CHAPTER III

LOG-PERIODIC ANTENNA THEORY AND DESIGN

3-1. Introduction

The object of this work is to study the way of designing a log-periodic antenna which is suited for use in domestic circuit; i.e., the one with frequency range from 3 to 10 Mc. The log-periodic dipole antenna is considered for this purpose because it is made of conventional linear dipole element which is easy to construct for such HF band, and it also serves quite well operation.

3-2. General Characteristics[3]

The geometry of log-periodic structures is chosen so that the electrical properties must repeat periodically with the logarithm of the frequency.

A precise definition of logarithmically periodic structures may be obtained by considering the transformation

$$Z = \ln w \qquad (3\text{-}1)$$

where w and Z are complex numbers. Letting $w = \rho e^{j\theta}$, and $Z = x + jy$, it is easily shown that

$$\rho = e^{x} \quad \text{or} \quad x = \ln \rho \qquad (3\text{-}2)$$

$$\theta = y \qquad (3\text{-}3)$$

With this transformation, a logarithmically periodic structure is formed by introducing periodic variations on the parallel strips in the Z plane and then transforming to the w plane. A few examples are shown in Fig. 3-1. Figures 3-1(a) and (b) show logarithmically periodic slot and tooth structures. The teeth in Fig. 3-1 (c) are formed by sinusoidal curves in the Z plane. It will be noticed that in the w plane all dimensions involved in the definition of a logarithmically periodic structure are proportional to their distance from the origin or feed point. In Fig. 3-1(a), the slots are bounded by the radii R_{n-1}, R_n, R_{n+1}, ... from (or) a geometrical sequence of terms where the geometric ratio is defined by

$$\tau = \frac{R_{n+1}}{R_n} \qquad (3\text{-}4)$$

The radii r_{n-1}, r_n, r_{n+1}, ... form a similar sequence having the same geometric ratio. The width of the slot is defined by

$$\sigma = \frac{r_n}{R_n} \qquad (3\text{-}5)$$

It can be seen that infinite structures of this type having the property that, when energized at the vertex, the fields at a frequency f will be repeated at all other frequencies given by $\tau^n f$. When plotted on a logarithmic scale, these frequencies are equally spaced with a seperation or period of $\ln \tau$; hence the name logarithmically periodic structures.

3-3. Log-Periodic Dipole Antenna[4]

The practical value of the log-periodic approach was enhanced even further when DuHamel and co-workers demonstrated that successful log-periodic antennas could be made with wire structures as well as sheet

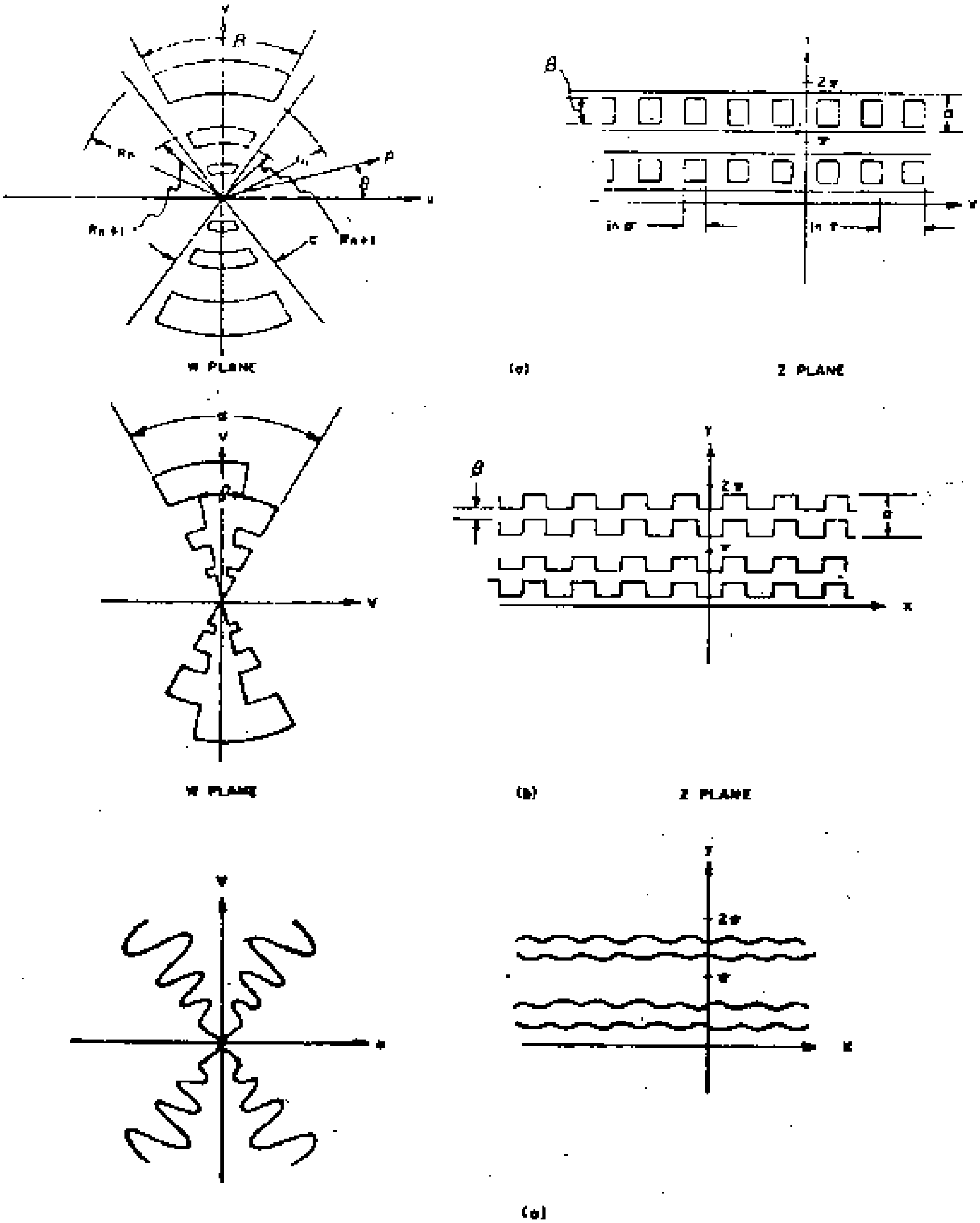


Fig. 3 – 1.
Logarithmically periodic structures.

structures. This development extended the range of application down from microwaves through the high-frequency band. The log-periodic dipole array of Fig. 3-2 is a kind of wire structure. As with all log-periodic geometries, all dimensions are increased by a constant ratio in moving outward from the origin. Thus the lengths and spacings of adjacent elements must be related by a constant scale factor $\tau < 1$, as follows:

$$\frac{l_n}{l_{n-1}} = \frac{d_n}{d_{n-1}} = \tau \qquad (3-6)$$

A line through the ends of the dipole elements on one side of the antenna subtended an angle α with the center line of the antenna at the virtual apex O. The spacing factor σ is defined as the ratio of the distance between two adjacent elements to twice the length of the larger element, and is a constant for a given antenna. The geometry of the antenna relates σ to τ and α.

$$\sigma = \frac{1}{4}(1 - \tau)\cot\alpha \qquad (3-7)$$

The largest element is called element number 1. The half length of element n is denoted by h_n. Therefore

