CHAPTER VI

ARRAYS OF LINEAR ANTENNAS

6-1. Array of Two Driven $\frac{1}{2}$ - wavelength Element, End - fire Case.

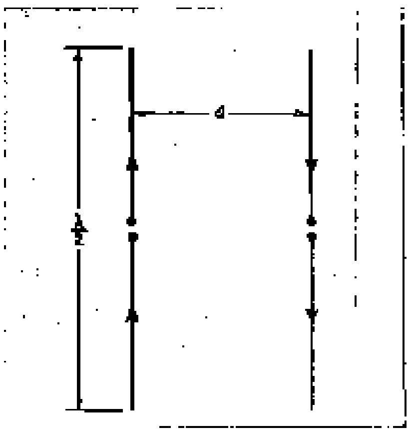

Fig. 6-1. End - fire array of two $\frac{1}{2}$ - wavelength elements with currents of equal magnitude but opposite phase.

Consider an array of two center-fed vertical $\frac{1}{2}$ - wavelength elements in free space arranged side by side with a spacing d and equal currents in opposite phase as in Fig. 6-1. The analysis will be divided into 2 subsections on the field patterns, driving - point impedance.

6-1a. Field Patterns. The field intensity as a function of ϕ at a distance D in a horizontal plane (x - y or ϕ plane in Fig. 6-2a) from a single element is

$$E_1 (\phi) = k I_1 \qquad (6\text{-}1)$$

where k = a constant involving the distance D

I_1 = the terminal current

The pattern in the horizontal plane of two isotropic point sources of equal amplitude but out - of - phase is given by

$$E_{iso}(\phi) = 2E_o \sin\left(\frac{d_r \cos\phi}{2}\right) \qquad (6\text{-}2)$$

Putting $E_o = E_1(\phi) = I_1$, and applying the principle of pattern multiplication. The field intensity $E(\phi)$ as a function of ϕ in the horizontal plane at a large distance D from the array is

$$E(\phi) = 2k\, I_1 \sin\left[\frac{d_r \cos\phi}{2}\right] \qquad (6\text{-}3)$$

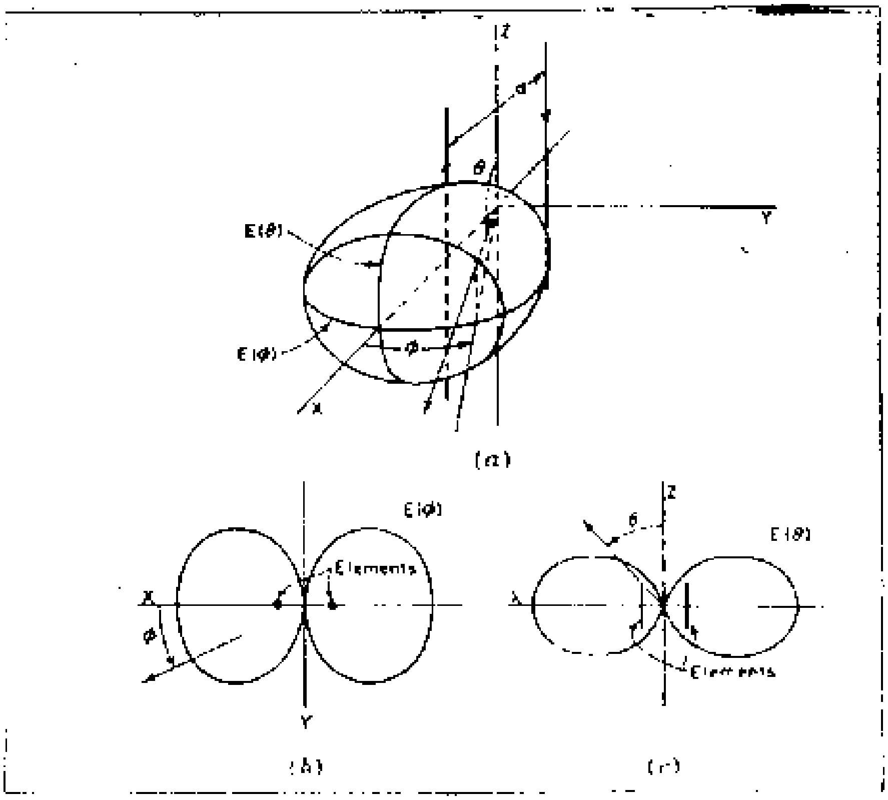

Fig. 6-2. Patterns for end - fire array of two linear out - of - phase ½ - wavelength elements with spacing d of ½ wavelength.

