CHAPTER VII

THEORY OF MODELS OF ELECTROMAGNETIC SYSTEMS

7-1. Introduction

Whenever a new aerial is developed or an established aerial is used in a new way, there are three methods by which the sign parameters can be optimized. They are calculation, experiment on full size aerials and experiment on small scale models of the aerials.

Of the three, a simple calculation is the easiest and cheapest. But in most cases, the precise calculations are long and laborious. Some aerials have outrun theory and are for all practical purposes incalculable.

Experiment on full scale aerials gives realistic results but is the most expensive and least accurate method. The accuracy is such that only coarse optimization may be carried out. Refinements which change the gain by less than 1 or 2 db may not make a noticeable difference unless protracted measurements are made, Ground and site effects mask small changes in radiation patterns.

The remaining technique, scale modelling, avoids the principal difficulties of the other methods. It can be applied to aerials for almost all frequency bands but is particularly useful for aerials working below 300 Kc. Models can range in scale from $\frac{1}{3}$ to 1/500 of full size and are invariably cheaper to construct than full size aerials. Tests are carried out at proportionately shorter wavelengths and can be confined to smaller areas, under more carefully controlled conditions.

7-2. Theory

7-2a. Types of Models. The usual model-antenna-pattern measurements are made on a type of model which may be called a qualitative or geometrical model; i.e., a model in which only the geometrical configurations of the lines of force in the field are modeled, no attempt being made to simulate power levels of the full scale system. A geometrical model directly yields data on those properties of the system which do not depend on power level (such as impedance, polarization, relative antenna patterns, etc.)

If, in addition to simulating the configurations of the lines of force in the field, the power level of the full-scale system is also simulated in the model, the model may be termed a quantitative or absolute model. This type of model is capable of yielding quantitative data on all electromagnetic properties of the system, so that, for example, measurements could be made of field intensity, and the like.

Both of the above types of models generally require some sort of mechanical model of the material portions of the full-scale system. A mechanical model is thus one in which there is geometrical simularity in the shapes of corresponding material parts.

7-2b. Requirements for Accurate Simulation of a System. The possibility of constructing a model of a given electromagnetic system arises from the linearity of the differential equations (Maxwell's equations) which describe the fields in any electromagnetic system. Therefore, for a model to be possible, it is necessary to exclude from the system nonlinear media (such as ferromagnetic media and ionized media in the presence of magnetic fields as in the ionosphere). It is not necessary to exclude nonhomogeneous media, since Maxwell's equations are valid for nonhomogeneous as well as for homogeneous media. Such media must be linear, however, so that, although the parameters which describe the media may vary from point to point throughout space, they must be independent of time.

The simple rule of preserving the physical dimensions of a system in wavelengths in the construction of a model applies only to perfectly conducting antennas in free space. In the conditions for an absolute model, we have to scale the conductivity, permeability, and dielectric constant. Proper scaling factors may be obtained from Maxwell's equations.

$$\text{curl } E = -\mu \frac{\partial}{\partial t} H , \quad \text{curl } H = \sigma E + \epsilon \frac{\partial}{\partial t} E \qquad (7\text{-}1)$$

Let primes denote various quantities in a model, then

$$\text{curl}' E' = -\mu' \frac{\partial}{\partial t'} H' , \quad \text{curl}' H' = \sigma' E' + \epsilon' \frac{\partial}{\partial t'} E' \qquad (7\text{-}2)$$

where the prime after curl denotes differentiation with respect to the scaled coordinates. Suppose that

$$E = \alpha E' , \quad H = \beta H' , \quad d = pd' , \quad t = \gamma t' \qquad (7\text{-}3)$$

where α = the scale factor for electric intensity

β = the scal factor for magnetic intensity

p = the mechanical scale factor

γ = the scale factor for time

It may be noted that four scale factors (α , β, γ, p) are all that are needed, since there are only four fundamental units (mass, length, time, and charge) required to describe any electromanetic quantity.

