CHAPTER III

SYSTEM CONTROL METHODS

As state before, by the action of the speed governor the system automatically accommodates the new load. But, if there is no signal input to the speed changer, the speed changer position does not change its position, the operation of the system is now at a new frequency other than the nominal frequency. This procedure is referred to as the uncontrolled case. In the complete control system, the frequency error must be eliminated.

Response of the Uncontrolled Case

Before prodeeding to a discussion of how to control the system it is useful to see the response of the uncontrolled case. Consider Fig. 7 with no command input to the speed changer that is $\Delta P_C = 0$. If a step load is applied to the system, there is a static frequency error which may be computed by using eq. (1.5) setting $\frac{d}{dt}\Delta f = 0$, and $\Delta P_G = -\frac{\Delta f}{R}$ or by using final-value theorem to Fig. 7 with $\Delta P_C = 0$. Thus,

$$\Delta f_{Static} = -\frac{K_P}{1 + \frac{1}{R} K_P} \Delta P_L = -\frac{\Delta P_L}{D + \frac{1}{R}} = -\frac{\Delta P_L}{B}$$

where
$$B \triangleq D + \frac{1}{R} \text{ pu } MW/H_Z$$
$$= \text{area frequency response characteristic}$$
$$= \text{AFRC.}$$

The physical significance of the AFRC can be stated as follows: A system operates alone (isolated) will, if uncontrolled and subjected to a step load change, experience a static frequency drop inversely proportional to its AFRC.

Control Specification

It is necessary to achieve much better frequency constancy than is obtained by the uncontrolled case. To accomplish this the control signals must be sent to the speed changer with some suitable control strategy. Before doing so, the control specification should be given. This control specification is that the static frequency error following a step load change must be quickly reduced to zero.

This requirement is of fundamental importance in a power system, since the operating performance of some equipments changes with frequency. The frequency is also the surest indicator of a serious fault, and thus by controlling it to the schedule under normal operating condition, a fault can be detected at an early stage.

The Conventional Control Strategy

The conventional control strategy to meet the above requirement is of linear, integral form

$$\Delta P_C = -K_I \int \Delta f \, dt \qquad (3.1)$$

The chosen strategy meets the above requirement for the following reason. Since after a step load change if a new static equilibrium exists, it can be achieved after the speed changer command ΔP_C has reached a constant value. This evidently requires that the integrand in eq. (3.1) must be zero, i.e.,

$$\Delta f = 0.$$

Or, the integral control will give rise to zero static frequency error following a step load change, for the following physical reason. As long as an error exists, the integrator output will increase, causing the speed changer to move. The generation is adjusted until the frequency error is zero then the speed changer attains a constant value. The gain constant K_I controls the signal to the integrator, and thus the speed of response of the system. The negative polarity of the integral controller is chosen so that it couses a positive frequency error to give rise to a negative or decrease command. The signal fed into the integrator is referred to as area control error ACE; or

$$\text{ACE} \triangleq \Delta f \,.$$

The Optimum System Control

In the conventional control method, the signal used to control the system is a function of only one variable, Δf. That is the system controller operates in response to the integral of ACE for that system. By dropping the restriction of a fixed control structure, a technique of optimal control theory can be applied. In this method the optimal controller is a linear function of all variables of the system. Before using the optimal control theory to develop the optimal controller, the system must be rewritten in state space form. The specifications that the system must satisfy are defined mathematically in the form of an integral cost that is to be minimized. The minimization of this cost yields an optimal controller which is a linear combination of all system states. By using this optimal controller allows one to find a new control method. This method gives the optimum response in the sense that the integral cost functional is minimized by the controller.

Development of the State Space Equation for the System

In this time the state variables are introduced and the system will be transformed into the state space or state model form. Consider block diagram of Fig. 7; then

$$\Delta F(S) = \left(\frac{K_P}{1 + ST_P}\right)(\Delta P_G(S) - \Delta P_L(S)) \qquad (3.2)$$

$$\Delta P_G(S) = \left(\frac{1}{1 + ST_T}\right)\Delta X_E(S) \qquad (3.3)$$

$$\Delta X_E(S) = \left(\frac{1}{1 + ST_G}\right)\left(-\frac{\Delta F(S)}{R} + \Delta P_S(S)\right) \qquad (3.4)$$

and

$$\Delta P_S(S) = \left(\frac{1}{1 + ST_K}\right)\Delta P_C(S)\ . \qquad (3.5)$$

Let

$$X_1 = \Delta f(t)$$
$$X_2 = \Delta P_G(t)$$
$$X_3 = \Delta X_E(t)$$
$$X_4 = \Delta P_S(t)$$
$$U = \Delta P_C(t)\ .$$