This is the absolute field pattern in the horizontal plane. The electric field at points in this plane is everywhere vertically polarized. The relative field pattern for the case where the spacing d is ½ -

wavelength is shown in Fig. 6-2b. and also partially in Fig. 6-2a. The maximum field intensity is at $\phi = 0^{\circ}$ and $\phi = 180^{\circ}$. Hence, the array is commonly referred to as an " end - fire " type.

The field intensity $E_1(\theta)$ as a function of θ from a single $\frac{1}{2}$ - wavelength element at a distance D in the vertical plane (x - z plane in Fig, 6-2a) is from (4-21) given by

$$E_1(\theta) = k I_1 \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} \qquad (6\text{-}4)$$

The pattern $E_{iso}(\theta)$ as a function of θ in the vertical plane for two isotropic sources in place of the two elements is from (6-2)

$$E_{iso}(\theta) = 2E_0 \sin\left(\frac{d_r \sin\theta}{2}\right) \qquad (6\text{-}5)$$

Note that θ is complementary to ϕ in (6-2), so $\cos\phi = \sin\theta$ Putting $E_0 = E_1(\theta)$ the field intensity $E(\theta)$ as a function of θ in the vertical plane at a large distance D from the array is

$$E(\theta) = 2k I_1 \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta} \sin\left(\frac{d_r \sin\theta}{2}\right) \qquad (6\text{-}6)$$

This is the absolute field pattern in the vertical plane. The relative pattern is illustrated in Fig. 6-2c, and also partially in Fig. 6-2a, for the case where the spacing is $\frac{1}{2}$ wavelength.

6-1b. <u>Driving - point Impedance</u>. Let V_1 and V_2 be the emf applied to the terminals of element 1 and 2 respectively. Then

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \qquad (6\text{-}7)$$

$$V_2 = I_2 Z_{22} + I_1 Z_{12} \qquad (6\text{-}8)$$

Since $I_2 = -I_1$. Then the terminal impedance Z_1 of element 1 is

$$Z_1 = Z_{11} - Z_{12} \quad (6\text{-}9)$$

of element 2 is

$$Z_2 = Z_{22} - Z_{12} \quad (6\text{-}10)$$

6-2. Horizontal Antennas Above a Plane Ground.[32]

The field of most antennas are affected by the presence of the ground. The change in the pattern from its free - space shape is of primary importance. The impedance relations may also be different than when the array is in free space, especially if the array is very close to the ground. In this section the effect of the ground on horizontal antennas is discussed.

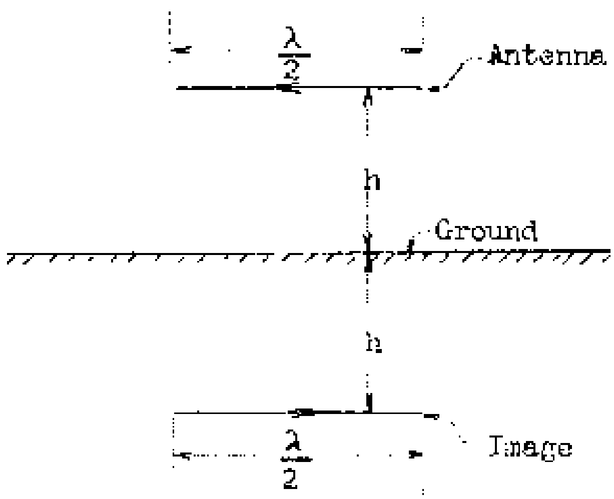

Fig. 6-3. ½ - wavelength antenna at height h above ground with image at equal distance below ground.

6-2a. Horizontal ½ - wavelength Antenna Above Ground.