$$h_n = h_1 \tau^{n-1} \qquad (3-8)$$

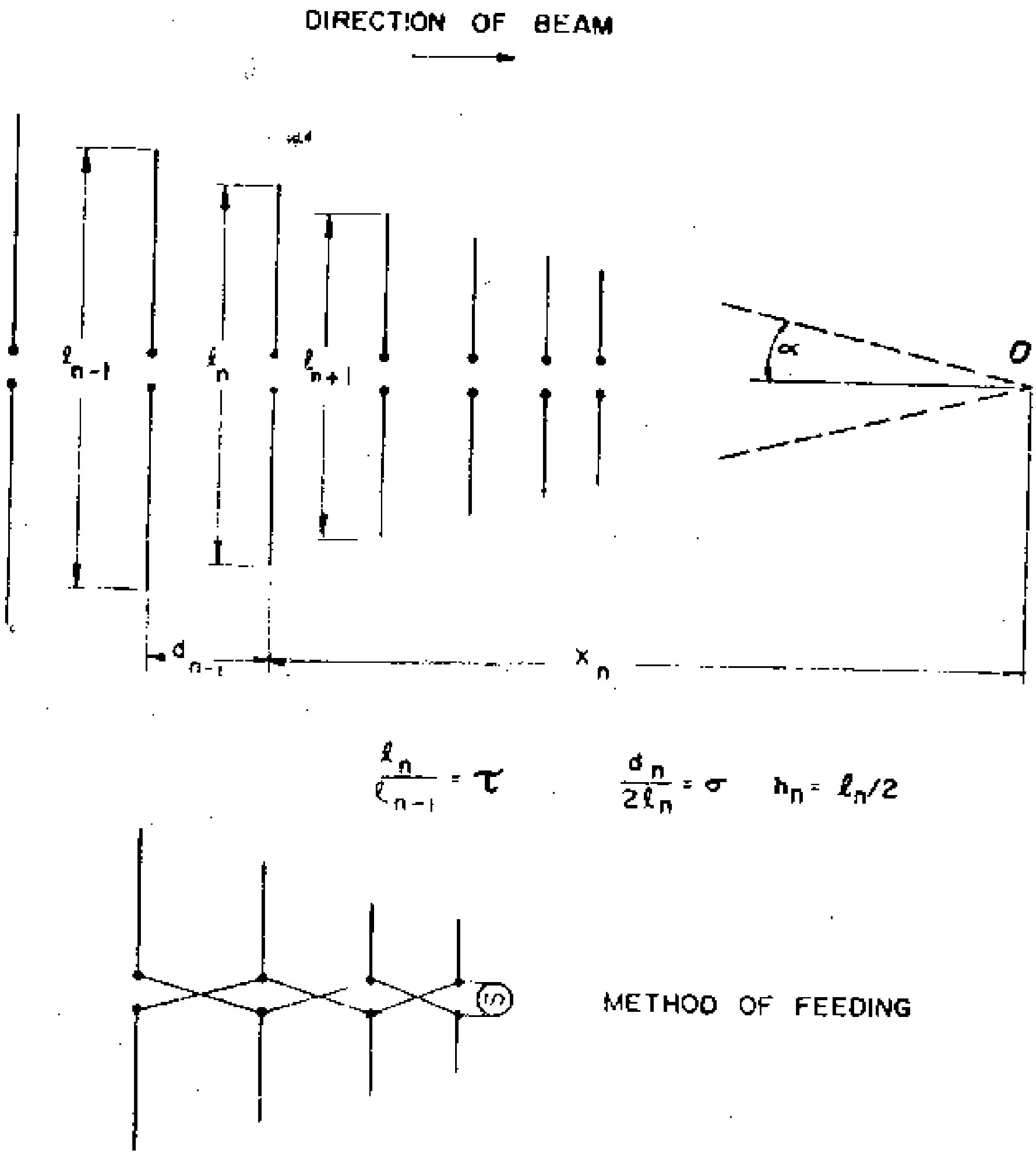
The distance d_n from element n to element n + 1 is given by

$$d_n = d_1 \tau^{n-1} \qquad (3-9)$$

If a_n is the radius of element number n, the a_n's are given by

$$a_n = a_1 \tau^{n-1} \qquad (3-10)$$

The ratio of element height to radius is the same for all elements in a given antenna and will be denoted by h/a.



$$\frac{\ell_n}{\ell_{n-1}} = \tau \qquad \frac{d_n}{2\ell_n} = \sigma \qquad h_n = \ell_n/2$$

Figure 3-2. A schematic of the log-periodic dipole antenna, including symbols used in its description

The array must be fed with a transposition of the transmission line between adjacent dipole elements. Due to this alternation, one cell of the LPD antenna consists of two adjacent dipoles and two sections of feeder. Thus τ as defined above is the square root of the cell scaling factor. The antenna is caused to radiate in the backfire direction (i.e., toward the source), a condition which appears to be necessary for successful unidirectional frequency-independent or log-periodic operation.

3-3a. An Approximate Formula for Input Impedance of LPD.[5] Consider the approximate formula for the input impedance of a small dipole antenna of half-length h,

$$Z = -jZ_a \cot \beta h \qquad (3\text{-}11)$$

where β is the free space propagation constant. Z_a is called the average characteristic impedance of a dipole antenna

$$Z_a = 120(\ln h/a - 2.25) \qquad (3\text{-}12)$$

This is a modification of a formula in Jordan which was derived by Siegal and Labus. Replacing the cotangent function in (3-11) by its small argument approximation, the capacitance of the n^{th} dipole is given by

$$C_n \doteq \frac{h_n}{cZ_a} \qquad (3\text{-}13)$$

where c is the velocity of light in vacuum. Using the mean spacing at dipole n

$$d_{mean} = \sqrt{d_n d_{n-1}} = \frac{d_n}{\sqrt{\tau}} \qquad (3\text{-}14)$$

the capacitance per unit length is given by

$$\Delta C = \frac{C_n}{\text{length}} = \frac{h_n \sqrt{\tau}}{c d_n Z_a} \qquad (3\text{-}15)$$

But h_n/d_n is related to the spacing factor σ by

$$\sigma = \frac{1}{4} \frac{d_n}{h_n} \qquad (3\text{-}16)$$

hence

$$\Delta C = \frac{\sqrt{\tau}}{4c\,\sigma Z_a} \qquad (3\text{-}17)$$

The characteristic impedance of the unloaded feeder is given by

$$Z_o = \sqrt{\frac{L_o}{C_o}} \qquad (3\text{-}18)$$

and the characteristic impedance of the equivalent line is given by

$$R_o = \sqrt{\frac{L_o}{C_o + \Delta C}} \qquad (3\text{-}19)$$

Using $\sqrt{L_o C_o} = 1/c$ and substituting, one finds

$$R_o = Z_o / \sqrt{m} \qquad (3\text{-}20)$$

where

$$m = 1 + \frac{Z_o}{Z_a} \frac{\sqrt{\tau}}{4\sigma} \qquad (3\text{-}21)$$

Equation (3-20) is used to approximate the input impedance of the LPD.

3-3b. <u>Width and Location of the Active Region.</u>[6] For a given antenna, the usuable bandwidth for frequency independent operation depends on the relative distance the active region can move before it becomes distorted by the smallest or largest element. Thus the width of the active region, if properly defined, can be used to measure the bandwidth capability of a given antenna. Furthermore, the knowledge of the width of the active region is prerequisite to the design of an antenna to cover a given bandwidth. The lower cut-off frequency of a given antenna is determined by the length of the longest element. Conversely, if the lower cut-off frequency is given, the relative length of the longest required element in the active region be known to fix the length of the longest element on the antenna.

If the active region was very narrow, the operating bandwidth of the antenna would be given substantially by the ratio of the length of the largest to smallest element. This ratio will be called the structure bandwidth, B_s.

$$B_s = \frac{l_1}{l_N} = \tau^{1-N} \qquad (3\text{-}22)$$

Since, the active region has some width, it is apparent that the operating bandwidth B is always less than B_s by a factor which can be called the bandwidth of the active region, B_{ar}. Thus

$$B = B_s / B_{ar} \qquad (3\text{-}23)$$

The empirical formula of B_{ar} as a function of σ and τ is given by

$$B_{ar} = 1.1 + 30.7\sigma(1 - \tau) \qquad (3\text{-}24)$$

The empirical formula agrees with the computed and measured results for all but the lowest values of τ, so its use should be restricted to $\tau \geqslant 0.875$. For a fixed τ, the bandwidth of the active region increases as σ increases. This is an important design consideration, because the size of an antenna to cover a given band increases as B_{ar} increases.

3-3c. Input Impedance as a Function of Z_a and h/a.[7] Formula (3-20) can be inverted to find the feeder impedance required to achieve a given input impedance. In its most useful form Z_o is normalized with respect to R_o, resulting in

$$\frac{Z_o}{R_o} = \frac{1}{8\sigma' \frac{Z_a}{R_o}} + \sqrt{\frac{1}{\left(8\sigma' \frac{Z_a}{R_o}\right)^2} + 1} \qquad (3\text{-}25)$$

where $\sigma' = \sigma / \sqrt{\tau}$ is the mean spacing factor. As in (3-20), Z_a is the average characteristic impedance of a short dipole, given by

$$Z_a = 120(\ln h/a - 2.25) \qquad (3\text{-}26)$$

A graph of Z_a versus h/a is given in Fig. 3-3. Fig. 3-4 is a graph of the relative feeder impedance Z_o / R_o versus the relative characteristic dipole impedance Z_a / R_o, for several values of the mean spacing factor σ'. This graph can be used to design for a specific input impedance, given σ' and Z_a.

