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APPENDICES

APPENDIX A

This appendix is concerned with the derivative of the general solutions for a homogeneous poroelastic material undergoing axisymmetric vibrations. Consider the governing equations, equations (3.4) and (3.5) in Chapter III. These equations can be solved by introducing the displacement decomposition based on Helmholtz representation for an axisymmetric vector field, equations (3.6) to (3.9), together with the assumption that the motion is time-harmonic yield two sets of partial differrential equations for Φ_1, Φ_2 and Ψ_1, Ψ_2 as

$$[(\lambda^* + \alpha^2 M^* + 2)\nabla^2 + \delta^2]\Phi_1 = -(\alpha M^* \nabla^2 + \rho^* \delta^2)\Phi_2 \quad (A.1)$$

$$(\alpha M^* \nabla^2 + \rho^* \delta^2)\Phi_1 = (ib^*\delta - m^*\delta^2 - M^*\nabla^2)\Phi_2 \quad (A.2)$$

$$(\nabla^2 + \delta^2)\Psi_1 = -\rho^*\delta^2\Psi_2 \quad (A.3)$$

$$\rho^*\delta^2\Psi_1 = (ib^*\delta - m^*\delta^2)\Psi_2 \quad (A.4)$$

where the dimensionless parameters λ^*, M^*, ρ^*, m^* and b^* are defined as

$$\lambda^* = \frac{\lambda}{\mu},\ M^* = \frac{M}{\mu},\ \rho^* = \frac{\rho_f}{\rho},\ m^* = \frac{m}{\rho} \text{ and } b^* = \frac{ab}{\sqrt{\rho\mu}} \quad (A.5)$$

and ∇^2 is the Laplacian operator defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (A.6)$$

In addition, a dimensionless frequency, δ, is defined as

$$\delta = \sqrt{\frac{\rho}{\mu}}\omega a \quad (A.7)$$

Application of the zeroth-order Hankel transform to equations (A.1) to (A.4) yield the ordinary differential equations for Φ_1, Φ_2 and Ψ_1, Ψ_2 as

$$\left[(\lambda^* + \alpha^2 M^* + 2)\left(\frac{d^2}{dz^2} - \xi^2\right) + \delta^2\right]\bar{\Phi}_1 = \left[\alpha M^*\left(\xi^2 - \frac{d^2}{dz^2}\right) - \rho^*\delta^2\right]\bar{\Phi}_2 \quad (A.8)$$

$$\left[\alpha M^*\left(\frac{d^2}{dz^2} - \xi^2\right) + \rho^*\delta^2\right]\bar{\Phi}_1 = \left[ib^*\delta - m^*\delta^2 - M^*\left(\xi^2 - \frac{d^2}{dz^2}\right)\right]\bar{\Phi}_2 \quad (A.9)$$

$$\left(\xi^2 - \frac{d^2}{dz^2} - \delta^2\right)\overline{\Psi}_1 = \rho^* \delta^2 \overline{\Psi}_2 \tag{A.10}$$

$$\rho^* \delta^2 \overline{\Psi}_1 = \left(ib^* \delta - m^* \delta^2\right)\overline{\Psi}_2 \tag{A.11}$$

It can be shown that the general solutions of Hankel transforms of Φ_i and Ψ_i $(i = 1,2)$ can be expressed as

$$\overline{\Phi}_1 = Ae^{\gamma_1 z} + Be^{-\gamma_1 z} + Ce^{\gamma_2 z} + De^{-\gamma_2 z} \tag{A.12}$$

$$\overline{\Phi}_2 = \chi_1\left(Ae^{\gamma_1 z} + Be^{-\gamma_1 z}\right) + \chi_1\left(Ce^{\gamma_2 z} + De^{-\gamma_2 z}\right) \tag{A.13}$$

$$\overline{\Psi}_1 = Ee^{\gamma_3 z} + Fe^{-\gamma_3 z} \tag{A.14}$$

$$\overline{\Psi}_2 = \chi_3\left(Ee^{\gamma_3 z} + Fe^{-\gamma_3 z}\right) \tag{A.15}$$

where $A(\xi,\delta)$, $B(\xi,\delta)$,...., $F(\xi,\delta)$ are the arbitrary functions to be determined by using appropriate boundary and/or continuity conditions relevant to a given problem and