Substituting (7-3) in equations(5-2), we find

$$\frac{p}{\alpha} \text{ curl } E = -\mu' \frac{\gamma}{\beta} \frac{\partial}{\partial t} H \, , \quad \frac{p}{\beta} \text{ curl } H = \sigma' \frac{E}{\alpha} + \frac{\gamma}{\alpha} \epsilon' \frac{\partial}{\partial t} E \quad (7\text{-}4)$$

But equations (7-4) must be indentical with equations (7-1) hence

$$\left.\begin{aligned} \frac{\beta}{p\,\alpha}\sigma' &= \sigma & \text{(a)} \\ \frac{\beta\,\gamma}{p\,\alpha}\epsilon' &= \epsilon & \text{(b)} \\ \frac{\alpha\,\gamma}{p\,\beta}\mu' &= \mu & \text{(c)} \end{aligned}\right\} \quad (7\text{-}5)$$

In practice, experiments on models . are performed in free space; it is evident that the permeability of the model media cannot differ appreciably from the value of the permeability for free space. There fore, for all media

$\mu' = \mu = 4\pi \times 10^{-7}$ henry per meter, and it follows from (7-5) that

$$\frac{\alpha\,\gamma}{p\,\beta} = 1 \quad (7\text{-}6)$$

Furthermore, when waves in free space are involved in the full-scale system it is apparent that the medium in which they travel (air) must be correctly simulated in the model. It is general practice at the present time to simulate the air in the full-scale system with air in the model system. This is partly because of convenience and partly because the use of any other medium usually means higher attenuation of the waves than can be tolerated. Therefore, air is generally used, and it follows that, for these regions of the model.

$$\epsilon' = \epsilon \qquad (7\text{-}7)$$

But, since (7-5b) has to be satisfied everywhere in the model system (7-7) must therefore be true for all media. Hence,

$$\frac{\gamma \beta}{p \alpha} = 1 \qquad (7\text{-}8)$$

A comparision of the requirements imposed by (7-6) and (7-8) shows that these equations can only be satisfied simultaneously by choosing

$$\frac{\alpha}{\beta} = 1 \qquad (7\text{-}9)$$

Hence,

$$\alpha = \beta \qquad (7\text{-}10)$$

and

$$p = \gamma \qquad (7\text{-}11)$$

These conditions therefore require that the relationship between conductivities in (7-5a) take the form

$$\sigma' = p\sigma \qquad (7\text{-}12)$$

Thus it is apparent that for a practical model which is subject to the above restrictions there are actually only two scale factors (p and either α or β) which can be arbitrarily chosen. The other scale factor are then fixed by (7-10) and (7-11).

It should be noted that the condition for conductivities required by (7-12) is not necessarily satisfied by using air for the model to simulate air in the full-scale system. Actually, the air has a small conductivity which varies with frequency, and which therefore should be taken into account in designing the model and simulated according to (7-12). However, for most frequencies the air can be considered to be a perfect insulator and the error made in neglecting its conductivity is generally very small.

The restriction that $\alpha = \beta$, imposed by (7-10), can be illustrated by noting from (7-3) that the ratio α/β is the ratio of the impedance of space in the full-scale system to the impedance of the medium which simulates free space in the model. Since the same medium is to be used in both systems, the impedance of this medium must be the same in both systems. Furthermore, it should be noted that (7-9) fixes only the ratio of α to β , and does not restrict the choice of one of them. If a specific value is assigned to either α or β, the model becomes an absolute model and it can be used to obtain quantitative results for all quantities. Otherwise, the model is a geometrical model.

7-2c. Geometrical Models. If a definite value is assigned only to the mechanical scale factor p, so that the values of α and β are unknown (but their ratio being equal to unity), the model becomes a geometrical model. A value may be assigned to p which results in a model of convenient size for measurements. The important requirements to be satisfied in constructing a geometrical model are, therefore, the following.

$$x' = x/p$$

$$y' = y/p$$

$$z' = z/p$$

$$t' = t/p$$

$$\epsilon' = \epsilon$$

$$\sigma' = p\sigma$$

$$\mu' = \mu$$

The parameter p which determines the scale of the model is arbitrarity chosen to yield a model of convenient size. The relationships between various full-scale and model quantities which can be obtained directly from model measurements are listed in table 7-1. In addition to these properties of the system, it is possible to determine such characteristics as polarization, bandwidth, relative antenna patterns, and the like, which do not depend on a knowledge of power level.