Owing to the ground, the field at a distant point P is the resultant of a direct wave and a wave reflected from the ground as in Fig. 6-4. Assuming that the ground is perfectly conducting, the tangential component of the electric field must vanish at its surface.

To fulfill this boundary condition, the reflected wave must suffer a phase reversal of 180° at the point of reflection.

To obtain the field at a distant point P, Kraus has introduced the " method of images ". By taking the current in the image equal in magnitude but reversed in phase by 180° with respect to the antenna current, the problem can be transformed into the problem already treated in Sec. 6-1 of a so - called end - fire array.

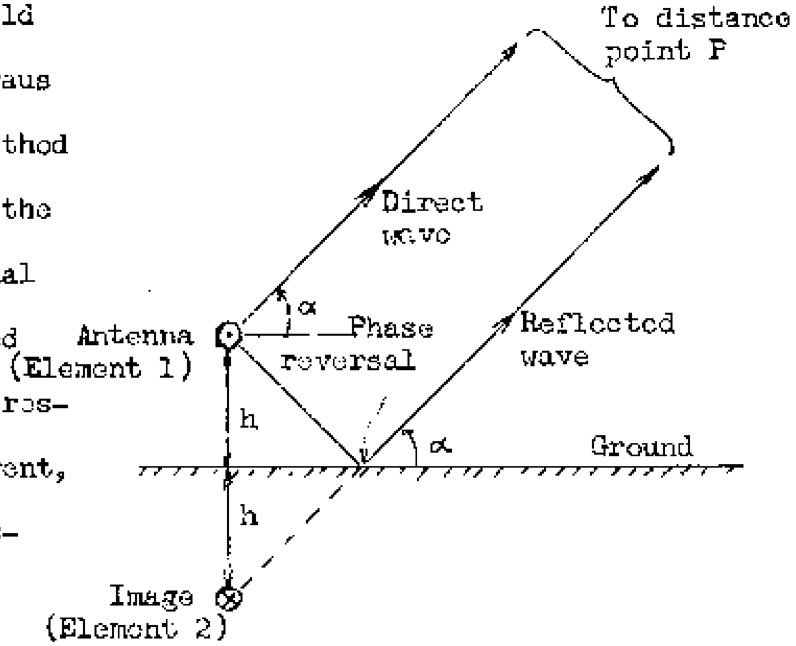

Fig. 6-4. Antenna above ground with image showing direct and reflected waves.

Owing to the presence of the ground, the driving point impedance of the antenna is, in general, different than its free-space value. Thus, the applied voltage at the antenna terminals is

$$V_1 = I_1 Z_{11} + I_2 Z_m \qquad (6\text{-}11)$$

where I_1 = the antenna current

I_2 = the image current

Z_{11} = the self - impedance of the antenna

Z_m = the mutual impedance of the antenna and its image at a distance of 2h

Since $I_2 = -I_1$, the driving - or feed - point impedance of the antenna is

$$Z_1 = \frac{V_1}{I_1} = Z_{11} - Z_m \qquad (6\text{-}12)$$

The real part of (6-12) or driving point radiation resistance is

$$R_1 = R_{11} - R_m \qquad (6\text{-}13)$$

The variation of this resistance at the center of the $\frac{1}{2}$ - wavelength antenna is shown in Fig. 6-5 as a function of the antenna height h above the ground. As the height becomes very large, the effect of the image on the resistance decreases, the radiation resistance approaching its free - space value.