3-3d. The Characteristic Pattern as a Function of τ and σ.[8] The scale factor τ and the relative spacing σ exercise primary control over the shape of the radiation patterns of LPD antennas. The directivity in decibels can be approximated using the formula from Kraus,

$$D = 10\log\frac{41253}{(BW_E)(BW_H)} \qquad (3\text{-}27)$$

BW_E and BW_H are the half-power beamwidths, in degrees. In Fig. 3-5 are plotted contours of constant directivity in decibels, as a function of τ and σ. A scale for the angle α is also given. A straight line connecting equal α indicates on the top (or right) and the bottom edges

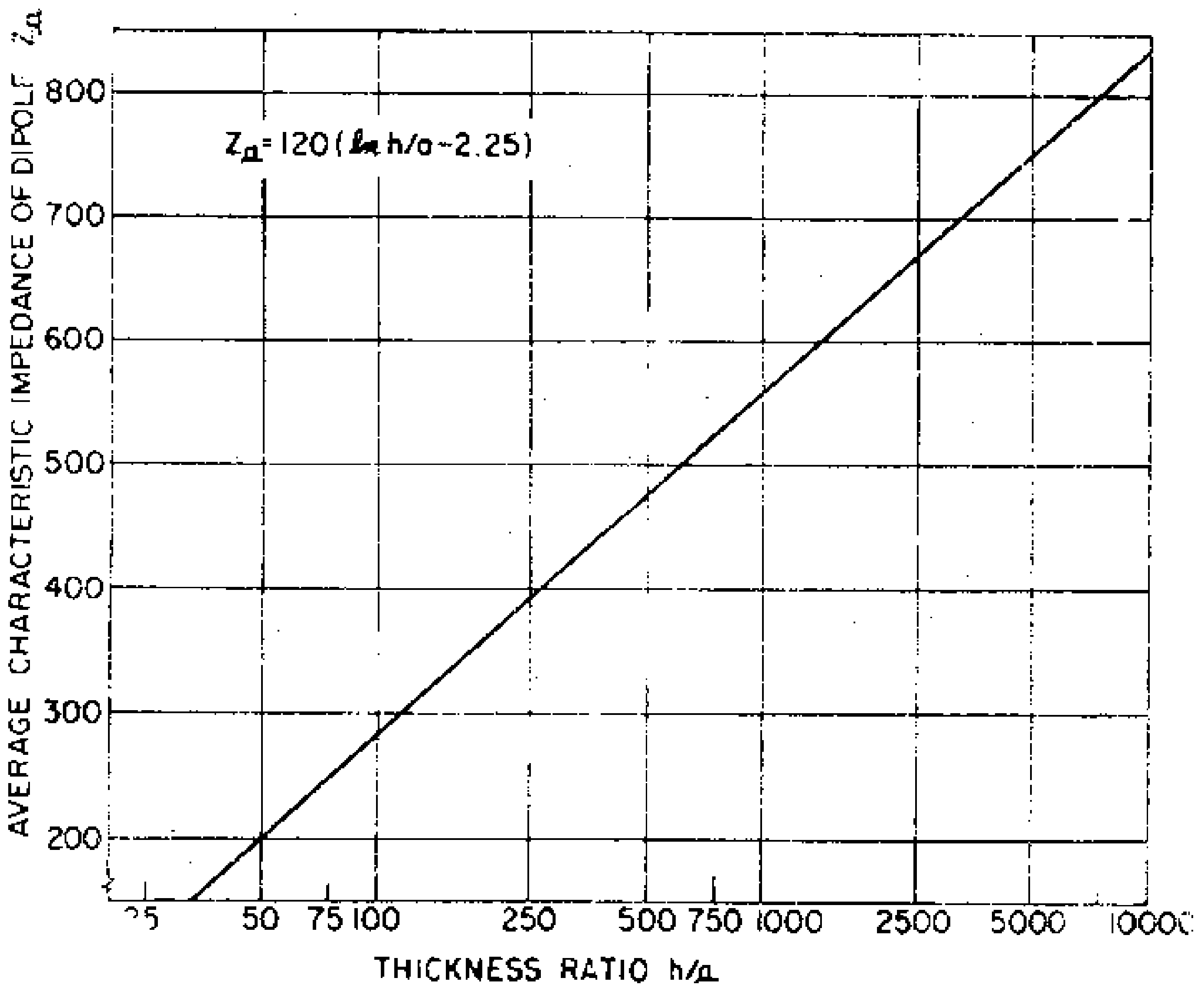


Figure 3-2. Average characteristic impedance of a dipole Z_a vs. height to radius ratio h/a.

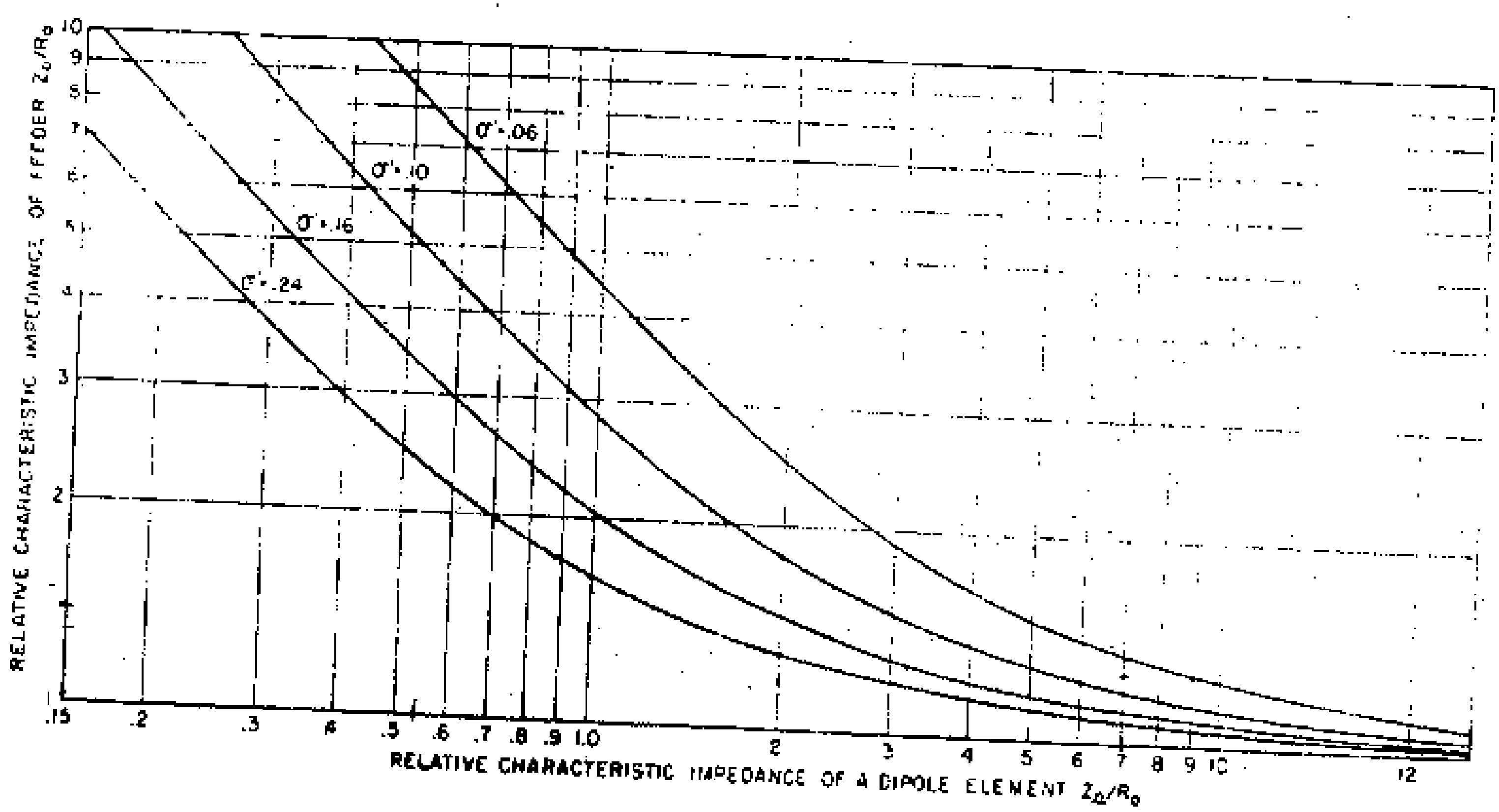


Figure 3-4 Relative feeder impedance Z_o/R_o vs. relative dipole impedance Z_a/R_o, from the approximate formula.

