulative normal distribution function is applied to compute these expected terms. Moreover, the random variable of the cumulative distribution function is the function of strike rate and volatilities.

However, those papers have some problems, especially in practice. The first problem is the volatility function is difficult to be specified in practice. In addition, the volatility function doesn’t have appropriate market data to calibrate and has problem with large volatilities. In my thesis, I also have the problem to find the volatility function, so I provide the sensitivity analysis of the volatility parameter instead.

2.2 The Range Accrual Note Pricing with Monte Carlo Simulation

Chalamandaris (2007) prices a range accrual note using Monte Carlo simulation with BGM (Brace, Gaterek and Musiela) model as an interest rate model. The simulation step is estimated using linear interpolation of the simulated reference rate. However, this paper has the same problems as the range accrual note pricing with closed-form solution.

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2.3 The Range Accrual Note Pricing with Trees

FinacialCAD Corporation (2005) presents the document that describes the functions for valuing range accrual notes using a one-factor short rate model (Hull-White or Black-Karasinski). The price of range accrual note is obtained by building a trinomial interest rate tree, constructing the market yield curve on each node of the tree, and re-valuing the range accrual note on each node. The model’s parameters which are volatility and mean reversion calibrated to caplets and/or floorlets with strike at or near the range accrual note boundaries, and the expiry dates at or near the future rate observation dates.

However, the trinomial tree is not suitable for pricing the range accrual note. Brigo and Mercurio (2006) suggest "For path-dependent products, we need to know the past history of the underlying variable to determine payoff at each final node, but this past history is not determined yet. (For normal products, we know the payoff at each final node, then we value it through discounting) Moreover, for path-dependent products, we have the problem about calibrating the trinomial tree to appropriate market data such as caps and swaptions prices." As a conclusion, using the trinomial tree of Hull-White model to price the range accrual note has problem about determining payoffs at each node and problem in calibration method.

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CHAPTER III

METHODOLOGY AND DATA

3.1 Methodology

3.1.1 <u>Range Accrual Note Terms and Conditions</u>

A Range Accrual Note (RAN) is one of the path-dependent products where the coupon depends on the performance of a reference rate, such as 3-month LIBOR, 6-month LIBOR and Thai Baht Interest Rate Fixing (THB Fix) etc. If LIBOR moves within the predetermined range during the life of note, investors will receive return more than return from fixed-rate deposits of comparable maturity and credit quality. Each coupon depends on number of days in the coupon period that LIBOR fixes within the predetermined range, while a lower coupon or zero interest is accrued any day that LIBOR fixes outside of the range.

Range Accrual Note data are issued at 18th April 2007 and maturity at 18th April 2017 (maturity period of ten years). A denomination of the note is USD 10,000. The coupon amount pays depending on the number of days the reference rate stays in the predetermined range, specifically; the coupon amount of each period is 7%*(*N/D*)*denomination, where *N* is the actual number of days in the respective calculation period for which the daily fixing of the reference rate (6M USD LIBOR) is at or below the upper boundary (7%) and at or above the lower boundary (0%), and *D* is the actual number of days in the respective calculation period.

3.1.2 <u>The Methods of Range Accrual Note Pricing</u>

This part I show the method how the range accrual note can be valued. Licht (2005) proposes how to value the range accrual note by using daily range digital call

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option. He also suggests that if we can estimate the reference rate in future, we can price the range accrual note by using his methods. He tries to approximate this function by using the Black's model to forecast the forward reference rate. The cumulative normal function *N(d)* is need for that simulation to determine the probabilities whether the reference rate stays in the predetermined range. I use the methods from this paper, but not directly. The difference is in the simulation part. Instead of using the cumulative normal function *N(d)*, I use the random process of short rate models to determine it. Finally, I adapt the applied methods to be instruments in order to use the short-rate models for pricing the range accrual note.

Before I present how to apply the short-rate models to price the range accrual note, I begin by considering following instruments which are the methods of range accrual note pricing:

1. European digital call option
2. European range digital call option
3. Single-period range accrual note
4. Multi-period range accrual note

Then, I use the concepts of these methods to value the range accrual note. Moreover, in order to price the note, it needs to focus on when the note is priced. If the note is valued on the date before period initiation, we don't know how many days the reference interest rate stays in the range. Therefore, it needs the short rate model to predict it. But if the valuation date is during the period, we can know the reference interest rate before that date. Then, we can know how much coupon we received, so the valuation is different from the case that valuation date is before period initiation. In this thesis I focus only on the valuation at range accrual note's issue date. The method of this case is the same as the valuation before issue date.

3.1.2.1 European Digital Call Option

Definition: A digital call option is an option that the payoff is paid in arrears. This option pays unit payoff at option's maturity if the reference rate at each date is above the strike rate and zero if it is below or equal to the strike rate.

Let T, $R(\tau;\alpha)$ and K denote the option's maturity, reference interest rate at each time τ with maturity $\tau+\alpha$ and strike rate, respectively. α is the tenor of the reference interest rate and, in case of 6M LIBOR, equals 6 months or 181 days. Then the payoff at maturity T with considering the reference rate at time t is given by

$$DC_T(\tau) = 1\,(R\,(\tau;\alpha) - K) \qquad (1)$$

Where the indicator function $1(R\,(\tau;\,\alpha) - K)$ equals 1 on $R\,(\tau;\,\alpha) > K$ and 0 otherwise. The timeline of function $R\,(\tau;\alpha)$ is shown in Figure 3.

3.1.2.2 European Range Digital Call Option

Definition: A range digital call is an option that the payoff is paid in arrears. This option pays unit payoff at option's maturity if the reference rate at each time t is within the range $(K_L,\,K_U]$ and zero outside this range. Where K_L is the lower strike and K_U is the upper strike.

The payoff at maturity T with considering the reference rate at time t is given by

$$RD_T(\tau) = \begin{cases} 1 & ;\, K_L < R\,(\tau;\,\alpha) \leq K_U \\ 0 & ;\, otherwise \end{cases} \qquad (2)$$

$$= 1(R\,(\tau;\,\alpha) - K_L) - 1(R\,(\tau;\,\alpha) - K_U) \qquad (3)$$

The timeline of function $RD_T(\tau)$ is shown in Figure 4.

Most of all range accrual notes pay coupons many periods before maturity, so they need to value by multi-period valuation. Before showing the multi-period valuation, I start by considering the basic single period range accrual note. The valuation of single period range accrual note allows an easy description of the multi-period range accrual note.

3.1.2.3 Single-Period Range Accrual Note

Definition: A fixed interest rate, interest rate range and period are specified on initiation of the single period range note. This note entitles the holder to a payment at the end of the period calculated by multiplying the fixed interest rate by the number of days during the period that a reference interest rate fell within the specified range. At the expiry of the contract the nominal is also, of course, paid back.

For the range accrual note, I focus on pricing at issue date. In this case we can't know the future interest rate, so the short rate model is required. In order to calculate the note's price, I use the function $R(\tau;\alpha)$ to compare with the predetermined range. Thus the replication of the range accrual note is collection of range digital call options giving

$$V(t;T_0,T_1) = \left[\frac{CN}{D} Z(t,T_1) \sum_{i=0}^{n} RD_{T_1}(t;T_0+i,T_1) \right] \tag{4}$$

Where C is coupon rate, D is the number of days in the year, t is the valuation date, T_0 is the initiation of the period ($t = T_0$), i is the i^{th} day after T_0, T_1 is the end of period, and $T_0 + n = T_1$. The timeline of single period range accrual note is shown in Figure 5.

3.1.2.4 Multi-Period Range Accrual Note

Definition: A multi-period range note is a successive series of single period range note with interest rate being paid at the end of each period and the nominal payment occurring at the end of the final period.

In this case we use the concept from single-period range note calculation by summing all the value of each period range accrual note.

$$P(t;T_0,T_m)=\sum_{j=1}^{m}V_j(t;T_{j-1},T_j)+Z(t,T_m)N \tag{5}$$

Where N is nominal, $T_j = T_{j-1} + n$. Then a m-period range accrual note P, with initiation date of first period T_0 and end date T_m of the m^{th} period. The timeline of multi-period range accrual note is shown in Figure 6.

3.1.2.5 Using Hull-White and CIR++ Model to Value RAN

After considering the instruments of range accrual note pricing, this part I provide more understanding to value the range accrual note step by step.

First, I focus on pricing the range accrual note at issue date. The calculation is started by summing all the value of each-period range accrual note.

$$P(t;T_0,T_m)=\sum_{j=1}^{m}V_j(t;T_{j-1},T_j)+Z(t,T_m)N \tag{6}$$

Where N is a nominal of the note.