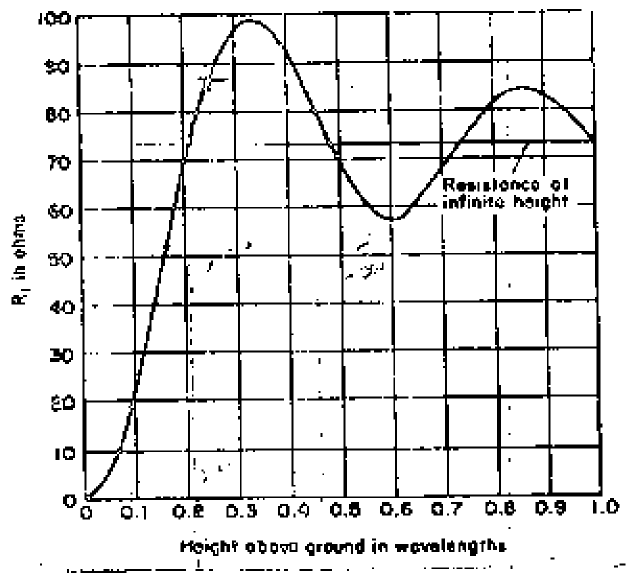

Fig. 6-5. Driving - or feed - point resistance R_1 at the center of a horizontal $\frac{1}{2}$ - wavelength antenna as a function of its height above a perfectly conducting ground.

6-2b. <u>The Vertical Field Pattern of Straight Horizontal Half - Wave Dipole Above Ground.</u>[33] The electric field around a $\lambda/2$ dipole in free space given by (4-22) to be

$$E = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \qquad (\theta \text{ measured from wire})$$

from which it can be seen that there is zero field intensity in the direction of the dipole and maximum everywhere at right angles to it. It is therefore a directive radiating system. In practice and when used over ground, some endwise radiation occurs because of reflections from ground and from the ionosphere, and in this direction the field is vertically polarized. Normal to the wire the field is horizontally polarized. In intermediate directions there are components of both.

The most important characteristic is its vertical-plane pattern normal to the wire. Over perfectly conducting earth, this has the equation (origin at dipole above ground)

$$F(\phi) = \cos(H \sin\phi \pm 90^\circ) \qquad (6\text{-}14)$$

where $F(\phi)$ = relative field intensity as a function of elevation angle ϕ.

H = electrical height of antenna above ground = $2\pi h/\lambda_o$

Fig. 6-6 shows a series of these patterns for various heights h up to $2\frac{1}{2}\lambda$. The relative polarity of the field in each lobe is indicated. Fig. 6-7 charts the angle of the maximums and the nulls for these patterns.

In reality, the ground is imperfect ground, the effect of ground loss on field pattern is in decreasing the amplitude of the maximums and the slight filling of the nulls due to incomplete cancellation of direct and reflected field. The angles of maximums and minimums are only slightly affected and for most practical uses may be considered to be the same.