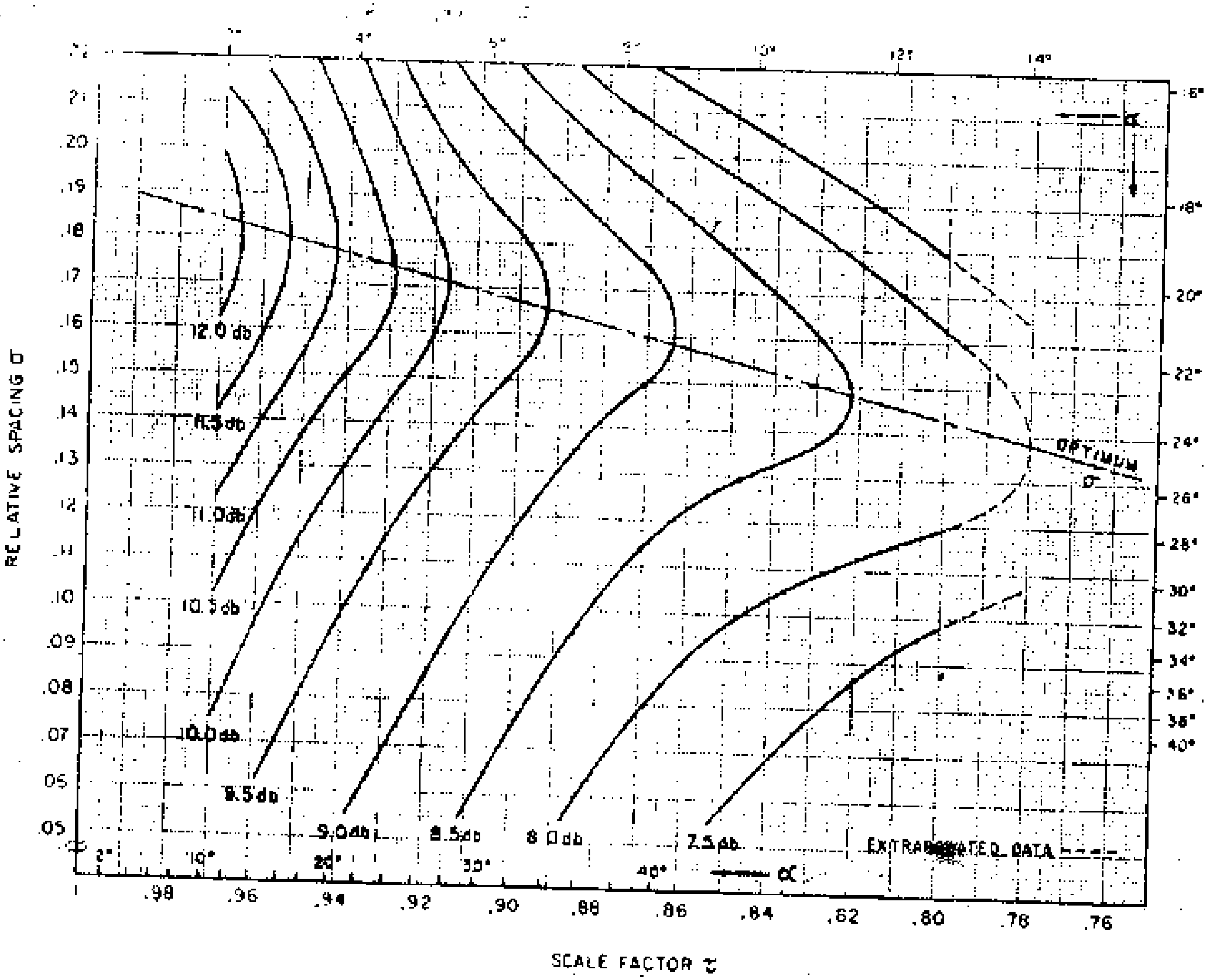


Figure 3-5. Computed contours of constant directivity vs. τ, σ, and α: $Z_o = 100$, Z_T = short at $h_1/2$, h/a = 177

of the graph describes combinations of τ and σ corresponding to the given angle α. All values of τ and σ which result in single-lobed frequency independent patterns fall within this graph. For σ less than 0.05 the directivity falls off rapidly and the front to back ratio decreases. Values of τ greater than 0.98 on the left side of Fig. 3-5 have not been extensively investigated because the number of elements required to achieve a given bandwidth becomes excessive. For values of σ greater than the optimum σ, the directivity falls off and the patterns either tend toward broadside or side lobes appear. In addition, the length of the antenna for a required bandwidth becomes excessive. For τ less than 0.8, only one dipole is near resonance at a given frequency, and it couples an insufficient amount of energy from the feeder, resulting in end effects which destroy the log-periodic performance. The element thickness controls the directivity to some extent. Fig. 3-5 is based on Z_o = 100 and h/a = 177, an adjustment should be made if h/a is much different from 177; i.e., the directivity should be decreased by about 0.1 db for each doubling of h/a. Hence, when h/a is increased to 2^n(177), where n is any number, then

$$\text{the modified directivity is} = D - 0.1n \qquad (3\text{-}28)$$

Hence for h/a $>$ 177, the constant directivity contours of Fig. 3-5 will read high. As with Z_o, the variation seems insignificant in the light of the approximations made, and for design purposes the directivity contours of Fig. 3-5 can be used directly.

3-3e. The Take-Off Angle as a Function of Height Above Ground. Since the vertex of the array would lie on the ground, the height of a resonant element in wavelengths is independent of frequency. The elevation radiation pattern is thus also independene of frequency. The take-off angle is determined by the product of the directional characteristic of the individual antenna elements and the array factor resulting from the seperation of the antenna from its image in the ground.[9] The rough approximation for the height of the active element above ground to give the desired take-off angle can be determined from the array factor.

The array factor of the antenna placed at a distance h above ground is given by

$$E = \sin\left(\frac{2\pi h}{\lambda} \cos\phi \right) \qquad (3\text{-}29)$$

The height h which gives optimum field strength at any specified take-off angle ϕ is given by

$$\left.\frac{dE}{dh}\right|_{\phi = \text{const.}} = 0 \qquad (3\text{-}30)$$

i.e.,

$$\cos\left(\frac{2\pi h}{\lambda}\cos\phi\right) = 0 \qquad (3\text{-}31)$$

or

$$h = \frac{\lambda}{4\cos\phi} \qquad (3\text{-}32)$$

The above height h can be used to determine the angle of the array with respect to ground which will give the take-off angle at ϕ degrees.

3-3f. The Phase Center as a Function of α.[10] The phase center of a log-periodic antenna does not lie at the vertex, rather it lies some distance x_p behind the vertex. For a fixed structure the distance as measured in wavelengths is essentially independent of frequency. The phase center lies on the center line of the half structure at a point near where a half-wave transverse element exists, and is given by

$$x_p \approx \frac{\lambda}{4} \cot \alpha \qquad (3\text{-}33)$$

3-4. The Design of Log-Periodic Dipole Antennas

The procedure in designing the LPD is outlined whereby the physical dimensions of an antenna which meets given electrical specifications can be determined by the use of graphs and nomograms.

3-4a. Review of Parameters and Effects. To varying degrees all the parameters which specify an LPD have an effect on the observed performance. Table 3-1 lists the parameters and qualitatively describes how each effects the performance.

TABLE 3-1

LPD Parameters and Their Effect on the Observed Performance[11]

Table entries denote the change in performance for an increase in the parameters of the first column.

LPD * Parameter	Bandwidth of Active Region B_{ar}	Input Impedance (always less than Z_0)	Directivity	Phase Center Distance to the Apex x_p	Boomlength L/λ_{max} for a fixed Operating Bandwidth B
τ (σ const.)	decrease	small decrease	increase	increase (depends on α)	Decrease to a point depending on B, then increase.
σ (α const.)	decrease	small decrease	small increase	independent	decrease

LPD Parameter *	Bandwidth of Active Region B_{ar}	Input Impedance (always less than Z_o)	Directivity	Phase Center Distance to the Apex x_p	Boomlength L/λ_{max} for a fixed Operating Bandwidth B
σ (τ const.)	increase	increase	increase	increase (depends on α)	increase
σ (α const.)	increase	increase	small decrease	independent	increase
Z_o	independent but location of AR moves toward apex	increase	small decrease	small decrease	small decrease
h/a	independent, but location of AR moves away from apex	increase	small decrease	small decrease	small increase

* The table entries hold true over the following range of parameter for which frequency independent operation has been verified: $0.875 < \tau < 0.98$, $0.05 < \sigma < \sigma_{optimum}$, $100 < Z_o < 500$, and $20 < h/a < 10000$. Any one of τ, σ, or Z_o may take on other values provided the remaining parameters are suitably restricted as explained in the text.

The directivity of an LPD depends primarily on the combination of τ and σ selected. Since an increase in directivity implies an increased aperture, it is not surprising that high directivity models are characterized by small α and large L/λ_{max}. For a given τ, σ and element thickness, the input impedance depends on the characteristic impedance of the feeder. Fortunately, the directivity is essentially independent of the feeder impedance. This makes it possible to design an antenna for a given directivity and then, in most cases, the input impedance can be adjusted to the required level.

Table 3-2 is a collection of the design equations. The most important relationship, that which relates the directivity to the antenna parameter, has not been put into equation form. For this information reference must be made to the graph of Fig. 3-5.

TABLE 3-2

Design Equations

Number of equations refer to the ones which are first introduced.

$$\sigma = \frac{1}{4}(1 - \tau)\cot\alpha\ ,\ \tau = 1 - 4\sigma\tan\alpha\ ,\ \alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) \qquad (3\text{-}7)$$

$$B_{ar} = 1.1 + 30.7\sigma(1 - \tau);\ B_{ar} = 1.1 + 122.8\sigma^2\tan\alpha$$

$$B_{ar} = 1.1 + 7.7(1 - \tau)^2\tan\alpha \qquad (3\text{-}24)$$

$$R_o = \frac{Z_o}{\sqrt{1 + \dfrac{Z_o}{4\sigma Z_a}}} \qquad (3\text{-}20)$$

$$\frac{Z_o}{R_o} = \frac{1}{8\sigma'\dfrac{Z_a}{R_o}} + \sqrt{\left(\frac{1}{8\sigma'\dfrac{Z_a}{R_o}}\right)^2 + 1} \qquad (3\text{-}25)$$

$$Z_a = 120(\ln h/a - 2.25) \qquad (3\text{-}26)$$

$$Z_o = 120\cosh^{-1}\frac{b}{2a} \qquad (3\text{-}42)$$

$$D_s = BB_{ar} \qquad (3\text{-}23)$$

$$L/\lambda_{max} = \frac{1}{4}\left(1 - \frac{1}{B_s}\right)\cot\alpha\ ,\ L/\lambda_{max} = \left(1 - \frac{1}{B_s}\right)\frac{1}{1-\tau} \qquad (3\text{-}36)$$

$$N = 1 + \frac{\log B_s}{\log \frac{1}{\tau}} \qquad (3-37)$$

$$x_p = \frac{\lambda}{4} \cot \alpha \qquad (3-33)$$

3-4b. Design Procedure. First is presented a method of finding the major design parameters τ and σ for a given value of directivity. Then it is shown how Z_c is adjusted to obtain the required input impedance.

