# CHAPTER I

# INTRODUCTION

A robot, from its very name, literally means "forced labor". To live up to this title, an ability to interact with surrounding is crucial. A robot should be able to manipulate objects. With a hand capable of grasping, a robot is blessed with greater degree of influent. Complex manipulation and maneuver are possible with grasping. Grasping also allows robots to use tools readily available for human. For example, instead of having a robot with various switchable specialized end effectors, we can build a robot with dexterous hands and make available tools usable by both robot and human. The later situation is more versatile and considered more intelligent than the first. Without a doubt, grasping is an essential ability of a robot, especially a humanoid one.

Colloquially, grasping means firmly holding of an object. There are many definitions of a firm grasp, either geometrically or mechanically. Many of grasping definitions, even though defined disparately, are eventually shown to be equivalent. The study of these definitions constitutes the ground theory of grasping. The essence of these definitions is the same: an object being grasped should be secured by contact points of a robot, even under a presence of reasonable external disturbance[1]. For example, a grasping of a hammer is considered firmly when the hammer can be effectively used, i.e., the hammer does not fall off from the hand due to the gravity and the hammer does not fly off the hand when it is swung and hit against nails. The hammer should still be in the hand of the robot even someone forcefully tries to take it away. These properties informally define a firm grasp.

A firm grasp is not hard to achieved. It has been shown that, with a sufficient number of contact points, a firm grasp is always possible on any object (Markenscoff et al., 1990). An algorithm for synthesizing a grasp is also easy, i.e., taking linear time under very relaxed constraints (Mishra et al., 1987). However, not all grasps are born equal. In a simulated world where many issues are considered negligible, all grasps are considered the same but when a real practical grasping hand is considered, many constraints arise such that some grasps are more preferable than the others. This discrimination originates from the fact that the act of grasping can naturally be separated into two layers, the task aspect and the hand aspect. The task aspect indicates an objective of grasping, i.e., it indicates a use of grasps. The hand aspect imposes constraints on a geometrical process of grasping, e.g., a certain grasp is not possible with some particular hand. Situated between this two layers is a grasp planning. Grasp planning acts as a consolidator between these two layers. It

[1] Another variation of grasping is "caging", where an object being caged is not fixed but is bounded in a finite region. Caging, to some extent, is considered a weak form of grasping and have an extensive study in its own right

determines a grasp that is suitable to both aspects.

Layering helps decompose the problem. It allows a hardware practitioner to concentrate on creating a hand. At the same time, robot visionary can imagine any use of robot in manipulation. The problem is that, currently, we have so many tasks and so many hands. As we have stated before, grasping is quite a fundamental action. Its use comes in a broad range and there exist lots of robotic hands in the present. In the literature, a common approach is to choose a specific task and/or a specific hand and identify goodness evaluator that suits them. After that, an appropriate grasp planning algorithm is tailored such that a good performance is possible. Obviously, that algorithm works best on that particular setting but the performance is no longer guaranteed under different setting.

Currently, there is no algorithm that works best on any setting. This is mainly because grasping tasks are varying. A particular task may require a property that is a trade-off with the other. For example, let us consider grasping of a hammer. If we are to relocate a hammer, it is best to grasp it on its metal head to reduce the effect of moment. However, if we wish to hammer something, the hammer should be grasped on its handle so that the moment of its head is maximized. Additionally, there is no universally accepted model of a hand. Different hands come with different constraints. A priori knowledge of a task and a hand is necessary for a design of an appropriate grasp planning algorithm.

This problem of task/hand performance dependency is the inspiration of our work. We aim to derive an algorithm that is applicable with any task or any hand in the real world. The key concept is simple; a task and a hand impose on grasping algorithms many constraints, which we simply choose to neglect them. We believe that an algorithm for grasp planning should be separated from a grasping process. Without any assumption on a task or a hand, all grasps are treated without any prejudice. If a grasp satisfies the accepted definition of a firm grasp, it should be gracefully considered as a fully qualified grasp. Instead of grading and reporting just the best grasp, the algorithm should report as many solutions as possible. Solutions from the algorithm will be dependent only on the object being grasped, not with any predefined hand or task. After solutions are identified, when knowledge of a hand and a task is provided, we may then prioritize solutions accordingly. This can be done in linear time since all solutions are already computed. The same solution set can be reused with other combination of a hand and a task as well.