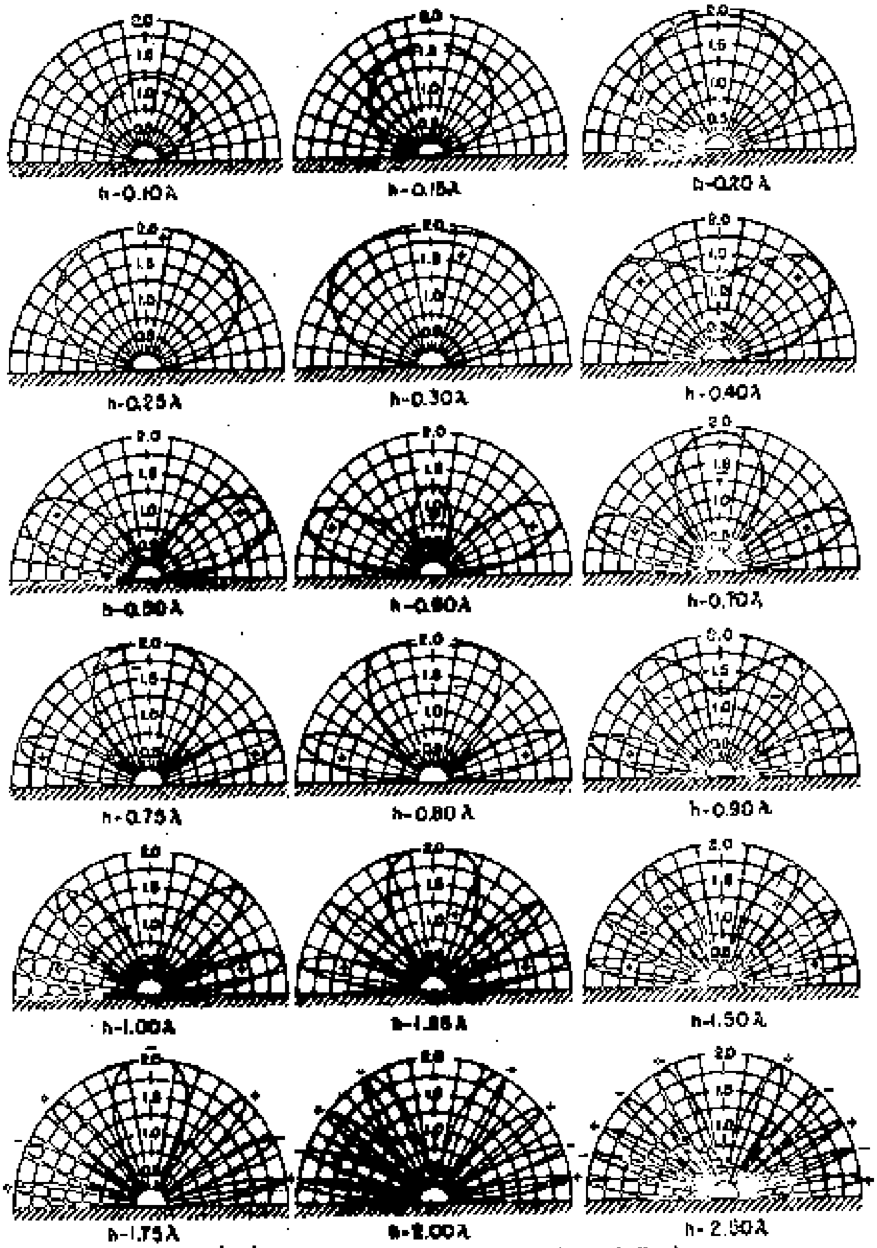

FIG. 6–6 Vertical polar diagrams for horizontal dipoles.

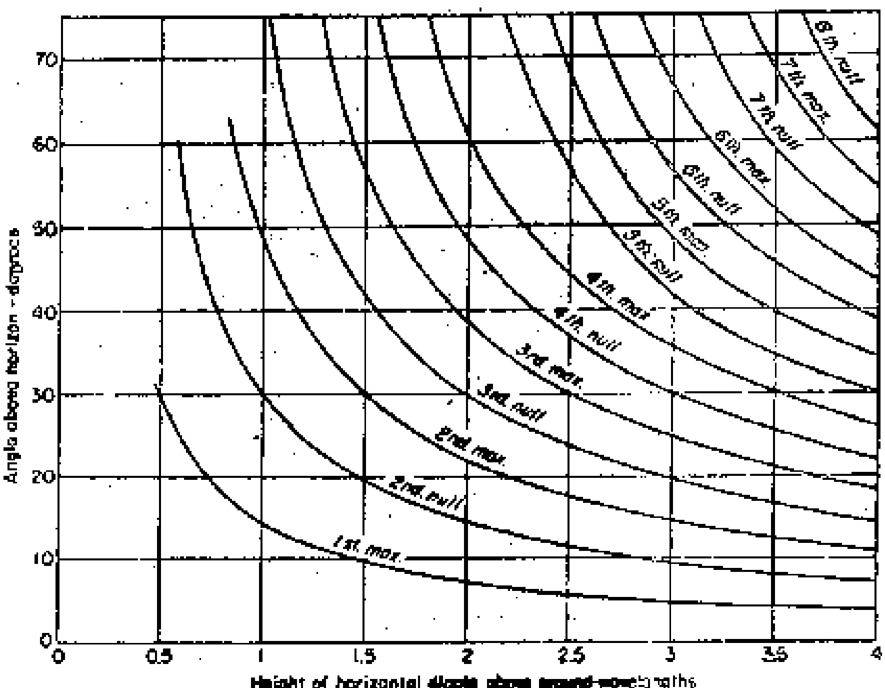

FIG. 6–7 Angles of maximums and minimums, horizontal dipole.

6-3. Arrays with Parasitic Elements.

The limited directive property of a half - wave antenna makes it necessary to use other types of antennas when it is desired to produce a concentration of r-f energy in a specific direction. One of the most common directive antennas is called a " parasitic " array.