3-4c. Choosing τ and σ To Obtain a Given Directivity.[12] For most applications, one is interested in designing an antenna which has a given directivity and input impedance over a given frequency band. Once these electrical characteristics are specified, one must decide the relative importance of minimizing the number of elements or the size of the antenna. These two properties are not independent. Thr number of elements is determined by τ; as τ increases the number of element increases. The antenna size is determined by the boom length (the distance between the smallest and the largest element), which depends primarily on α. As α decreases the size increases. From the graph of Fig. 3-5, it can be seen that a number of combinations of τ and σ lead to a given value of directivity. One of this set (τ , σ) leads to minimum boom length and another leads to a minimum number of elements.

With these facts in mind a preliminary choice of τ and σ can be made from the graph of Fig. 3-5. It is usually best to start with the optimum value of σ and then proceed to lower values. Knowing τ and σ, the value of the dependent variable α can be determined from

$$\tan \alpha = \frac{1 - \tau}{4\sigma} \qquad (3-34)$$

or from the nomograph of Fig. 3-6.

The structure bandwidth B_s must be found to determine the boom length and the number of elements. B_s depends on the required operating bandwidth B and the bandwidth of the active region B_{ar}. For the given values of τ, σ, and α, B_{ar} can be determined from the nomograph of Fig. 3-7. B_s is then given by

$$B_s = B\,B_{ar} \qquad (3\text{-}35)$$

Since the length of element number one is $\lambda_{max}/2$ in the preliminary design, the geometry of the LPD antenna provides an expression for the boom length relative to the longest operating wavelength.

$$\frac{L}{\lambda_{max}} = \frac{1}{4}\left(1 - \frac{1}{B_s}\right)\cot\alpha \qquad (3\text{-}36)$$

where L is the boom length between the longest and shortest elements. A nomograph of (3-36) is given in Fig. 3-8. The number of elements is found from the equation

$$N = 1 + \frac{\log B_s}{\log \frac{1}{\tau}} \qquad (3\text{-}37)$$

a nomograph of which is given in Fig. 3-9.

It is likely that the first estimate of τ and σ will not minimize L or N. Repeating the process for different values of τ and σ will establish the trend, and the minimum designs will become readily apparent.

3-4d. Designing for a Given Input Impedance.[13] Once the final values of τ and σ are found, the characteristic impedance of the feeder Z_o must be determined so as to give the required input impedance R_o. The ratio h/a