Therefore, we need to know the function $V_j(t;T_{j-1},T_j)$ and $Z(t,T_m)$. The function $Z(t,T_m)$ is discount factor which uses to discount the nominal of the note. This function can be derived as $Z(t,T_m)=\exp\left(-R(t,T_m)(T_m-t)\right)$ and $R(t,T_m)$ is observed from market swap curve at time t and maturity at time T_m. The function

$V_j(t;T_{j-1},T_j)$ is the value of single-period range accrual note which is the collection of range digital call option (*RD*) giving

$$V_j(t;T_{j-1},T_j) = \left[\frac{CN}{D} Z(t,T_j) \sum_{i=0}^{n} RD_{T_j}(t;T_{j-1}+i,T_j)\right] \quad (7)$$

Where *C* is coupon rate, *D* is the number of days in the year, *t* is the valuation date and $T_{j-1} + n = T_j$. The parameter *C, N* and *D* of this equation are known, so the key function is RD_T.

$$RD_T(\tau) = \{1\ (R\ (\tau;\ \alpha) - K_L) - 1\ (R\ (\tau;\ \alpha) - K_U)\} \quad (8)$$

From RD_T equation, the most important function is $R(\tau;\alpha)$. If we can estimate this function, the range accrual note can be priced. In the Simulation Model section, I show how to apply Hull-White model and CIR++ model to value function $R(\tau;\alpha)$.

3.1.3 Simulation Model

This part I show the characteristics and equations of the Hull-White model and CIR++ model in order to apply them to price the range accrual note. From The Methods of Range Accrual Note Pricing section, a function $R(\tau;\alpha)$ is needed for obtaining the function RD_T and then the price of the note. Both models start by collecting the market data to be models' inputs. Some data need to be interpolated before being those inputs. Then, the models obtain the realization from the random process of Monte Carlo method instead of the probabilities from cumulative normal function to determine whether the reference rate stays in the predetermined range. Finally, I take all inputs into the function $R(\tau;\alpha)$ of Hull-White model and CIR++ model and will obtain the range accrual note's price.

3.1.3.1 <u>The Hull-White Model</u>

Hull and White (1990) assumed that the instantaneous short-rate process evolves under the risk-neutral measure according to

$$dr = k\,[\theta - r(\tau)]\,d\tau + \sigma(\tau)dW(\tau) \tag{9}$$

Where $\sigma(\tau)$ are deterministic function of time. The function θ is selected so that the model exactly fits the initial term structure of interest rates being currently observed in the market. The parameter of volatility function $\sigma(\tau)$ are chosen to fit the market volatility curve.

The generalized Hull-White model contains many popular term structure models as special cases. When $k = 0$ and σ is constant it is the Ho-Lee (1986) model. When k is not zero it is the original Hull-White (1990) model. In both models, future interest rates of all maturities are normally distributed and there are many analytic solutions for the prices of bonds and options on bonds. When r is changed to be $ln(r)$, it is the Black-Karasinski (1991) model which is perhaps the most popular version currently in use. In this model the future short-rate is log-normally distributed and rates of all other maturities are approximately log normally distributed.

Next, I will describe the process to obtain $r(\tau)$.

First I set the current time to t which is the issue date of range accrual note and define a deterministic function φ, which satisfies

$$\varphi(\tau) = f^{M}(t,\tau) + \frac{\sigma^{2}\left(1-e^{-k\tau}\right)^{2}}{2k^{2}} \tag{10}$$

It can be shown that $f^{M}(t,\tau)$ is the market instantaneous forward rate at time t for the maturity τ, that is to say,

$$f^{M}(t,\tau) = -\frac{\partial \ln P^{M}(t,\tau)}{\partial \tau} \tag{11}$$

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With $P^M(t,\tau)$ is the market discount factor for the maturity τ. Then, we obtain $x(\tau)$ by defining a new variable x with the random process

$$dx(\tau) = k(\theta - x(\tau))d\tau + \sigma(\tau)dW(\tau) \tag{12}$$

This random process is the fluctuation of historical reference rates which can make the reference rates go up and down from mean reversion level. So that I can write

$$r(\tau) = x(\tau) + \varphi(\tau) \tag{13}$$

Where the function $r(\tau)$ is the instantaneous short rate at each time τ. It is clear that the function $r(\tau)$ consists of the random function $x(\tau)$ and the mean reversion function $\varphi(\tau)$. When I simulate function $r(\tau)$, each simulated path will be different. I focus on this method instead of using the cumulative normal function to determine the probabilities whether the reference rates stay in predetermined range. In addition, these rates $r(\tau)$ will be the random paths of the function $R(\tau;\alpha)$ (use $r(\tau)$ to generate $R(\tau;\alpha)$). Next, I show how to obtain the function $R(\tau;\alpha)$.

Brigo and Mercurio (2006) present a way to estimate the function $R(\tau;\alpha)$. This function cannot be observed in the financial market directly. For example, if I set the current time to t and I want the spot rate occurred at time t and maturity at time T, this spot rate can be observed from the market such as the yield curve and the swap curve. However, the time τ is between the time t and T, so the estimation is need for obtaining this rate $R(\tau;\alpha)$.

The function $R(\tau;\alpha)$ is derived from the method of zero-coupon bond pricing. The zero-coupon bond that is priced at time τ for the maturity T can be write as

$$P(\tau,T) = E\left\{\exp\left(-\int_{\tau}^{T} r(s)ds\right)\right\} \tag{14}$$

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From this expression it is clear that whenever I can characterize the distribution of $\exp\left(-\int_{\tau}^{T} r(s)ds\right)$ in terms of a chosen dynamics for *r*, I can compute bond prices *P*. Next, the price of a zero-coupon bond can be derived by computing this equation. We obtain

$$P(\tau, T) = A(\tau, T) \exp(-B(\tau, T)r(\tau)) \qquad (15)$$

Where

$$B(\tau,T) = \frac{1}{k}\left[1 - e^{-k(T-\tau)}\right]$$

$$A(\tau,T) = \frac{P^M(t,T)}{P^M(t,\tau)} \exp\left\{B(\tau,T) f^M(t,\tau) - \frac{(\sigma(\tau))^2}{4k}(1 - e^{-2k\tau})B(\tau,T)^2\right\}$$

After I have the equation to price a zero-coupon bond, I present the affine term structure model which can compute the function $R(\tau;\alpha)$ from it.

Affine term structure models are interest rate models where $R(\tau;\alpha)$ is an affine function in the instantaneous short rate $r(\tau)$. I can write

$$R(\tau; \alpha) = \alpha(\tau, T) + \beta(\tau, T)r(\tau) \qquad (16)$$

Where α and β are deterministic functions of time. When the zero-coupon bond price can be written in the form above, this relation is always satisfied. Since then clearly it is enough to set

$$\alpha (\tau, T) = - (\ln A(\tau, T)) / (T - \tau), \qquad \beta (\tau, T) = B(\tau, T) / (T - \tau).$$

When I combine all equations together, I obtain the formula to estimate the function $R(\tau;\alpha)$ as

$$R(\tau;\alpha) = -\frac{\ln A(\tau,T)}{T-\tau} + \frac{B(\tau,T)r(\tau)}{T-\tau} \qquad (17)$$

Where

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$$B(\tau,T)=\frac{1}{k}\left[1-e^{-k(T-\tau)}\right]$$

$$A(\tau,T)=\frac{P^M(t,T)}{P^M(t,\tau)}\exp\left\{B(\tau,T)f^M(t,\tau)-\frac{(\sigma(\tau))^2}{4k}(1-e^{-2k\tau})B(\tau,T)^2\right\}$$

Finally, this function $R(\tau;\alpha)$ can be used as reference interest rate in the method of pricing range accrual note. Then, the range accrual note can be valued.

For clearly application of the range accrual note pricing, I show in the estimation of models' parameters and inputs section.

3.1.3.2 CIR++ Model

Brigo and Mercurio (2001) present how to extend time-homogeneous short-rate model to a model which can reproduce any observed yield curve, through a procedure that preserves the possible analytical tractability of the original model. The Cox-Ingersoll-Ross (1985) model is the most relevant case to which their procedure can be applied, referred to as CIR++. The short-rate dynamics is then given by

$$dr(\tau)=k(\theta-r(\tau))d\tau+\sigma(\tau)\sqrt{r(\tau)}dW(\tau) \quad (18)$$

with k *and* θ positive constants.

Next, I will describe the process to obtain $r(\tau)$.

The process to obtain $r(\tau)$ of CIR++ model has the same method as Hull-White model, that is

$$r(\tau) = x(\tau) + \varphi(\tau) \quad (19)$$

But the process to obtain $x(\tau)$ and $\varphi(\tau)$ of CIR++ model are different from Hull-White model. $\varphi(\tau)$ of CIR++ model can be write as

$$\varphi(\tau)=f^M(t,\tau)-f^{CIR}(t,\tau) \quad (20)$$

$$f^{CIR}(t,\tau)=\frac{2k\theta(\exp\{\tau h\}-1)}{2h+(k+h)(\exp\{th\}-1)} \tag{21}$$

$$h=\sqrt{k^2+2\sigma^2} \tag{22}$$

Then, the process to obtain *x(τ)* can be computed from the random process

$$dx(\tau)=k(\theta-x(\tau))d\tau+\sigma(\tau)\sqrt{x(\tau)}dW(\tau) \tag{23}$$

where the random distribution function of CIR++ model is the chi-square distribution which is different from the normal distribution of Hull-White model.