From eq. (3.2) ;

$$\Delta F(S) + \Delta F(S) ST_P = K_P(\Delta P_G(S) - \Delta P_L(S))$$

$$S\Delta F(S) = \frac{1}{T_P}\left[K_P(\Delta P_G(S) - \Delta P_L(S)) - \Delta F(S)\right]\ .$$

Taking the inverse Laplace transform and substituting the state variables defined above, then

$$
\begin{aligned}
\dot{X}_1 &= \frac{1}{T_P}\left[K_P(X_2 - \Delta P_L) - X_1\right] \\
&= \frac{1}{\frac{2H}{f^{\bullet}D}}\left[\frac{1}{D}(X_2 - \Delta P_L) - X_1\right]
\end{aligned}
$$

or;

$$\dot{X}_1 = -\frac{X_1}{\frac{2H}{f^{\bullet}D}} + \frac{X_2}{\frac{2H}{f^{\bullet}}} - \frac{\Delta P_L}{\frac{2H}{f^{\bullet}}} \;. \tag{3.6}$$

Similarly; it can be shown that

$$\dot{X}_2 = -\frac{X_2}{T_T} + \frac{X_3}{T_T} \tag{3.7}$$

$$\dot{X}_3 = -\frac{X_1}{T_G R} - \frac{X_3}{T_G} + \frac{X_4}{T_G} \tag{3.8}$$

and

$$\dot{X}_4 = -\frac{X_4}{T_K} + \frac{U}{T_K} \;. \tag{3.9}$$

The last four equations can be written in matrix form as follows :

$$
\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix}
=
\begin{bmatrix}
-\frac{f^{\bullet}D}{2H} & \frac{f^{\bullet}}{2H} & 0 & 0 \\
0 & -\frac{1}{T_T} & \frac{1}{T_T} & 0 \\
-\frac{1}{T_G R} & 0 & -\frac{1}{T_G} & \frac{1}{T_G} \\
0 & 0 & 0 & -\frac{1}{T_K}
\end{bmatrix}
\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_K} \end{bmatrix} U +
\begin{bmatrix} -\frac{f^{\bullet}}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_L \;.
$$

Thus, the state model of the system is

$$\dot{\bar{X}} = A\bar{X} + BU + \Gamma \Delta P_L \tag{3.9}$$

where

$$\dot{\bar{X}} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} \quad ; \quad \bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{f^{\bullet}D}{2H} & \frac{f^{\bullet}}{2H} & 0 & 0 \\ 0 & -\frac{1}{T_T} & \frac{1}{T_T} & 0 \\ -\frac{1}{T_G R} & 0 & -\frac{1}{T_G} & \frac{1}{T_G} \\ 0 & 0 & 0 & -\frac{1}{T_K} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_K} \end{bmatrix} \quad ; \text{ and } \Gamma = \begin{bmatrix} -\frac{f^{\bullet}}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} .$$

The Γ matrix is called the disturbance matrix. ΔP_L, the incremental load disturbance is a known constant. It is necessary to change the state model into suitable form by eliminating the disturbance matrix Γ. This can be done by introducing new variables.[9] The procedure is shown as follows. The terminal conditions to be satisfied are

$$\Delta f(\infty) = 0, \quad \Delta P_G(\infty) - \Delta P_L = 0, \Delta X_E(\infty) - \Delta P_L = 0$$

and $$\Delta P_S(\infty) - \Delta P_L = 0 \; . \tag{3.10}$$

Note ΔX_E is the incremental change in the governor valve position in pu MW.

A change of variables is introduced :

$$\bar{X}' \triangleq \bar{X} - \rho \Delta P_L \tag{3.11}$$

where

$$\rho = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} .$$

That is,

$$\bar{X}' = \begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ X_4' \end{bmatrix} = \begin{bmatrix} X_1 - 0 \\ X_2 - \Delta P_L \\ X_3 - \Delta P_L \\ X_4 - \Delta P_L \end{bmatrix} = \begin{bmatrix} \Delta f - 0 \\ \Delta P_G - \Delta P_L \\ \Delta X_E - \Delta P_L \\ \Delta P_S - \Delta P_L \end{bmatrix} .$$