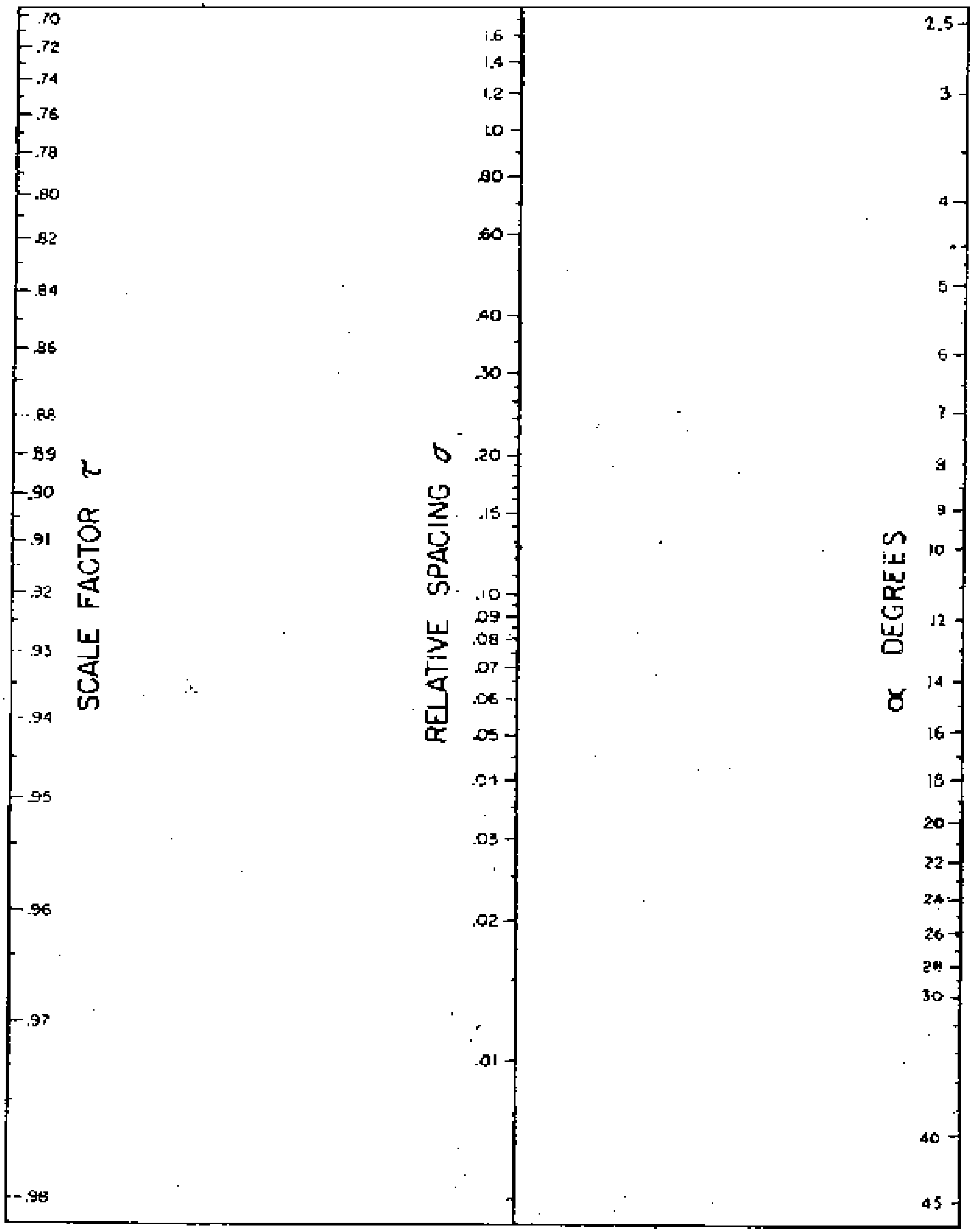


Figure 3-6. Nomograph, $\sigma = \frac{1}{4}(1 - \tau)\cot\alpha$

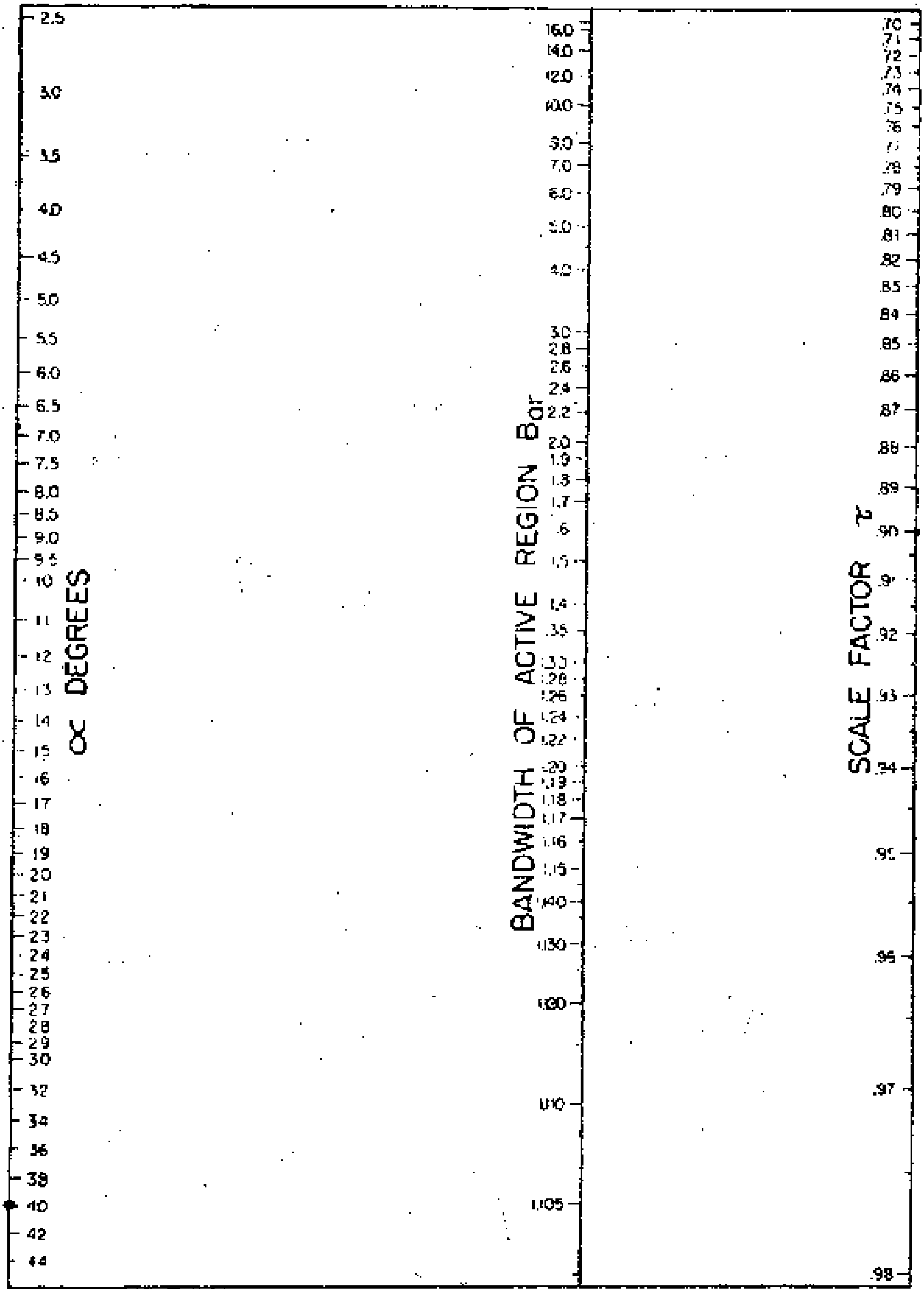


Figure 3-7. Nomograph, $B_{ar} = 1.1 + 7.7(1-\tau)^2 \cot \alpha$

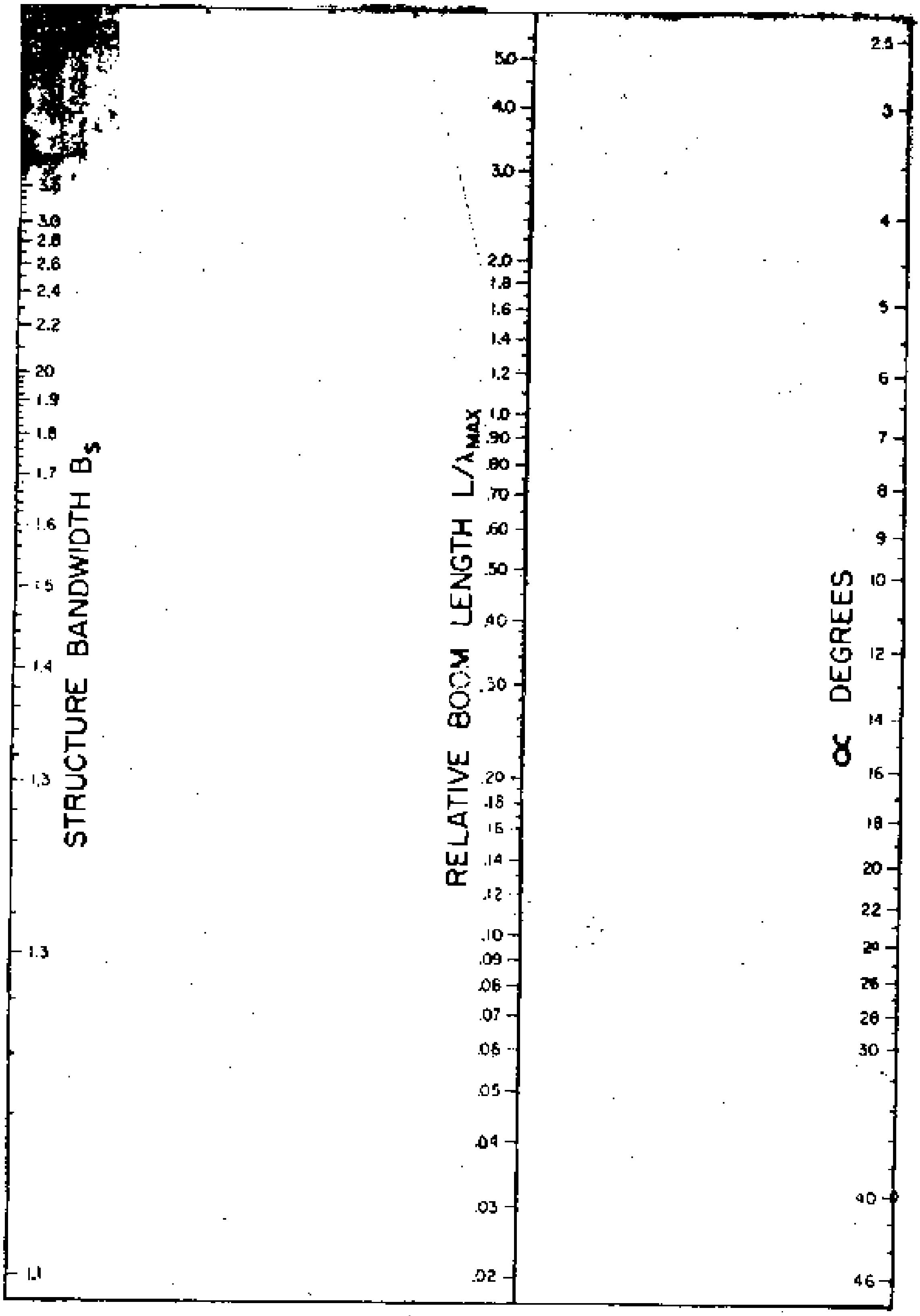


Figure 3-8 Nomograph, $\frac{L}{\lambda_{max}} = \frac{1}{4}(1 - \frac{1}{B_s})\cot \alpha$

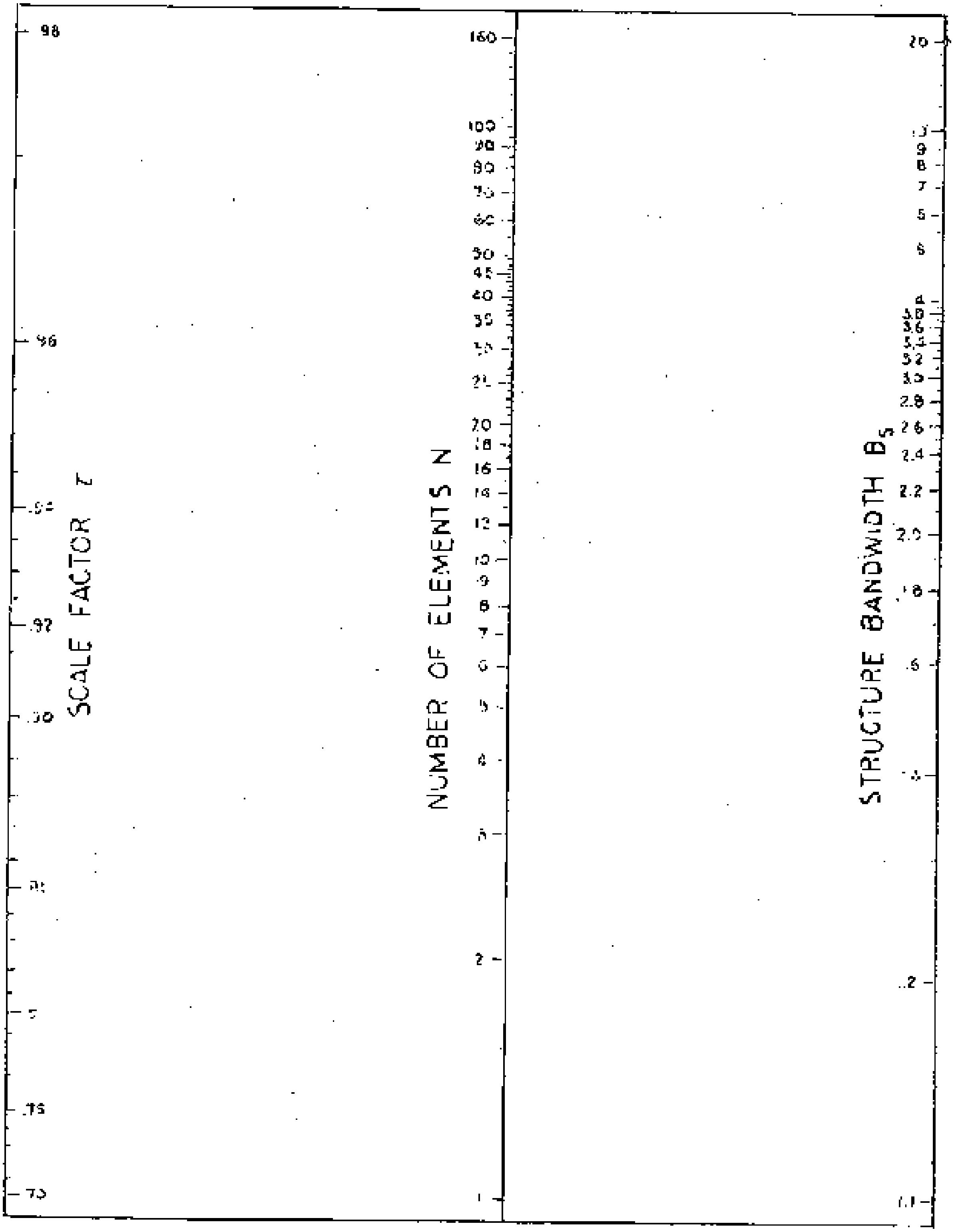


Figure 3-9. Nomogram, $N = 1 + \frac{\log B_s}{\log \frac{1}{\tau}}$

is determined from structural considerations, and ideally should be the same for each element. Practically, the element diameters can be scaled in groups, and in the computation of input impedance the average h/a in a group should be used. The average characteristic impedance of a dipole element Z_a can be found from