Moreover, the function $R(\tau;\alpha)$ of CIR++ model has additional terms comparing to the function $R(\tau;\alpha)$ of Hull-White model. Therefore, the function $R(\tau;\alpha)$ can be write as

$$R(\tau;\alpha)=-\frac{\ln A(\tau,T)}{T-\tau}+\frac{B(\tau,T)r(\tau)}{T-\tau}+\frac{\ln\dfrac{P^M(t,T)A(t,\tau)}{P^M(t,\tau)A(t,T)}}{T-\tau}-\frac{B(\tau,T)\varphi(\tau)}{T-\tau} \tag{24}$$

Where

$$A(\tau,T)=\left[\frac{2h\exp\{(k+h)(T-\tau)/2\}}{2h+(k+h)\exp\{(T-\tau)h\}-1}\right]^{2k\theta/\sigma^2}$$

$$B(\tau,T)=\frac{2(\exp\{(T-\tau)h\}-1)}{2h+(k+h)(\exp\{(T-\tau)h\}-1)},$$

$$h=\sqrt{k^2+2\sigma^2}.$$

Finally, this function $R(\tau;\alpha)$ can be used as reference interest rate in the method of pricing range accrual note. Then, the range accrual note can be valued.

For clearly application of the range accrual note pricing, I show in the estimation of models' parameters and inputs section.

3.2 Data

3.2.1 Daily 6M USD LIBOR

The historical data of daily 6M USD LIBOR is obtained from Reuters 3000Xtra. It spans ten years from 18th April 1997 – 18th April 2007 which are 5.36% on the issue date of the note. The numbers of observations are 2,547 samples. The mean of this data is 4.11%. The highest value is 7.1% in 2000, and the lowest value is 1.1% in 2003. This data are used to determine the mean reversion parameter and speed of mean reversion parameter of the Hull-White model and CIR++ model. The graph of this data is shown in Figure 1.

3.2.2 Weekly U.S. Swap Curves

The weekly U.S. swap curves which are collected from DataStream are shown in Figure 2. They are collected from 18th April 2000 to 18th April 2007 spanned ten years to maturity such as 1, 3, 5, and 10 year swap rates. In Figure 2, the graph needs the interpolation technique to obtain the swap rates with everyday maturity. This technique is presented next. These data are the input of those models and also used for computing the volatility parameter which depends on time.

3.2.3 The Estimation of Models' Parameters and Inputs

3.2.3.1 The Process to Obtain Parameter k, $\sigma(\tau)$ and θ

There are many studies of estimation of the short rate models' parameters. One of the popular methods is the calibration of these parameters to the market traded options prices. Hull and White (2000) present the process of determining the parameters that are used in the term structure model. It is analogous to selecting them when implementing the Black-Scholes model to price equity options. The procedure is to choose the parameters so that the tree implementation of the term structure model replicates the market prices of actively traded options. Then, they minimize the sum of the squares of the differences between the model prices and market prices for these options. The most common source of option prices for calibration purposes are quotes that are available from brokers on European-style swap options and caps and floors. Brigo and Mercurio (2006) also present this method again, especially for the calibration to the real market, and the comparable results of several short rate models.

Shen, Huang, and Tao (2002) propose the methods to estimate the parameters of interest-rate model. The parameters estimated by this method are the mean reversion and the speed of mean reversion. All of these parameters are used for computing the random process x, the instantaneous short rate $r(\tau)$, and also the function $R(\tau;\alpha)$.

For the Range Accrual Note pricing, its reference rate is the daily 6M USD LIBOR. They suggest using the historical data of daily 6M USD LIBOR for estimating those parameters. Then, I collect these data from 18th April 1997 to 18th April 2007 (2,547 samples) and run the linear regression of equation

$$dr_t = \beta_1 + \beta_2 r_t$$

Comparing with the equation

$$E[dr] = k\theta d\tau - krd\tau \ ,$$

I can calculate the speed of mean reversion parameter k as

$$k = -\frac{\beta_2}{d\tau}$$

and the mean reversion level parameter θ as

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$$\theta = -\frac{\beta_1}{\beta_2}$$

Where the $d\tau$ is 1/365 for 365 days in a year.

From the table 1, the values of best fitted parameters are 0.03 and 0.055 for the mean reversion level and the speed of mean reversion. Both parameters from this method are constant. Except for the volatility parameter, it needs to depend on time. Why does the volatility parameter must depend on time? Eric Benhamou, Goldman Sachs International, says that it is important to observe this parameter depending on time when pricing the range accrual note. An investor, who wants a strategy with a certain carry or pick-up, may be interested in investing in a range accumulation bond. Investors are like for many other structures playing the game of increasing their risk to get higher returns. If their forecast were right, their strategy would turn out to be very profitable. Obviously, investors don't expect short-term rates be outside the range. However, along with the level of LIBOR, investors must have keep a watchful eye on volatility levels, because a combination of increasing rates and increasing volatility levels may have significant impact on the security's market price.

Next, I show how to obtain the volatility parameter which is computed from the method of interpolating the volatility curve. Dai and Singleton (2000) propose the exploration of differences and relative goodness-of-fits of affine term structure models. They suggest the implied term structure of volatility curve is defined as the historical sample standard deviation of weekly changes of swap curve. This method is one of many methods which can compute the implied term structure of volatility curve.

For the range accrual note pricing, I use the historical weekly swap rates during 18th April 2000 - 18th April 2007 to compute the volatility curve. It can be written as

$$\sigma(\tau) = SD\left\{\frac{Swap_{i+1}(\tau) - Swap_i(\tau)}{Swap_i(\tau)}\right\}$$

where τ is maturity and i is week.

Finally, I can obtain the term structure of historical volatility which is exponential shape with a peak at initial date. The volatility starts at 6 bps and ends at 2.2 bps. This graph shows that the volatility of short-term swap rate is higher than the volatility of long-term swap rate. The volatility curve is shown in Figure 7.

3.2.3.2 The Process to Obtain Function $f^M(t,\tau)$ and $P^M(t,\tau)$

The function $f^M(t,\tau)$ is the market instantaneous forward rate at time t for the maturity τ and the function $P^M(t,\tau)$ is the market discount factor for the maturity τ. Both functions are used for being the inputs of Hull-White model and CIR++ model in order to compute the function $R(\tau;\alpha)$. The U.S. swap curve which is collected on 18th April 2007 spanned ten years to maturity is the essential data for obtaining those functions. The range accrual note has ten years to maturity, so it needs the reference rate which is forecasted to the end of its maturity. However, the LIBOR are short term rate and has only a maturity of one year or less, but the swap rates are the rates which have one year to thirty years maturities.

Nevertheless, the LIBOR and swap rates which I observed don't have everyday maturity until ten years. The LIBOR has only one month to twelve months maturities and the swap rates conclude only two, three, four, five, seven, and ten years to maturities. Therefore, they must be interpolated in order to build the curve. There are many studies about the interpolation methods such as linear discrete-time interpolation, cubic spline-based discrete-time interpolation, the Nelson and Siegel (1987), the Diament (1993) model, and the four-parameter model of Mansi and Phillips (2001). The interpolation method I used in this thesis is the cubic spline interpolation proposed in Rendleman (2004), because it is suitable for swap curve. After interpolating the observed data of LIBOR and swap rates, I obtain the swap curve at 18th April 2007. This curve is shown in Figure 8.

Next, I describe how to compute function $P^M(t,\tau)$ which is the market discount factor and one of the short rate models' inputs. When we calculate the present value of some future cash flow, we are said to discount that cash flow. A discount factor is the factor by which the future cash flow must be multiplied to obtain the present value. Discount factors can be calculated from spot rates. For example, 6M USD LIBOR at 18th April 2007 is 5.36%, the discount factor is

$$\exp\left(-0.0536*(181/365)\right)=0.9741$$

When I completely calculate all of spot rates from interpolated swap curve, I obtain the discount curve of function $P^M(t,\tau)$ shown in Figure 9.

Furthermore, I show how to compute the function $f^M(t,\tau)$ which is the market instantaneous forward rate at time t for the maturity τ. This function is also one of the short rate models' inputs and calculated by

$$f^M(t,\tau)=-\frac{\partial \ln P^M(t,\tau)}{\partial \tau}$$

Where $\partial\tau = 1/365$

After I have obtained the discount curve, I can compute $f^M(t,\tau)$ curve at 18th April 2007 and be shown in Figure 10.

Finally, I've got the function $P^M(t,\tau)$ and $f^M(t,\tau)$ for all ten-year maturities.

3.2.4 The Accuracy of the Result Given by Monte Carlo Simulation

The accuracy of the range accrual note's prices given by Monte Carlo simulation depends on the number of trials. It is usual to compute the standard deviation and the mean of the range accrual note's prices given by simulation trials. Denote the mean of all simulation's prices by μ and the standard deviation by ω. The parameter μ is the simulation's estimate of the range accrual note's price. The standard error of the estimate is

$$\frac{\omega}{\sqrt{M}}$$

where M is the number of trials. A 95% confidence interval for the range accrual note's price P is given by

$$\mu - \frac{1.96\omega}{\sqrt{M}} < P < \mu + \frac{1.96\omega}{\sqrt{M}}$$

This shows that the uncertainty about the range accrual note's price is inversely proportional to the square root of the number of trials.

When the range accrual note is priced by Monte Carlo simulation, the results can show the mean prices (μ), the maximum prices, and the minimum prices. In this thesis, I price the range accrual note by using 1,000 iterations of Monte Carlo method. I expect that the 1,000 numbers of iterations are enough to show the maximum and minimum prices are less than 5% from the mean prices. If the mean price is USD 10,000, a 95% confidence interval for the range accrual note's price is [USD 9,500, USD 10,500]. Moreover, the models' volatilities may affect the range of prices. So, this method should explain the effect of volatility to the note's prices better.