$$\chi_i = \frac{\left(\lambda^* + \alpha^2 M^* + 2\right)L_i^2 - \delta^2}{\rho^* \delta^2 - \alpha M^* L_i^2}, \quad i = 1,2 \tag{A.16}$$

$$\chi_3 = \frac{\rho^* \delta}{ib^* - m^* \delta} \tag{A17}$$

$$\gamma_i = \sqrt{\xi^2 - L_i^2}, \quad i = 1,2 \tag{A.18}$$

$$\gamma_3 = \sqrt{\xi^2 - S^2} \tag{A.19}$$

$$L_1^2 = \frac{w_1 + \sqrt{w_1^2 - 4w_2}}{2} \tag{A.20}$$

$$L_2^2 = \frac{w_1 - \sqrt{w_1^2 - 4w_2}}{2} \tag{A.21}$$

$$S^2 = \left(\rho^* \chi_3 + 1\right)\delta^2 \tag{A.22}$$

$$w_1 = \frac{\left(m^* \delta^2 - ib^* \delta\right)\left(\lambda^* + \alpha^2 M^* + 2\right) + M^* \delta^2 - 2\alpha M^* \rho^* \delta^2}{\left(\lambda^* + 2\right)M^*} \tag{A.23}$$

$$w_2 = \frac{\left(m^* \delta^2 - ib^* \delta\right)\delta^2 - \left(\rho^*\right)^2 \delta^4}{\left(\lambda^* + 2\right)M^*} \tag{A.24}$$

In view of equations (3.1)-(3.3), (3.6)-(3.9) and (A.12)-(A.15), the general solutions for Hankel transforms of displacements u_i and w_i (i = r, z), stresses σ_{ij}

and excess pore pressure p, given in equations (3.12) to (3.17), can be obtained. In addition, the variables η_i, β_i and S_1 are defined by

$$\eta_i = (\alpha + \chi_i) M^* L_i^2, \quad i = 1,2 \tag{A.25}$$

$$\beta_i = 2\gamma_i^2 - \lambda^* L_i^2 - \alpha\eta_i \ , \ i = 1,2 \tag{A.26}$$

$$S_1 = \xi^2 + \gamma_3^2 \tag{A.27}$$

APPENDIX B

This appendix is given the non-zero arbitrary functions appearing in the general solutions given by equations (3.12)-(3.17) for different loading cases.

B.1 Arbitrary Functions for Vertical Loading:

$$A_1 = \frac{\eta_2 e^{-\gamma_1 z'}}{2\mu N_1}\overline{T}_z(\xi) \tag{B.1}$$

$$B_1 = \frac{\eta_2\left(v_2 e^{-\gamma_1 z'} + 2\xi^2 v_3 e^{-\gamma_2 z'} - 4\xi^2 S_1 v_1 e^{-\gamma_3 z'}\right)}{2\mu N_1 R}\overline{T}_z(\xi) \tag{B.2}$$

$$C_1 = -\frac{\eta_1 e^{-\gamma_2 z'}}{2\mu N_1}\overline{T}_z(\xi) \tag{B.3}$$

$$D_1 = \frac{\eta_2\left(2\xi^2 v_4 e^{-\gamma_1 z'} - v_6 e^{-\gamma_2 z'} + 4\xi^2 S_1 v_1 e^{-\gamma_3 z'}\right)}{2\mu N_1 R}\overline{T}_z(\xi) \tag{B.4}$$

$$E_1 = \frac{\xi v_1 e^{-\gamma_3 z'}}{2\mu\gamma_3 N_1}\overline{T}_z(\xi) \tag{B.5}$$

$$F_1 = \frac{\xi v_2\left(v_4 e^{-\gamma_1 z'} - v_3 e^{-\gamma_2 z'}\right) + \xi v_1 v_7 e^{-\gamma_3 z'}}{2\mu\gamma_3 N_1 R}\overline{T}_z(\xi) \tag{B.6}$$