$$Z_a = 120 \left(\ln h/a - 2.25 \right) \qquad (3\text{-}38)$$

or from the graph of Fig. 3-3. Inverting (3-20), gives the characteristic impedance of the feeder relative to R_o,

$$\frac{Z_o}{R_o} = \frac{1}{8\sigma' \frac{Z_a}{R_o}} + \sqrt{\left(\frac{1}{8\sigma' \frac{Z_a}{R_o}} \right)^2 + 1} \qquad (3\text{-}39)$$

where Z_a / R_o is the average characteristic impedance of a dipole element with respect to the required input impedance R_o, and σ' is the mean relative spacing.

$$\sigma' = \sigma / \sqrt{\tau} \qquad (3\text{-}40)$$

A graph of (3-39) is given in Fig. 3-4.

The major parameters of the required design have now been determined. It remains to find the size of the first element relative to the maximum operating wavelength. The half-length of the longest element is given by

$$h_1 = S \frac{\lambda_{max}}{4} \qquad (3\text{-}41)$$

where S is the shortening factor as read from the graph of Fig. 3-10. Knowing τ, σ, N, Z_o, and h_1, one can find the dimensions of all other parts of the antenna.

The impedance Z_T which terminates the feeder has an effect only at the lowest operating frequencies. In practice, the feeder is terminated in a short circuit a distance of $\lambda_{max} / 8$ or less behind the largest element, so that Z_T remains inductive at the lowest frequencies.

3-5. Application of the Design Procedure

Specifications

Frequency range.	3 to 10 Mc
Polarization.	Horizontal
Azimuth-plane beamwidth.	90° between half power points.
Vertical-plane beamwidth.	60° between half power points.

$$\text{Directivity} = 10 \log \frac{41253}{90 \times 60} \approx 9 \text{ db}$$

Take-off angle 45°

h / a 5,000

Input impedance 50 ohms

In entering the graph of Fig. 3-5, an adjustment should be made because h / a = 5,000 instead of 177.

From (3-28), the correction factor $\bar{n}$ can be found by:

$$2^n (177) = 5{,}000$$

then

$$n \approx 5$$

Then the constant directivity contours of Fig. 3-5 will read 0.5 db high. Hence the contour designated 9.5 db will read 9.0 db for this case. Starting with a value of σ greater than optimum (to clearly illustrate the trend), the set (τ, σ) which yields a directivity of 9.0 db is recorded from Fig. 3-5 in Table 3-3.

The values of B_{ar}, L / λ_{max}, and N were determined from the nomographs. The minimum-element design is (.894, .166). The minimum beam length design cannot be attained for values of τ and σ within the operable range. A compromise between length and number of elements is given by $\tau = 0.90$ and $\sigma = 0.153$. This design has $L / \lambda_{max} = 1.2$ and $N = 17$. The average characteristic dipole impedance for $h/a = 5{,}000$ is about 750 ohms (from Fig. 3-3) so $Z_a / R_o = 750 / 50 = 15$, $\sigma' = \sigma / \sqrt{\tau}$ is 0.1615, and from the graph of Fig. 3-4, Z_o / R_o is approximately 1.02, therefore the feeder impedance Z_o is 1.02 X 50 = 51 ohms. The half-length of element one is set equal to $1.12_{14} \lambda_{max} / 2$ because the shortening factor is 1.12 (see Fig. 3-10).

In compiling a table of the dimensions of the antenna, it is best to start with a tabulation of power of τ which is accurate to a least four decimal places. The half lengths of the elements h_n, the distance from the apex to each element, x_n; the distance from the front of the antenna to each element, D_n; and the element diameters are then computed, as shown in Table 3-4. The half length of the largest element is 28.0 m because the desired low frequency cut-off is 3 Mc. The resulting antenna is shown in Fig. 3-11.

The feeder is spaced to give a characteristic impedance of 51 ohms according to the formula,

$$Z_o = 120 \cosh^{-1} \frac{b}{2a} , \qquad (3\text{-}42)$$

where b is the center to center spacing and 2a is the diameter of the feeder conductors.

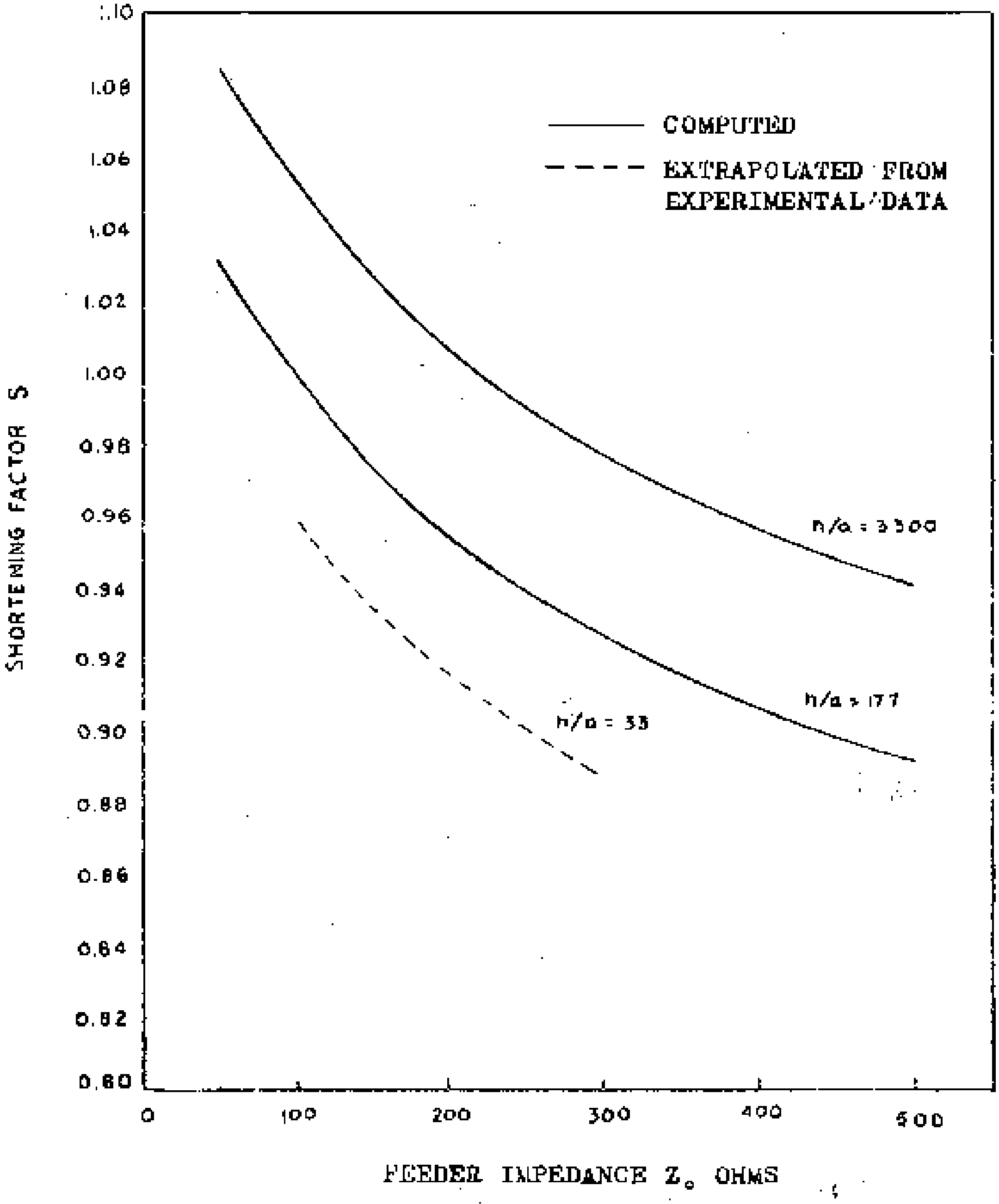


Figure 3-10 Shortening factor S. vs. Z_o and h/a .