CHAPTER IV

RESULTS

The result section is divided into two subsections which are the result of range accrual note pricing and a comparison of Hull-White model and CIR++ model.

4.1 Range Accrual Note Pricing Result

This part shows the results of Range Accrual Note pricing. The reference rates are simulated by using function $R(\tau;\alpha)$ of Hull-White model and CIR++ model. The interval of the simulation is one day and the number of iterations is 1,000.

In addition, this part presents the behavior of both models by providing the sensitivity analysis of the speed of the mean reversion of the reference rate, the mean reversion level of the reference rate, and the volatilities.

4.1.1 Sensitivity analysis of the mean reversion and the speed of the mean reversion of Hull-White model and CIR++ model

This part, I present the sensitivity analysis of the mean reversion and the speed of mean reversion. When I price the range accrual note, I change the mean reversion parameter and speed of mean reversion parameter. The mean reversion is set to be 0.01, 0.03, 0.05, 0.07, and 0.09. The speed of mean reversion is set to be 0.01, 0.055, and 0.1. But I fix other parameters which are range of strike rate, volatility, and initial short rate. The range of strike rate is set to be 0%-7%. The volatility is the historical volatility. The initial short rate is set to be 5.36%.

This case the term structure of volatility is observed from the market shown in Figure 7. From result table 2, the ranges of prices of the note at issue date are USD [10929.73, 11389.50] and USD [11161.76, 11631.29] by using CIR++ model and Hull-White model, respectively (at θ=0.03, k=0.055, and σ=market volatility). The

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difference of both mean prices is 2.08%. For each parameter of each model, the prices of the note are not much different. The Hull-White model's prices are higher than the CIR++ model's prices. Even the historical of reference rate is rising (it has been rising since 2004), but both models forecast the reference rate will decrease. Therefore, instead the price of the note is lower than its selling price; the simulation shows the note's price is higher.

As a conclusion, the mean reversion of reference rate and speed of mean reversion of reference rate should be considered together. For the Hull-White model and CIR++ model, the different values of mean reversion level and speed of mean reversion don't make the note's prices change much in case of low volatility.

4.1.2 Sensitivity analysis of volatility of Hull-White and CIR++ model

This part, I present the sensitivity analysis of volatility. When I price the range accrual note, I change the term structures of volatilities which are set to be flat, linear, and historical volatility. But I fix other parameters which are range of strike rate, initial short rate, the mean reversion parameter and speed of mean reversion parameter. The range of strike rate is set to be 0%-7%. The mean reversion is set to be 0.03. The speed of mean reversion is set to be 0.055. The initial short rate is set to be 5.36%.

This part shows the sensitivity analysis of the volatilities of Hull-White model and CIR++ model. The volatilities of both models are set to be different in order to obtain the effect of volatilities to the note's prices. The term structures of volatilities are flat, linear, and observed from the market. The note's price is very sensitive to volatility. The results of the Monte Carlo simulation (1,000 iterations) show the range of maximum and minimum prices of the note.

From result table 3, in case of flat term structure of volatility, the prices of the note of both models are directly affected by the different values of volatilities. The more volatility's value is, the more range between maximum and minimum price is. The high volatility makes the value of random path rise, and then the reference rate will be more volatile. This situation makes high chances that the entire reference rates stay in predetermined range or almost reference rates stay outside. Moreover, the Hull-White model has a range between maximum and minimum price higher than the CIR++ model. It seems that the Hull-White model has the higher effect of volatilities' values to the note's prices.

In the case of linear term structure of volatility, the negative slope volatility begins from 90 bps to 1 bps. The mean of these volatilities are 45 bps. From simulation, the ranges of prices of the note at issue date are USD [10401.32, 12391.74] and USD [10380.19, 12412.87] by using CIR++ model and Hull-White model, respectively. These prices are closely to the prices between 30 bps and 50 bps of flat volatilities. It seems that the linear volatility provides the note's prices are the same as the flat volatility which has a value of linear volatility's mean.

As a conclusion, the volatility highly affects the note's price. The higher volatility affects the note's price more than the lower volatility.

4.1.3 Sensitivity analysis of strike rate and initial rate of Hull-White and CIR++ Model

This part shows the sensitivity analysis of the strike rate and initial instantaneous short rate of Hull-White model and CIR++ model. When I price the range accrual note, the volatilities, strike rates, and initial rate of both models are set to be different in order to obtain the effect of them to the note's prices. The term structures of volatilities are flat (extreme volatility σ equals 1bps and 90 bps), linear, and historical volatility. The strike rates are divided into 4 ranges which are 0%-1%, 0%-3%, 0%-5%, and 0%-7%. The initial rates are 3%, 5.36%, and 7%. But I fix other parameters which are the mean reversion parameter and speed of mean reversion parameter. The mean reversion is set to be 0.03. The speed of mean reversion is set to be 0.055. The results of the Monte Carlo simulation (1,000 iterations) show the mean prices of the note.

From result table 4, the note's prices of both models are highly affected by the changing of strike rates. In case of 3% initial short rate, the narrow range of strike rates (0%-1% and 0%-3%) makes the note's prices be less than the wide range of strike rates (0%-5% and 0%-7%). It shows that the wide range has more chances the reference rate stays in this range. Moreover, the note's prices are very sensitive in 0%-3% range, because the reference rate moves between 1% and 5%. The prices of the note with narrow range (0%-1%) are around USD 7,500 which is lower than the note's face value for all volatility cases. That means the range accrual note pays only face value. In case of 5.36% initial short rate, the note's prices are the same way as the prices of 3% initial short rate, but the note's prices are very sensitive in 0%-5% range, because the reference rate moves between 3% and 7%. Finally, in case of 7% initial short rate, the note's prices are the same way as the prices of 3% initial short rate and 5.36% initial short rate, but the note's prices are very sensitive in 0%-7% range, because the reference rate moves between 5% and 9%.

When comparing the range accrual note's prices of Hull-White model and CIR++ model, the prices of Hull-White model are higher than the prices of CIR++ model. To prove them, I use the histograms of reference rates of both models to see the frequency distribution of them. For each model, I collect the 3,650-days reference rates of one realization to build the graph of histogram. I set the initial short rate to be

3%, but I set the volatilities are historical volatility (mean=3bps) and extremely high volatility (90 bps). Then, they show how reference rates fall into each of several interest rate categories. The histograms, which are shown in Figure 14 and 15, show that the reference rates of Hull-White model move inside the range of strike rate more than the reference rates of CIR++ model. So, these histograms prove that the prices of Hull-White model are higher than the prices of CIR++ model. The reason why Hull-White model's prices are higher than CIR++ model's prices is the difference of random distribution functions. The Hull-White model uses normal distribution function, while the CIR++ model uses chi-squares distribution function.

As a conclusion, the changing of range of strike rates highly affects the prices of range accrual note. The narrow range provides the note's prices are lower than the wide range for all volatility cases. Moreover, range of strike rates which is near the movement of reference rate makes the note's prices more sensitive than other ranges. Therefore, the initial short rate highly affects the range accrual note's prices too. Lastly, the normal distribution function of Hull-White model makes the range accrual note's prices are higher than the chi-squares distribution function of CIR++ model.

4.2 A Comparison of Hull-White model and CIR++ Model after Simulation

4.2.1 Effect of mean reversion and speed of reference rate to both models

For the mean reversion of the reference rate, does it affect the Hull-White model and CIR++ model? The mean reversion level means the future reference rate tends to reach that level. Considering from the result of Range Accrual Note pricing, in case of low volatility, all the note's prices of CIR++ model and Hull-White model in same speed of mean reversion don't change much when the mean reversion changes. But, in case of high volatility, the entire note's prices of both models in same speed of mean reversion decrease when the mean reversion increases.

For the speed of mean reversion of the reference rate, does it influence both models? Evaluating from the result of Range Accrual Note pricing, in case of low volatility, all the note's prices of CIR++ model and Hull-White model in same mean reversion don't change much when the speed of mean reversion changes. But, in case of high volatility, the entire note's prices of both models in same mean reversion increase when the speed of mean reversion increases.

As a conclusion, the mean reversion parameter and speed of mean reversion parameter of the CIR++ model and the Hull-White model should be considered in case of high volatility.

4.2.2 Effect of volatility to both models

For the volatility of the reference rate, it absolutely affects the Hull-White and CIR++ model. The volatility indicates how volatile the reference rate goes up or down from the mean. Considering from the result of Range Accrual Note pricing, the note's prices of the CIR++ model and the Hull-White model have high effect from the change of volatilities. The higher volatility makes the note's prices have higher range of maximum and minimum price.

As a conclusion, the effect of volatility to the CIR++ model and the Hull-White model is very high. The higher volatility provides the higher range of maximum and minimum prices.