In spite of the fact that the power level being used in a geometrical model may be unknown, it is still possible to obtain quantitative results from such a model in a number of ways. The most common method consists of using a standard model whose performance is known, and comparing the model under test with this standard. The performance of the standard model may be obtained from calculations, from full-scale measurements, or by any other means. The comparision between the test model and the standard model must be made at the same power level (although the actual level need not be known) or at power levels whose ratio is known.

TABLE 7-1

Conditions for a Practical Geometrical Model

Name of Quantity	Full-Scale System	Model System
Length	l	$l' = l/p$
Time	t	$t' = t/p$
Conductivity	σ	$\sigma' = p\sigma$
Inductive capacity	ϵ	$\epsilon' = \epsilon$
Permeability	μ	$\mu' = \mu$

Name of Quantity	Full-Scale System	Model System
Frequency	f	$f' = pf$
Wavelength	λ	$\lambda' = \lambda/p$
Phase velocity	v	$v' = v$
Propagation const.	k	$k' = pk$
Resistance	R	$R' = R$
Reactance	X	$X' = X$
Impedance	Z	$Z' = Z$
Capacitance	C	$C' = C/p$
Inductance	L	$L' = L/p$
Antenna Gain	g	$g' = g$

p = ratio of any full-scale length to the corresponding model length.

For antenna model measurements, the standard model is usually a half-wave dipole antenna whose characteristic are known from calculations. By comparing the field produced by the test model antenna with the field produced by a half-wave dipole when fed with an equal amount of power, the performance of the full-scale test antenna can be determined.

Another method for converting relative measurements of field intensities of antennas to absolute values makes use of the properties of the Poynting vector. From relative measurements of field intensity, the value of the Poynting vector at each point in space can be determined, except for a constant multiplier which depends only on the power levels. But, if the Poynting vector is integrated over the surface of a sphere of large radius, the result must equal the power radiated by the antenna. Hence, the actual value of the unknown multiplying factor which must be used to convert the relative Poynting vector to absolute values may be obtained by choosing it to make the integral equal to the known value of full - scale radiated power.

7-3. Choice of Scale

In choosing the scale, the size of the available aerial test site, the constructional facilities, the available equipment; and the nature and accuracy of the information required must all be considered.

True radiation patterns can only be measured in the far field of the aerial. The far field is usually regarded as the region beyond the critical radius given by

$$R = \frac{2D^2}{\lambda} \qquad (7\text{-}13)$$

where D is the aerial aperture and λ the wavelength

Lastly, scale depends upon the type of measurements and the accuracy required. Thus a scale factor of as much as 500 : 1, making the h. f. band 1,500 to 15,000 Mc (20 to 2 cm), may be accepted for radiation patterns of simple aerials. On the other hand for precise impedance measurements on wire aerials 4 : 1 may be just acceptable and 10 : 1 enough to cause serious inaccuracies.

The scale finally chosen is clearly a compromise and it may in fact be necessary to use two different scales in order to satisfy all the above requirements.

7-4. Conclusions

How closely do model results foretell the performance of a full size aerials? For h. f. aerials, gains, azimuth beamwidths and side lobe levels agree well. Impedance tend to be a little better on the model than on the full size aerial, probably because of the higher standards of construction which are possible on models.

It is therefore suggested that one should not ask how valid modelling is, but rather how nearly the full size installation approaches the ideal conditions of the model. Only if the site is equally unobstructed can one hope to achieve the same side lobe level. Only if the dimensions are as accurate, in proportion, and the joints as well bonded, can one hope to reproduce the same impedance match. Finally, the model work can give a more complete and accurate picture of the performance of the aerial than any limited measurement programme that one can hope to carry out on the real thing.