$$B_2 = B_1 - A_1 e^{2\gamma_1 z'} \tag{B.7}$$

$$D_2 = D_1 - C_1 e^{2\gamma_2 z'} \tag{B.8}$$

$$F_2 = F_1 + E_1 e^{2\gamma_3 z'} \tag{B.9}$$

where

$$v_1 = \eta_1 - \eta_2 \tag{B.10}$$

$$v_2 = \eta_1\beta_2 - \eta_2\beta_1 \tag{B.11}$$

$$v_3 = 4\eta_1\gamma_2\gamma_3 \tag{B.12}$$

$$v_4 = 4\eta_2\gamma_3\gamma_1 \tag{B.13}$$

$$v_5 = S_1 v_2 - \xi^2 \left(v_3 + v_4\right) \tag{B.14}$$

$$v_6 = S_1 v_2 + \xi^2 \left(v_3 + v_4\right) \tag{B.15}$$

$$v_7 = S_1 v_2 + \xi^2 \left(v_3 - v_4\right) \tag{B.16}$$

and

$$N_1 = 2\xi^2 v_1 - v_2 \tag{B.17}$$

$$R = -S_1 v_2 + \xi^2 \left(v_3 - v_4\right) \tag{B.18}$$

In the above equations, $\overline{T}_z(\xi) = p_o a J_1(\xi a)/\xi$ is the zeroth-order Hankel transform of the applied axisymmetric vertical load over a circular area of radius a and uniform intensity p_o at $(z = z')$.

B.2 Arbitrary Functions for Applied Fluid Pressure:

$$A_1 = \frac{-\left(\lambda^* + 2\right) L_2^2 e^{-\gamma_1 z'}}{2\mu N_1} \overline{P}(\xi) \tag{B.19}$$

$$B_1 = \frac{-\left(\lambda^* + 2\right)\left[v_5 L_2^2 e^{-\gamma_1 z'} + 2\xi^2 \eta_2 \left(\beta_3 e^{-\gamma_2 z'} - \beta_5 e^{-\gamma_3 z'}\right)\right]}{2\mu N_1 R} \overline{P}(\xi) \tag{B.20}$$

$$C_1 = \frac{-\left(\lambda^* + 2\right) L_1^2 e^{-\gamma_2 z'}}{2\mu N_1} \overline{P}(\xi) \tag{B.21}$$

$$D_1 = \frac{\left(\lambda^* + 2\right)\left[v_6 L_1^2 e^{-\gamma_2 z'} - 2\xi^2 \eta_1 \left(\beta_4 e^{-\gamma_1 z'} + \beta_5 e^{-\gamma_3 z'}\right)\right]}{2\mu N_1 R} \overline{P}(\xi) \tag{B.22}$$

$$E_1 = -\frac{\xi\left(\lambda^* + 2\right)\left(L_1^2 - L_2^2\right) e^{-\gamma_3 z'}}{2\mu \gamma_3 N_1} \overline{P}(\xi) \tag{B.23}$$

$$F_1 = -\frac{\xi\left(\lambda^* + 2\right)\left[v_2 \left(\beta_4 e^{-\gamma_1 z'} - \beta_3 e^{-\gamma_2 z'}\right) + v_7 \left(L_1^2 - L_2^2\right) e^{-\gamma_3 z'}\right]}{2\mu \gamma_3 N_1 R} \overline{P}(\xi) \tag{B.24}$$

$$B_2 = B_1 - A_1 e^{2\gamma_1 z'} \tag{B.25}$$

$$D_2 = D_1 - C_1 e^{2\gamma_2 z'} \tag{B.26}$$

$$F_2 = F_1 + E_1 e^{2\gamma_3 z'} \tag{B.27}$$

where

$$\beta_3 = 4\gamma_2\gamma_3 L_1^2 \tag{B.28}$$

$$\beta_4 = 4\gamma_3\gamma_1 L_2^2 \tag{B.29}$$

$$\beta_5 = 2S_1\left(L_1^2 - L_2^2\right) \tag{B.30}$$

In the above equations, $\overline{P}(\xi) = p_o a J_1(\xi a)/\xi$ is the zeroth-order Hankel transform of the applied fluid pressure over a circular area of radius a and uniform intensity p_o at $(z = z')$.

BIOGRAPHY

Mr. Yasothorn Sapsathiarn was born in Ayudhaya 1977. He graduated from Faculty of Engineering, Chulalongkorn University in 1998. He continued his study for Master Degree in Civil Engineering at Chulalongkorn in 1999.