CHAPTER V

CONCLUSION

5.1 Conclusion

This thesis studies how to apply the Hull-White model and CIR++ model to price the range accrual note by using Monte Carlo simulation. Moreover, I also study the behavior of both models by providing the sensitivity of the note's price, when changing models' parameters (the mean reversion, the speed of mean reversion, and the volatility), the range of strike rates, and the initial instantaneous short rate. From the results, the method used in this thesis is well applied to price the range accrual note by using Monte Carlo simulation. These models' prices are little affected by changing their mean reversion and speed of mean reversion in case of low volatility, but they are highly affected by changing them in case of high volatility. Moreover the prices are highly affected by changing their range of strike rates and initial short rates. Lastly, the Hull-White model makes the range accrual note's prices are higher than the CIR++ model. Some limitations of this thesis are the method to compute the volatility parameter and the range of strike rates. The volatility parameter is computed from historical data and the range of strike rates is assumed to be constant until maturity date. The problem found in this study is it takes much time for running Monte Carlo simulation at long maturity and at high number of iterations. It can be solved by creating well-designed programming.

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APPENDIX

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Table 1 Result (OLS Regression to obtain parameter *k* and *θ*)

Regression Statistics	
Multiple R	0.01482221
R Square	0.000219698
Adjusted R Square	-0.000173297
Standard Error	0.00028065
Observations	2546

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	4.40319E-08	4.4E-08	0.559034	0.454718694
Residual	2544	0.000200376	7.88E-08		
Total	2545	0.00020042			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	6.49216E-06	1.32642E-05	0.489449	0.624566	-1.95176E-05	3.25E-05
X Variable 1	-0.000219243	0.000293228	-0.747686	0.454719	-0.000794233	0.000356

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Table 2 Result (Sensitivity Analysis of parameter *k* and *θ*)

This table shows the sensitivity analysis of the mean reversion and the speed of the mean reversion of Range Accrual Note pricing. Table 2a shows the maximum and minimum prices and table 2b shows the mean prices. Table 2c shows the mean prices of extreme volatility. The maturity is 10 years. The coupon is paid 7% annually. The volatility is observed from the market. The values of best fitted parameters are 0.03 and 0.055 for the mean reversion level and the speed of mean reversion, respectively. The number of iterations is 1,000.

Table 2a

CIR++ Model

k \ theta	0.01	0.03	0.05	0.07	0.09
0.01	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50
0.055	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50
0.1	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50	10929.73 - 11389.50

HW Model

k \ theta	0.01	0.03	0.05	0.07	0.09
0.01	11151.97 - 11621.08	11151.97 - 11621.08	11151.97 - 11621.08	11151.97 - 11621.08	11151.97 - 11621.08
0.055	11161.76 - 11631.29	11161.76 - 11631.29	11161.76 - 11631.29	11161.76 - 11631.29	11161.76 - 11631.29
0.1	11161.96 - 11631.49	11161.96 - 11631.49	11161.96 - 11631.49	11161.96 - 11631.49	11161.96 - 11631.49

Table 2b

CIR++ Model

k \ theta	0.01	0.03	0.05	0.07	0.09
0.01	11159.61	11159.59	11159.57	11159.55	11159.54
0.055	11159.62	11159.62	11159.62	11159.62	11159.62
0.1	11159.62	11159.62	11159.62	11159.61	11159.61

HW Model

k \ theta	0.01	0.03	0.05	0.07	0.09
0.01	11386.24	11386.24	11386.24	11386.24	11386.24
0.055	11396.53	11396.53	11396.53	11396.53	11396.53
0.1	11396.73	11396.73	11396.73	11396.73	11396.73

Table 2c

	CIR++ Model			HW Model		
	k \ theta	0.01	0.09	k \ theta	0.01	0.09
Low Vol. σ = 1 bps	0.01	11159.62	11159.6	0.01	11396.53	11396.51
	0.1	11159.62	11159.59	0.1	11396.53	11396.5
	k \ theta	0.01	0.09	k \ theta	0.01	0.09
Historical Vol. σ ≈ 3 bps	0.01	11159.61	11159.54	0.01	11386.24	11386.24
	0.1	11159.62	11159.61	0.1	11396.73	11396.73
	k \ theta	0.01	0.09	k \ theta	0.01	0.09
High Vol. σ = 90 bps	0.01	11141.49	10998.29	0.01	11378.4	11235.2
	0.1	11159.44	11158.02	0.1	11396.35	11394.97

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Table 3 Result (Sensitivity Analysis of Volatility)

This table shows the sensitivity analysis of volatility of range accrual note pricing. The maturity is 10 years. The coupon is paid 7% annually. The volatility is set to be flat and linear comparing with market volatility. The mean reversion level and the speed of mean reversion are 0.03 and 0.055. The number of iterations is 1,000.

CIR++ Model

Flat

volatility	0.01		0.03		0.05		0.07		0.09	
	min	max	min	max	min	max	min	max	min	max
price	11137.29	11181.96	11092.61	11226.63	11047.94	11271.29	11003.26	11315.96	10958.59	11360.62
	11159.62		11159.62		11159.61		11159.61		11159.60	

Flat

volatility	0.1		0.3		0.5		0.7		0.9	
	min	max	min	max	min	max	min	max	min	max
price	10936.25	11382.95	10489.51	11829.34	10042.76	12275.38	9596.02	12721.07	9149.37	13166.31
	11159.60		11159.42		11159.07		11158.54		11157.84	

Linear

volatility	Linear	
	min	max
price	10401.32	12391.74
	11159.16	

Market

volatility	Market	
	min	max
price	10929.73	11389.5
	11159.62	

HW Model

Flat

volatility	0.01		0.03		0.05		0.07		0.09	
	min	max	min	max	min	max	min	max	min	max
price	11374.19	11418.87	11329.52	11463.54	11284.84	11508.22	11240.17	11552.89	11195.5	11597.56
	11396.53		11396.53		11396.52		11396.52		11396.51	

Flat

volatility	0.1		0.3		0.5		0.7		0.9	
	min	max	min	max	min	max	min	max	min	max
price	11173.16	11619.9	10726.41	12066.65	10279.67	12513.39	9832.926	12960.13	9386.182	13406.88
	11396.51		11396.33		11395.98		11395.45		11394.65	

Linear

volatility	Linear	
	min	max
price	10380.19	12412.87
	11396.07	

Market

volatility	Market	
	min	max
price	11161.76	11631.29
	11396.53	

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Table 4 Result (Sensitivity Analysis of Strike Rate and Initial Short Rate)

This table shows the sensitivity analysis of strike rate and initial instantaneou rate of range accrual note pricing. The maturity is 10 years. The coupon is p annually. The volatility is set to be flat and linear comparing with historical vo The mean reversion level and the speed of mean reversion are 0.03 and 0.0! number of iterations is 1,000.

r_0 = 3%

CIR++ Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	11359.57	11359.47	11358.65	11357.18
0% - 5%	11359.57	11359.47	11358.65	11357.18
0% - 3%	9283.32	9184.71	8658.20	8066.20
0% - 1%	7407.53	7407.51	7407.39	7407.20

Volatility	Average
Const. 1bps	1 bps
Historical	3 bps
Linear	45 bps
Const.90bps	90 bps

HW Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	11596.75	11596.65	11595.83	11594.36
0% - 5%	11596.75	11596.65	11595.83	11594.36
0% - 3%	9520.50	9421.89	8895.38	8303.38
0% - 1%	7644.71	7644.69	7644.57	7644.38

r_0 = 5.36%

CIR++ Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	11159.62	11159.52	11158.70	11158.15
0% - 5%	8504.80	8406.19	7879.68	7287.68
0% - 3%	5851.76	5851.74	5851.64	5851.57
0% - 1%	5849.98	5849.96	5849.86	5849.79

HW Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	11396.53	11396.43	11395.61	11395.05
0% - 5%	8741.71	8643.10	8116.59	7524.59
0% - 3%	6088.67	6088.65	6088.55	6088.48
0% - 1%	6086.89	6086.87	6086.77	6086.70

r_0 = 7%

CIR++ Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	7619.72	7521.11	6994.60	6402.60
0% - 5%	4966.68	4966.66	4966.56	4966.49
0% - 3%	4964.90	4964.88	4964.78	4964.71
0% - 1%	4964.90	4964.88	4964.78	4964.71

HW Model

Strike	Volatility			
	Const. 1bps	Historical	Linear	Const.90bps
0% - 7%	7856.63	7758.02	7231.51	6639.51
0% - 5%	5203.59	5203.57	5203.47	5203.40
0% - 3%	5201.81	5201.79	5201.69	5201.62
0% - 1%	5201.81	5201.79	5201.69	5201.62

Figure 1 Historical Reference Rate (6M USD LIBOR)

This 6M USD LIBOR is observed between 18 April 1997 and 18 April 2007 which is 5.36% on the issue date of the note. The mean of this data is 4.11% with highest at 7.1% in 2000 and lowest at 1.1% in 2003. This data used for determining the mean reversion parameter and speed of mean reversion parameter of the Hull-White model and CIR++ model.