TABLE 3-3

Values of τ, σ and α, which give 9.0 db directivity over 3.33 : 1 band.

τ	σ	α	B_{ar}	$B_s = DB_{ar}$	L/λ_{max}	N
.92	.205	5.7	1.59	5.30	2.07	21
.91	.1925	6.75	1.63	5.43	1.75	19
.90	.1825	7.9	1.65	5.50	1.50	17
.894	.166	9.2	1.63	5.43	1.28	16
.90	.153	9.25	1.56	5.20	1.20	17
.91	.1425	8.9	1.49	4.96	1.25	18
.92	.128	9.0	1.42	4.73	1.22	19
.93	.115	8.76	1.34	4.46	1.23	21
.94	.098	8.75	1.28	4.26	1.24	24
.95	.079	9.10	1.22	4.06	1.16	28

TABLE 3-4

Antenna Dimensions in Meters

n	τ^{n-1}	h_n	x_n	D_n	$2a_n$	Diameter of wire used
1	1.0000	28.0	172.2	140.3	.0112	.011
2	0.9000	25.2	155.0	123.1	.01008	.011
3	0.8100	22.7	139.5	107.6	.00908	.010
4	0.7290	20.4	125.6	93.7	.00816	.008
5	0.6561	18.35	113.0	81.1	.00734	.008
6	0.5905	16.51	101.7	69.8	.006604	.0065
7	0.5315	14.87	91.5	59.6	.00595	.0060
8	0.4784	13.40	82.4	50.5	.00536	.005
9	0.4306	12.04	74.1	42.2	.00482	.005
10	0.3875	10.84	66.7	34.8	.00434	.004
11	0.3488	9.78	60.0	28.1	.00390	.004
12	0.3139	8.79	54.0	22.1	.00352	.0035
13	0.2825	7.92	48.8	16.9	.00317	.003
14	0.2543	7.13	43.8	11.9	.00285	.003
15	0.2289	6.42	39.4	7.5	.00257	.0025
16	0.2060	5.77	35.4	3.5	.00231	.0025
17	0.1854	5.20	31.9	0	.00208	.002

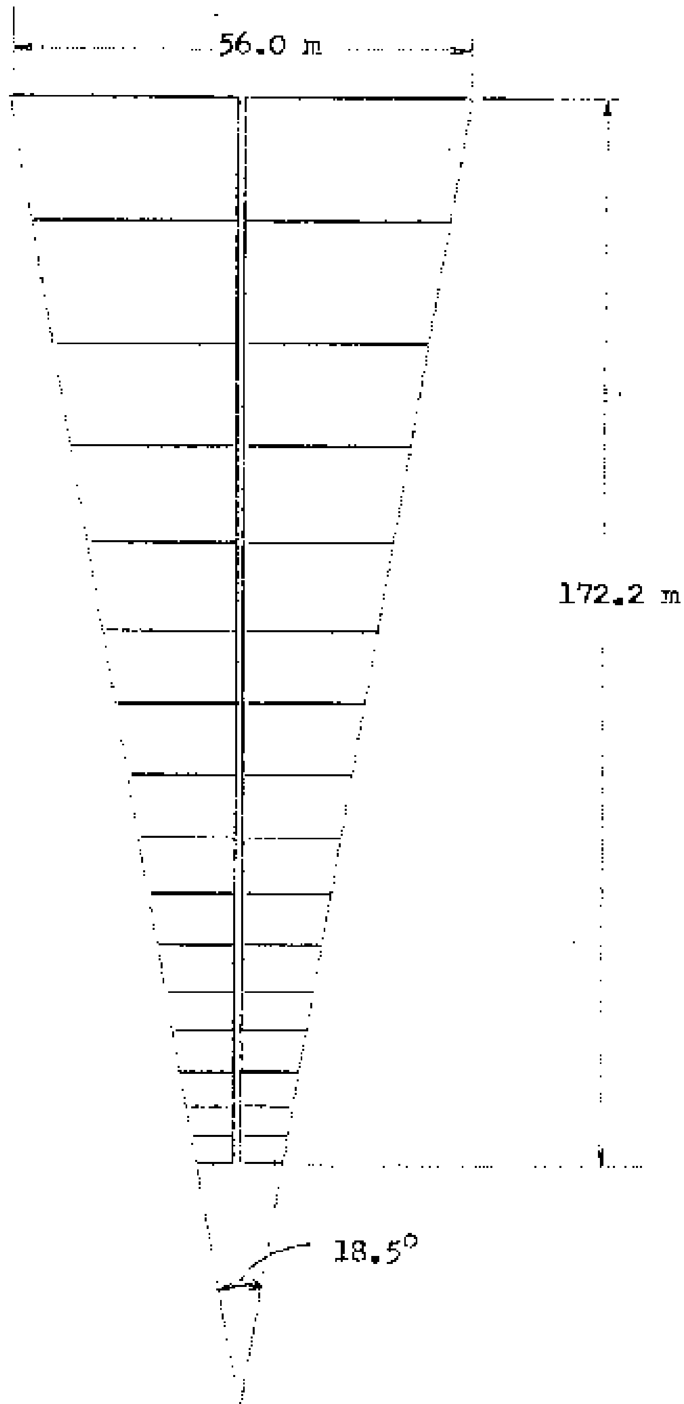


Fig. 3-11. Log-periodic dipole antenna for frequency range 3 to 10 Mc, τ = 0.9, σ = 0.153.

If the diameter of the feeder conductor is 1 cm; then the center to center spacing is 1.0925 cm.

For take-off angle at 45°, the approximate angle that the array is inclined to ground can be found as follow:

$$h_{max} = \frac{\lambda_{max}}{4 \cos \phi} = \frac{\lambda_{max}}{4 \cos 45^\circ} = \frac{\lambda_{max}}{2\sqrt{2}}$$

$$= 0.3535\, \lambda_{max}$$

$$h_{min} = \frac{3}{10} \times 0.3535\, \lambda_{max}$$

$$\text{boom length} = 1.2\, \lambda_{max}$$

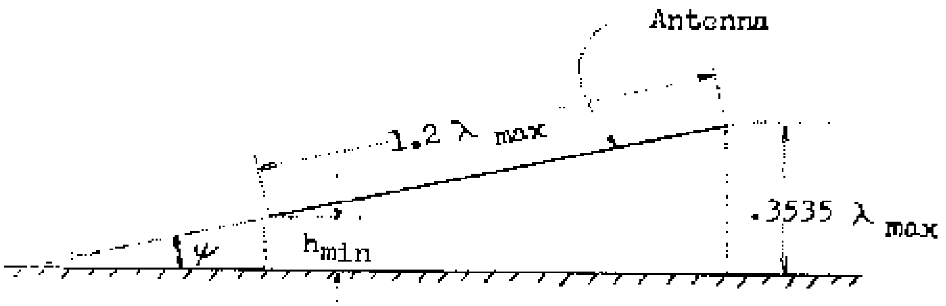


Fig. 3-12. LPD above ground.

then,

$$\text{angle } \psi = \sin^{-1} \frac{\frac{7}{10} \times 0.3535}{1.2} = \sin^{-1} 0.206$$

$$= 11.9^\circ$$

The antenna's setting is shown in Fig 3-12.

At this angle of inclination, each element is elevated the same fractional part of its resonant wavelength, the take-off angle will be independent of the operating frequency. If, on the other hand, part of the elements are placed at different phase distances above ground the pattern will vary with frequency.

3-6. Conclusions

In this chapter, a step by step procedure which enable one to design a log-periodic dipole antenna over a wide range of input impedance, bandwidth, directivity and antenna size. Although the design method is cut and try, a rough approximation to an optimum design may be obtained by the given procedure.

Of course, the outlined procedure is only one of many, and variation will become apparent to those who gain experience in the design of LP antennas. The graphs and nomograms are particularly useful because they allow one to achieve many preliminary designs without resorting to tedious computations.

In practical, the scaled model of the antenna should be constructed first and the characteristics be investigated. A scale factor of as much as 500 : 1, making the HF band 1,500 to 15,000 Mc (20 to 2 cm), may be acceptable for radiation pattern measurement. The inclination of the antenna with respect to ground to give the desired take off angle both independent and dependent on frequency can easily be controlled. The measurement of the input impedance in the scaled model is unnecessary because it can easily be adjusted by simply increase or decrease the spacing between the feeders in the full size antenna to get the desired value.

The typical LPD for use in domestic circuit has already been designed. The radiating elements should be rugged enough to withstand tension, the appropriate radiators should be made of seven-strand Calsum bronze wire, the numbers are according to the diameter sizes. The various components of the antenna can easily be constructed and could be set up with minimum cost. The computed design equations are based on certain engineering approximations such as sinusoidal current distribution along straight conductors of small cross section, and the applicability of circuit theory concepts to certain parts of an antenna structure that are small compared to wavelength. However, it is believed that a good agreement between measured and computed results may exist. This design is felt to be useful.