A parasitic antenna is coupled to a driven antenna by the mutual impedance between the two antennas. A voltage induced in the parasitic element by the current in the driven antenna will cause a current to flow in the parasitic. This current will produce a radiation field which will combine with the radiation field of the driven antenna. The resultant pattern of such an array will usually have increased directivity over the pattern of the driven antenna, depending upon the spacing between the driven and parasitic elements and the dimensions of the parasitic. Parasitic arrays are usually designed with close spacing between elements. The directive properties of a particular parasite and driven antenna combination are determined by the relative phase and magnitude of the driven and induced currents. Specially, for close spacings, if the phase of the current in the parasite lags the phase of the current in the driven antenna, the direction of maximum radiation will be in the direction from the driven element toward the parasitic and the parasitic is called a director. If the phase of the current in the parasite leads the phase of the current in the driven antenna, the direction of maximum radiation will be in the direction from the parasitic element toward the driven element and the parasite is called a reflector.

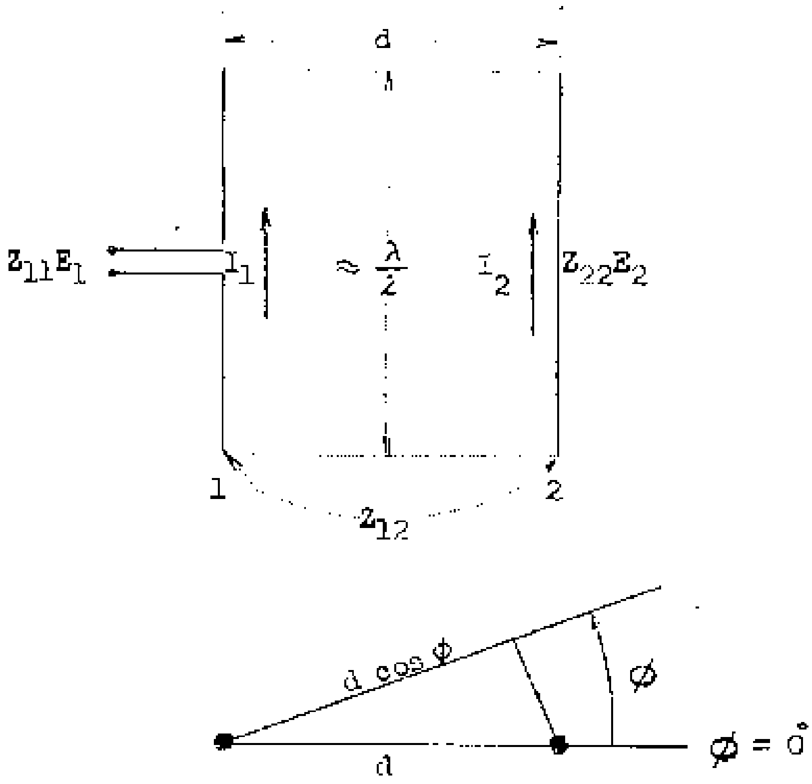

Fig. 6-8. Parasitic Antennas.

The equations describing the relationships between one driven and one parasitic element, as in Fig. 6-8, are as follows[34]:

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \qquad (6\text{-}15)$$

$$0 = I_1 Z_{12} + I_2 Z_{22} \qquad (6\text{-}16)$$

From (6-16) the current in element 2 is

$$I_2 = - I_1 \frac{Z_{12}}{Z_{22}} \qquad (6\text{-}17)$$

$$= I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \underline{/ \pi + \theta_{12} - \theta_{22}} \qquad (6\text{-}18)$$

where $\theta_{12} = \arctan \frac{X_{12}}{R_{12}}$

$\theta_{22} = \arctan \frac{X_{22}}{R_{22}}$

The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k\,(I_1 + I_2 \underline{/ + d_r \cos\phi}\,) \qquad (6\text{–}19)$$

where $d_r = \beta d = \frac{2\pi}{\lambda} d$ (6–20)

Substituting (6–18) for I_2 in (6–19)

$$E(\phi) = k\,I_1\,\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \underline{/\pi + \theta_{12} - \theta_{22} + d_r \cos\phi}\right) \qquad (6\text{–}21)$$