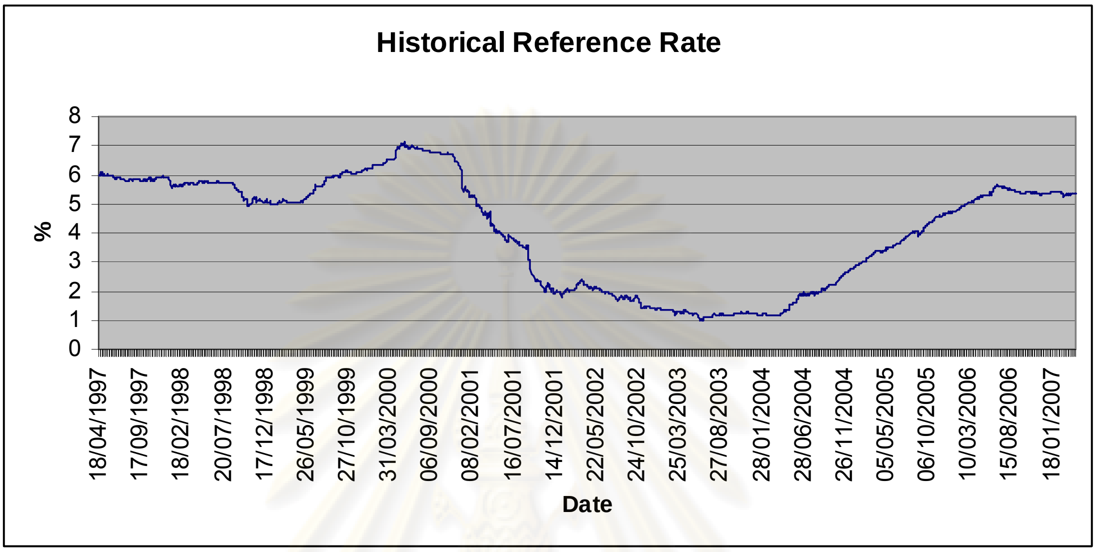

Figure 2 Historical Swap Curve

The weekly U.S. swap curves are collected from 18th April 2000 to 18th April 2007 concluded ten years to maturity such as 1, 3, 5, and 10 year swap rates. The graph needs the interpolation technique to obtain the swap rates with everyday maturity. This technique is presented in the methodology section. These data are the input of those models and also used for computing the volatility parameter which depends on time.

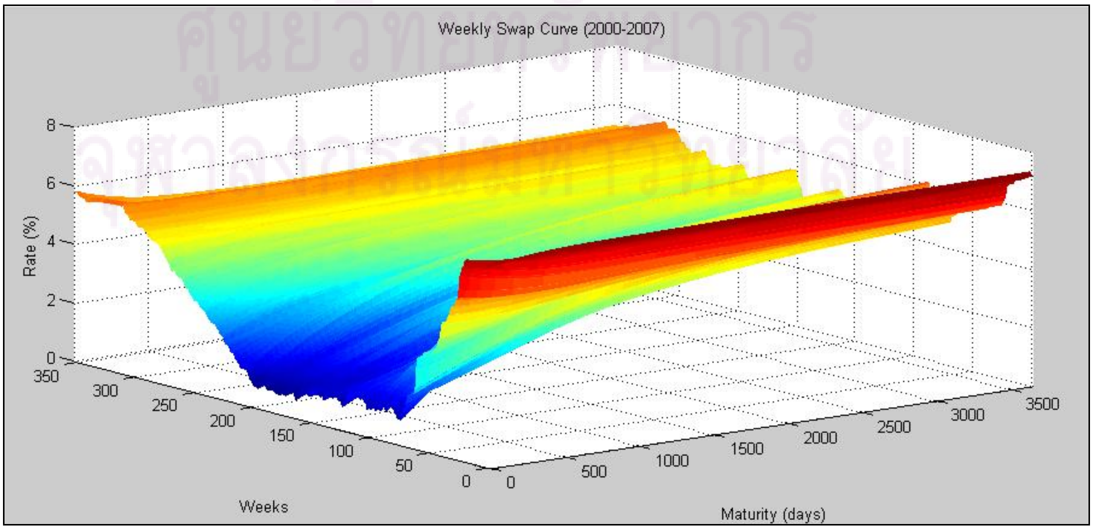

Figure 3 The Timeline of Function *R(τ ;α)*

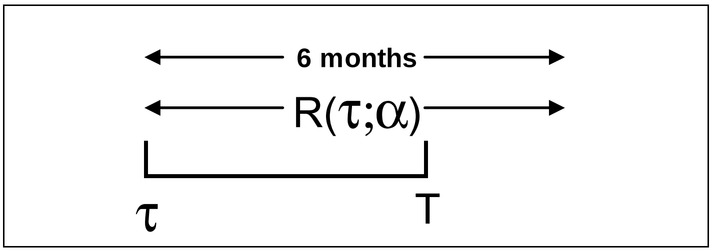

Figure 4 The Timeline of Function RD_T

I assume the valuation date of range accrual note is the same as the date of first period of range accrual note. If the reference rate $R\ (\tau\ ;\alpha)$ at day τ stays in the predetermined range, the holder will receive the payoff 1 at maturity T_1. Finally, the RD_T of day τ is one at maturity T_1.

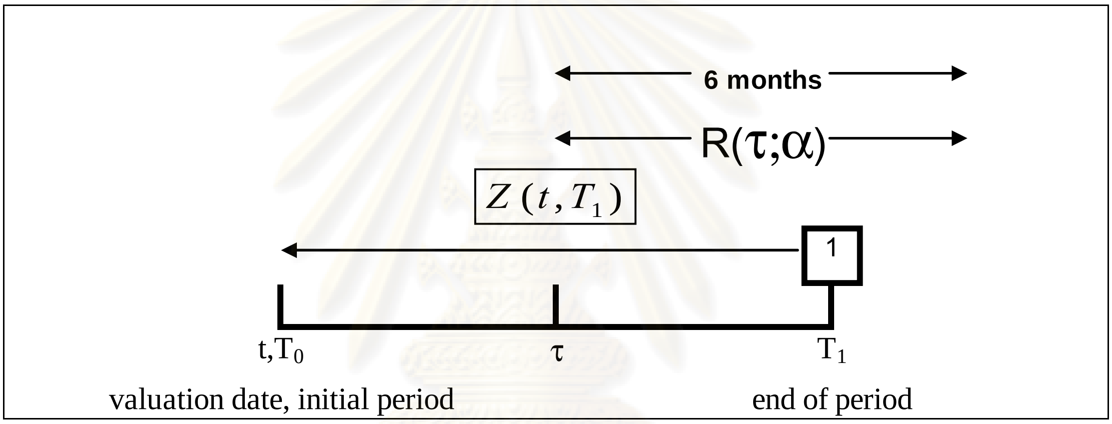

Figure 5 The Timeline of Single Period Range Accrual Note

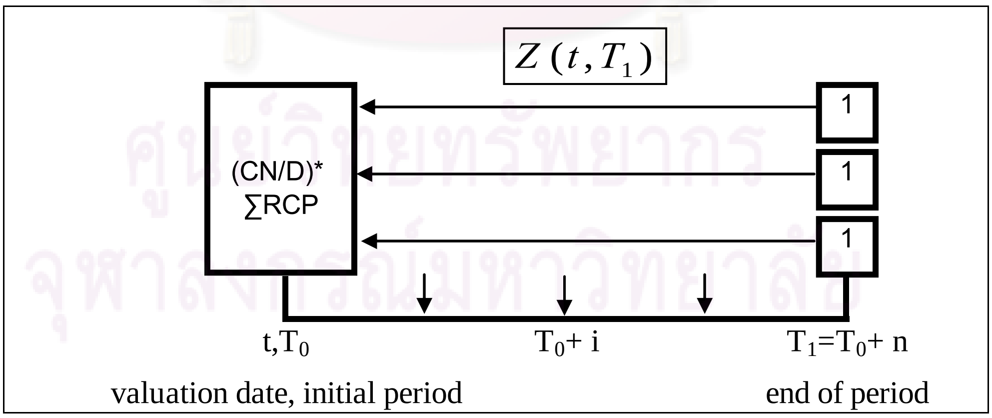

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Figure 6 The Timeline of Multi-Period Range Accrual Note

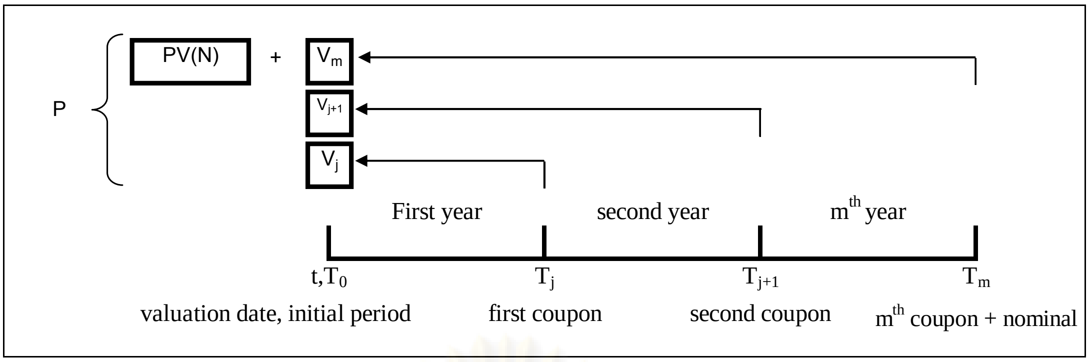

Figure 7 The Volatility Curve

This graph shows the volatility curve at 18th April 2007. It is created by computing the standard deviations of weekly change of swap curves.