Or, $\bar{X}' = \bar{X} - \bar{X}_{SS}$; where $\bar{X}_{SS}$ = steady state values of $\bar{X}$. From the conditions in eq. (3.10) it is seen that $\bar{X}'(\infty) = 0$, which is the desired terminal conditions. Substituting eq. (3.11) into eq. (3.9);

$$\dot{\bar{X}}' = A(\bar{X}' + \rho\Delta P_L) + BU + \Gamma\Delta P_L ;$$

$$\dot{\bar{X}}' = A\bar{X}' + BU + (A\rho + \Gamma)\Delta P_L . \qquad (3.12)$$

$$\text{Since} \quad A\rho + \Gamma = \begin{bmatrix} \frac{f^{\bullet}}{2H} \\ 0 \\ 0 \\ -\frac{1}{T_K} \end{bmatrix} + \begin{bmatrix} -\frac{f^{\bullet}}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{T_K} \end{bmatrix} = -B .$$

Therefore from eq. (3.12) ;

$$\dot{\bar{X}}' = A\bar{X}' + BU - B\Delta P_L$$

or

$$\dot{\bar{X}}' = A\bar{X}' + BU' \qquad (3.12a)$$

where

$$U' = U - \Delta P_L \qquad (3.12b)$$

and

$$\bar{X}'(0) = -\bar{X}_{SS} .$$

Equation (3.12a) is a suitable form for the application of the optimal control theory. The state model has been changed from

$$\dot{\bar{X}} = A\bar{X} + BU + \Gamma\Delta P_L \quad \text{to}$$

$$\dot{\bar{X}}' = A\bar{X}' + BU'$$

by the introduction of the new variables

$$\bar{X}' = \bar{X} - \bar{X}_{SS} .$$

Redefining the states in terms of their steady state values is equivalent to shifting the reference position of the system. It is important to note that matrices A and B remain unchanged. In the further analysis the superscript " ' " will be neglected for simplicity.

Optimum Control Strategy

The next step is to find the optimum control strategy for the system model. First, the system cost functional to be minimized by the optimal controller must be defined. Then, the optimal control theory known as the state regulator or linear regulator problem is employed to develop the optimal control.[1]

Problem statement. Given a linear time - invariant system represented by the state variable differential equations

$$\dot{\bar{X}} = A \bar{X} + B\bar{U} \tag{3.13}$$

$$\bar{Y} = C \bar{X}$$

where $\bar{X}$ = $n \times 1$ state vector

$\bar{U}$ = $r \times 1$ control vector

A = $n \times n$ A matrix

B = $n \times r$ B matrix

C = $m \times n$ C matrix, and

$\bar{Y} \quad = \quad m \times 1$ output vector.

Find the control $\bar{U}$ which minimizes the cost functional

$$J = \frac{1}{2}\int_{0}^{\infty} \left[\bar{X}^T Q \, \bar{X} + \bar{U}^T R \, \bar{U} \right] dt \qquad (3.14)$$

where $Q \quad = \quad n \times n$ positive semidefinite matrix

$R \quad = \quad r \times r$ positive definite matrix, and

$\bar{U} \quad = \quad r \times 1$ unconstrained control vector.

The performance of the system is specified in term of a cost functional J that is to be minimized by the controller. In order that the minimum cost functional J is finite the system must be completely controllable.[1] It is quite conceivable that if the system were uncontrollable and unstable then the cost would be infinite for all controls since the control interval is infinite. The check for controllability is shown in Appendix A. The elements of Q and R matrices are to be chosen according to what one wishes the system to perform.[7] For example, if $R = 0$ but Q is nonzero, that is no charge for the control effort used. Hence the best control strategy would be in the form of infinite impulse. This control would drive the state to zero in the shortest possible time with the greatest effort. Once J is chosen, the results of the optimal system are unique. The purpose of minimizing J is to minimize the error response and the control effort of the system after a disturbance.

The system requirement is that the static frequency error following a step load change must be zero. Thus the cost functional J to be minimized must contain the Δf^2 term. In transformation of the system into the state model, the state X_1 represented for Δf. So Q matrix is chosen as follows :

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} .$$

To penàlize the control effort by adding the term U^2 requires

$$R = \begin{bmatrix} 1 \end{bmatrix} .$$

For $Q \triangleq C^T C$; then $C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$,

which gives Δf as the output response of the system.

<u>Optimal Controller</u>

The optimal controller that minimizes the cost functional J is a function of the system states, i.e.,

$$\hat{\bar{U}} = - R^{-1} B^T K \bar{X} . \tag{3.15}$$

See Appendix A for more details.

The K motrix is the unique solution of the nonlinear time invariant matrix algebraic equation called the Riccati equation :

$$- A^T K - KA + KBR^{-1}B^T K - Q = 0 . \tag{3.16}$$