The total pattern will be the pattern as determined from (6–21) multiplied by the pattern of the $\lambda/2$ driven antenna. Solving (6–15) and (6–16) for the driving - point impedance Z_1 of the driven element, we get

$$Z_1 = Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_{11} - \frac{|Z_{12}^2| \; \underline{/2\theta_{12}}}{|Z_{22}| \; \underline{/\theta_{22}}} \qquad (6\text{–}22)$$

The real part of Z_1 is

$$R_1 = R_{11} - \left|\frac{Z_{12}^2}{Z_{22}}\right| \cos(2\theta_{12} - \theta_{22}) \qquad (6\text{–}23)$$

The gain in field intensity (as a function of ϕ) of the array with respect to a single $\frac{1}{2}$ - wavelength antenna with the same power input is

$$G_f(\phi)\left|\frac{A}{H.W.}\right| = \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} - \left|Z_{12}^2 / Z_{22}\right| \cos\left(2\,\theta_{12} - \theta_{22}\right)}}\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \underline{/\pi + \theta_{12} - \theta_{22} + d_r \cos\phi}\right) \quad (6\text{-}24)$$

where R_{1L} = effective loss resistance

6-3a. <u>Self - Resonant Parasitic Element</u>. When the parasitic element is the same length as the radiator (driven element), it is called self - resonant and the spacing determines whether it is a director or reflector. A parasitic element of this length spaced less than 0.14λ acts as a director, producing a relatively high antenna gain. A parasitic element of this length spaced greater than 0.14λ acts as a reflector, reproducing a relatively high antenna gain. At a spacing of 0.14λ, the parasitic element causes radiation equally in both directions, producing a relatively small antenna gain. Fig. 6-9 shows gain and radiation resistance as a function of the element spacing for this case calculated from Eqs. (6-23), (6-24) (from analysis by G. H. Brown)[35]. The front - to - back ratio at any spacing is the difference between the values given by curves A and B at that spacing.

6-3b. <u>Tuned Parasitic Element</u>. When the parasitic element is shorter (tuned to a higher frequency) than the driven element, it is called a director and reinforces radiation in the direction of a line pointing

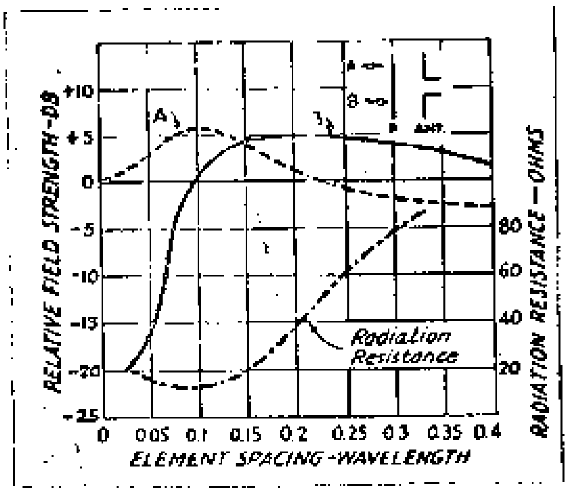

Fig. 6-9. Gain in a two - element parasitic array as a function of element spacing when the parasitic element is self - resonant.

toward itself from the driven element. See Fig. 6-10a

When the parasitic element is longer (tuned to a lower frequency) than the driven element it is called a reflector and reinforces radiation in the direction of a line pointing away from itself toward the driven element. See Fig. 6-10b.

The spacing is the chief factor in determining the gain of an antenna - reflector or an antenna - director combination, while the lengths of the parasitic elements