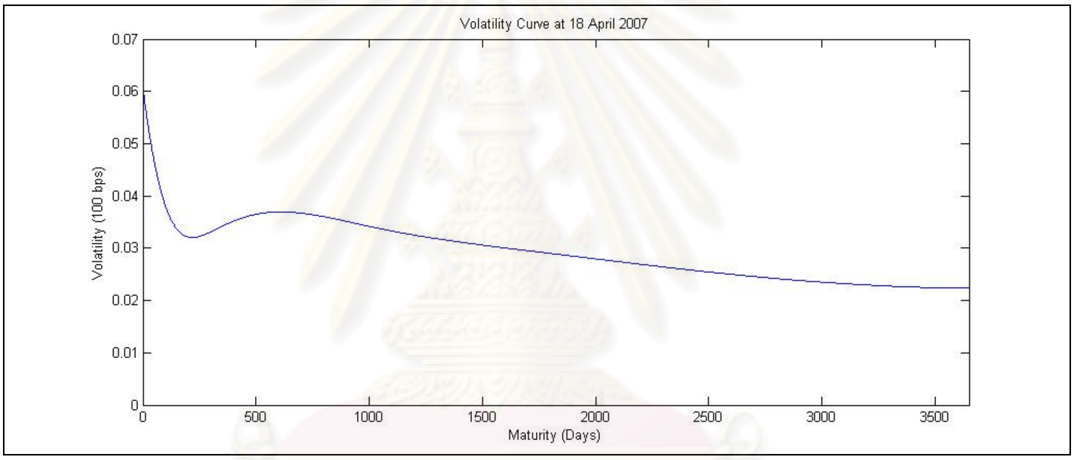

Figure 8 The Swap Curve

This graph is the swap curve at 18th April 2007. It is interpolated by cubic spline interpolation.

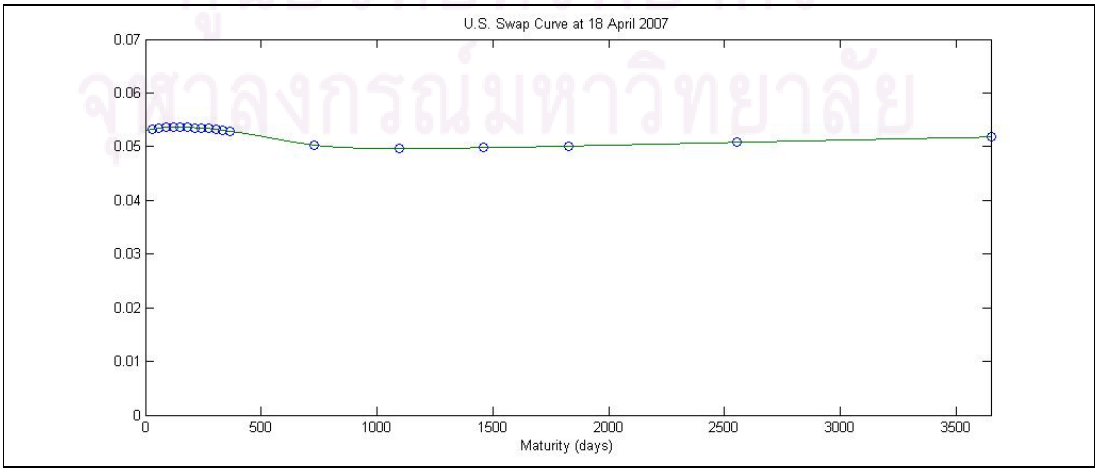

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Figure 9 The Discount Curve

This graph is the discount curve at 18th April 2007 which is computed from the discount factor of swap curve at 18th April 2007.

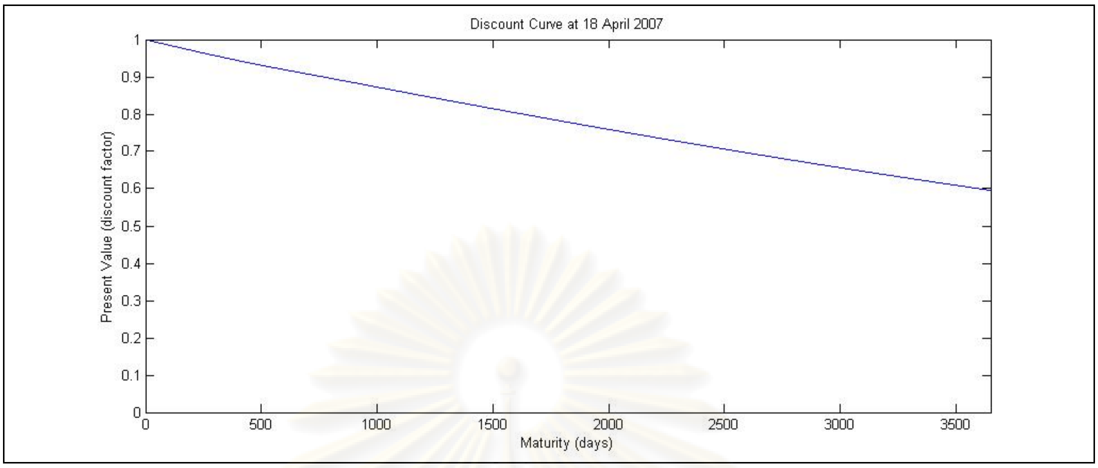

Figure 10 The Instantaneous Forward Rate Curve

This graph is the instantaneous forward rate curve at 18th April 2007 which is computed from the discount curve at 18th April 2007.

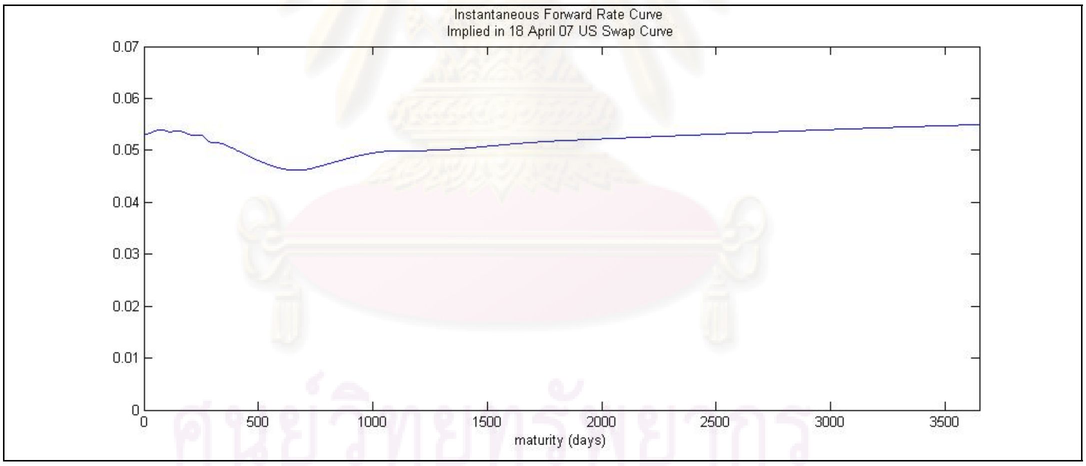

Figure 11 The Timeline of Instantaneous Forward Rate

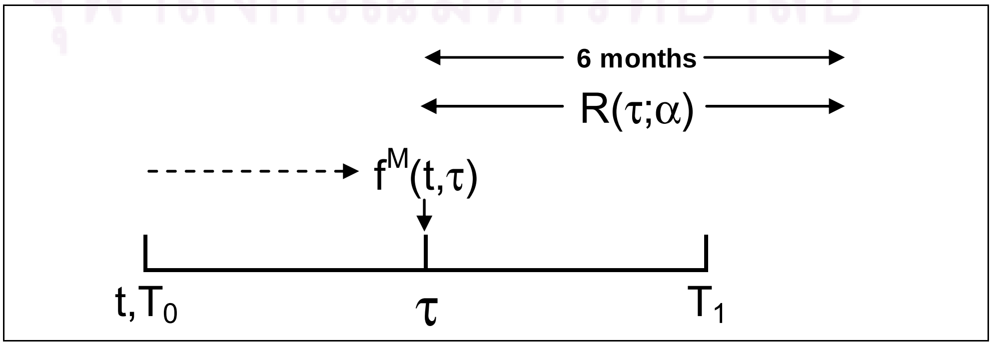

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Figure 12 The Flat Volatility Curves

This graph shows the flat volatility curves. Each curve has a constant value during range accrual note's period for ten years. It concludes the volatility's values 1, 3, 5, 7, 9, 10, 30, 50, 70, 90 basis points.

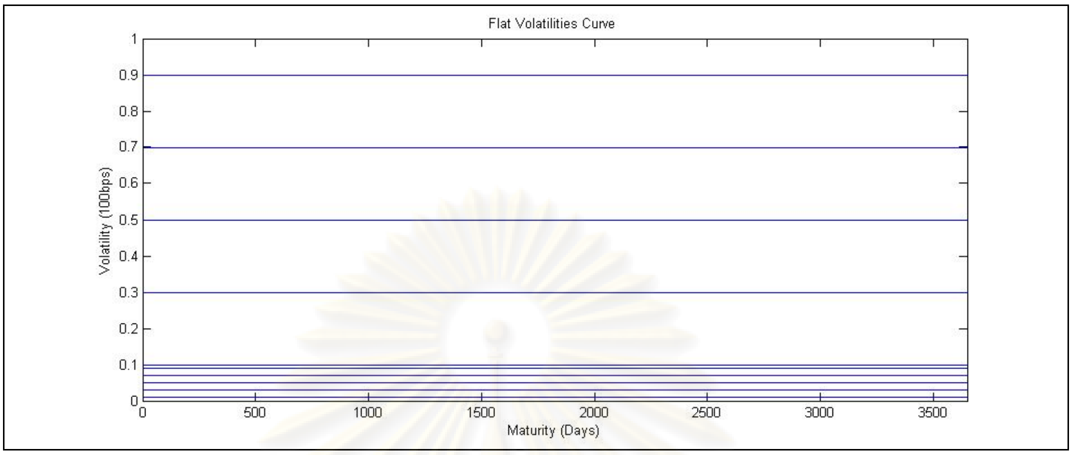

Figure 13 The Linear Volatility Curve

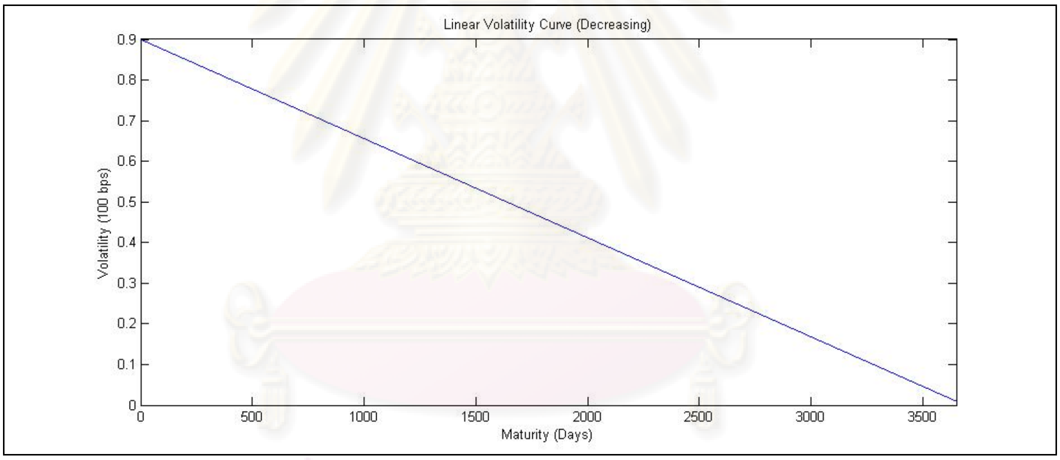

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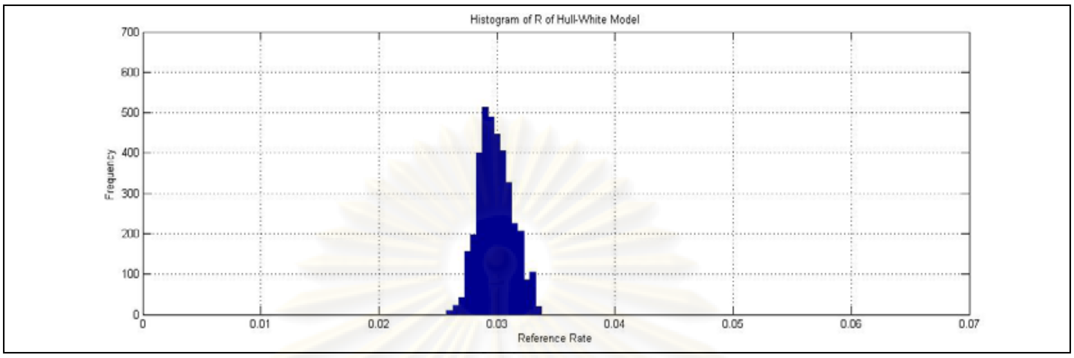
Figure 14 Histogram of Reference Rate (Low Volatility)

These graphs show the histogram of reference rate of Hull-White model and CIR++ model after simulation. They present the frequency distribution of reference rate. First graph shows the histogram of Hull-White model and second graph shows the histogram of CIR++ model. The initial short rate is set to be 3% and the volatility is historical volatility (mean = 3 bps).



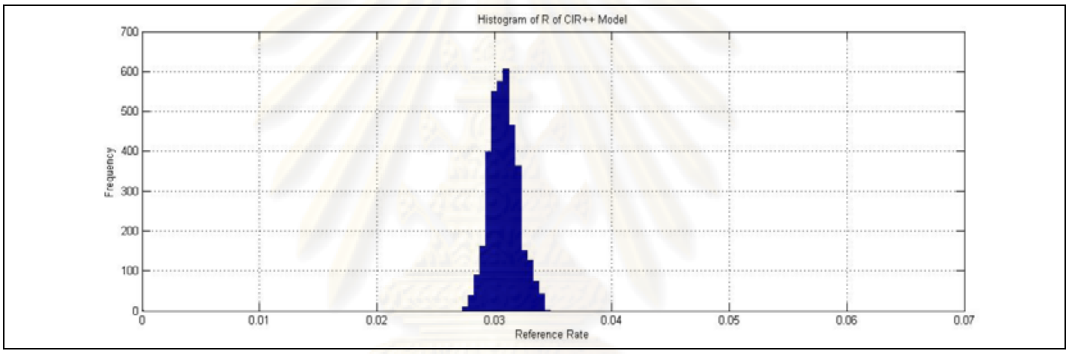


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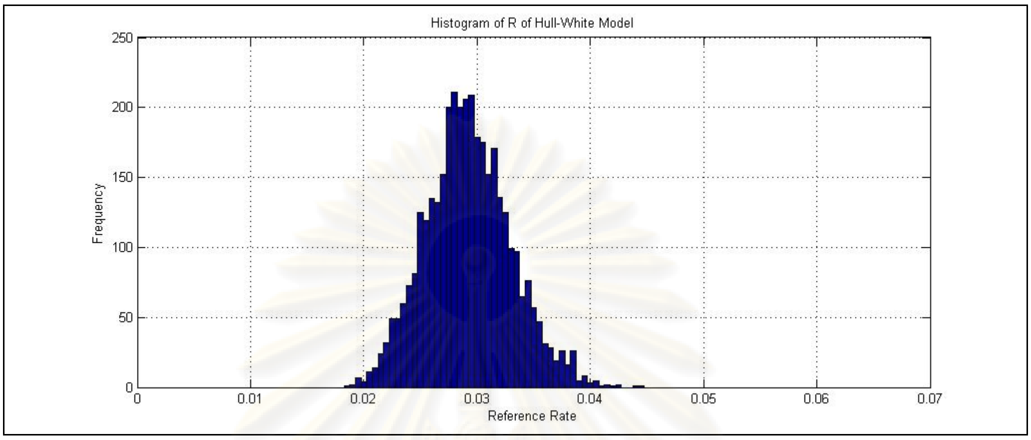
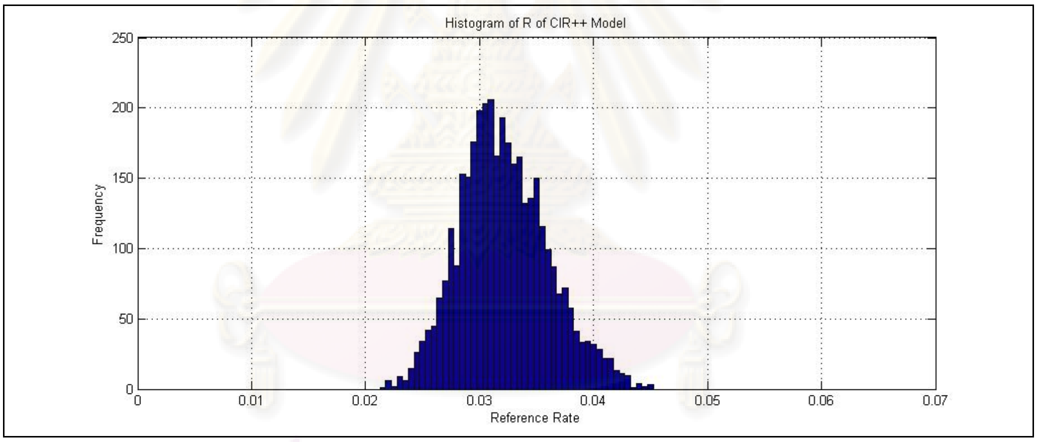
Figure 15 Histogram of Reference Rate (High Volatility)

These graphs show the histogram of reference rate of Hull-White model and CIR++ model after simulation. They present the frequency distribution of reference rate. First graph shows the histogram of Hull-White model and second graph shows the histogram of CIR++ model. The initial short rate is set to be 3% and the volatility is extremely high volatility (90 bps).





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BIOGRAPHY

Mr. Chawalit Kajkumjorndej was born in Jan 5, 1985 in Bangkok. At the secondary school, he graduated from Assumption College. At the undergraduate level, he graduated from the Faculty of Engineering, Chulalongkorn University in May 2006 with a Bachelor of Engineering degree, majoring in Electrical Engineering. He joined the Master of Science in Finance program, Chulalongkorn University in June 2006.

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