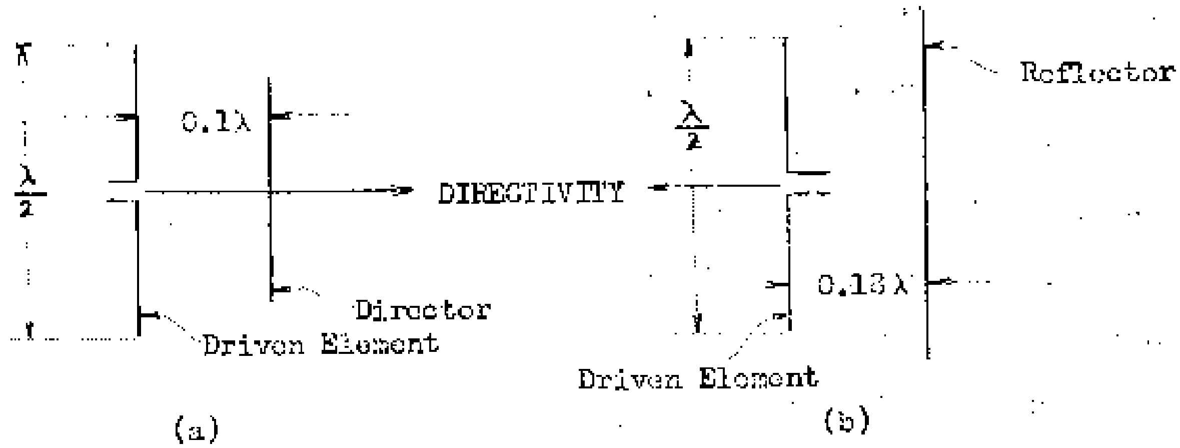

Fig. 6-10. A parasitic element shorter than the driven element acts like a director (a). A parasitic element longer than the driven element acts like a reflector.(b).

determine the sharpness of resonance of the multi - element array. The maximum gain theoretically obtainable with a single parasitic element, as a function of spacing is shown in Fig. 6-11, as calculated from Eq. (6-24) (from analysis by G. H. Brown[36]). The two curves

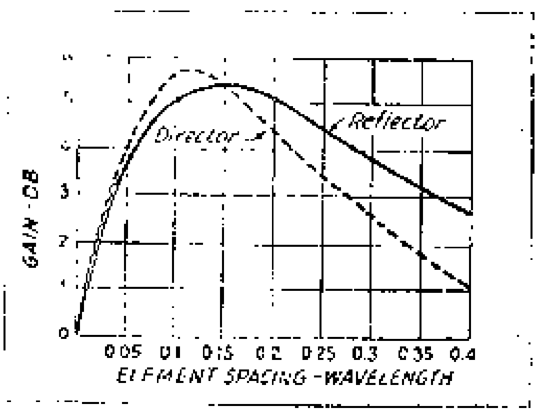

Fig. 6-11. The maximum possible gain obtainable with a parasitic element over a half - wave antenna alone, assuming that the parasitic element tuning is adjusted for greatest gain at each spacing.

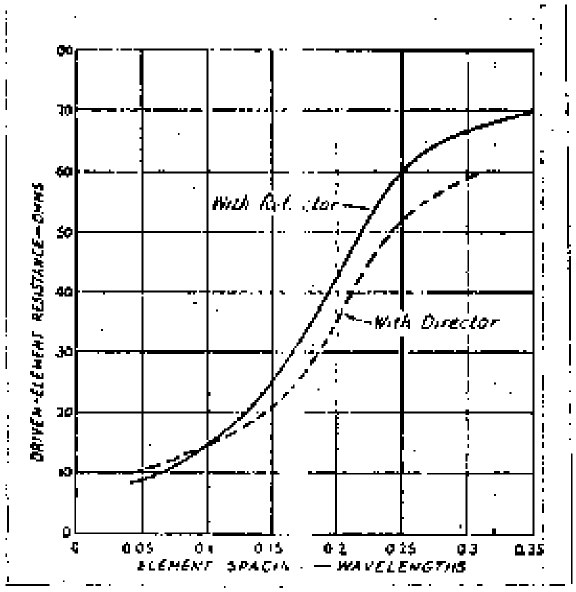

Fig. 6-12. Radiation resistance at center of driven element as a function of element spacing, when the parasitic element is adjusted for the gains given in Fig. 6-11.

show the greatest gain to be expected when the element is tuned for optimum performance either as a director or reflector. The radiation resistance at the center of the driven element, calculated from Eq. (6-22), varies as shown in Fig. 6-12 for the spacing and tuning conditions that give the gains indicated by the curves of Fig. 6-11.