Providing multiple solutions is also beneficial to regrasping problem. Sometimes, a grasp is needed to be modified, i.e., contact positions ares needed to be relocated while a grasp should still firmly hold an object. The act of changing a grasping configuration while maintaining a firm grasp is called regrasping. With multiple grasping solutions at hand, we can identify a grasp to be changed easily.

Still, the problem of identifying all possible grasps without biased task/hand constraints is not challenging enough. Most works in the literature assume some geometrical model of the object being grasped with a linear model being at the apex of popularity. This is mainly because a linear model allows efficient or analytical formulation for characterizing grasps on a given object. Many works propose algorithms that do not consider the issue of selecting a grasping facet, they aim solely on deriving a grasp planning algorithm that works on a set of polygonal faces whose number is equal to the number of contact points. When the number of faces of the object exceeds the number of contact points, an exhaustive search is performed. By assuming that the object can be modeled with a polygon with minuscule number of faces, the algorithm undoubtedly works acceptably well. There are only a few works that bear the goal of polygonal face selection as a main objective, much less are the works that applicable on the object as a whole.

Obviously, linear model cannot accurately represent every real world object. In practice, to apply such methods, we have to sense an object and then the sensed data has to be approximated by a polygon. With a limited number of edges in the polygonal model, there are always some objects (such as curved objects) that may not fit well. The resulting inaccurate model could then lead to unreliable resulting grasps. Of course, modeling accuracy can be improved simply by using more polygonal edges. This action, however, increases the model's complexity which results in higher computational cost in selecting combination of edges for which good grasps can be found. An alternative that addresses this issue assumes the curved object model in computing grasps. Although a curved model may better represents the shape of some objects, the cost of model fitting and grasp computation are significantly higher than using polygonal models. Only few works in the literature derive a method for curved objects.

In this work, a different approach is pursued. Our method does not operate on any boundary model; the input of the method is a set of points on the object's boundary together with the corresponding contact normals. Clearly, with our approach, there is no need for fitting the sensed object with a specific boundary model. Objects in any shape can therefore be handled in the same manner with the same accuracy. Nevertheless, from a practical standpoint, it is legitimate to ask about the performance of the proposed approach. To accurately model an object, a large number of boundary points is required. The number of points might be as many as a hundred to a few thousand points. This poses a great challenge to the problem, considering that a brute force algorithm would take time in the order of $O(n^k)$ where $k$ is the required number of contact points. A work that ignores the issue of face selection obviously suffers a severe performance problem.

However, using contact point as in input will result in a solution with much greater accuracy and it should be applicable in the real world.

The obvious problem of this approach is that we will be overwhelmed with plethora of grasps. Constraints imposed by a task or a hand provide a way to eliminate second-rated solutions. Without them, we have to consider every grasp. It is of crucial importance that we can compute solutions with speed. This is the goal of our work: to compute as many grasps as possible and as fast as possible under modeless assumption of the object being grasped.

## 1.1 Related Works

Works on grasping and fixturing have received lots of attention during the last two decades. Many works have been published since the pioneering work of Salisbury and Roth (Salisbury, 1982; Salisbury and Roth, 1982). This section review some of related works. Interested readers should refer to (Bicchi and Kumar, 2000; Bicchi, 2000) for recent survey and to (Mishra and Silver, 1989; Pertin-Troccaz, 1989) for more detailed on preceding works.

### 1.1.1 Hands

The problem of hand dependency stems from the fact that there is no universally-accepted model of a mechanical hand. Obviously, the diversity of hands is beneficial. Different task requires different design of hand. The more a robotic hand is designed to resemble the actual human hand, the more complex it is. Only a few robotic hands are available at the present, due to high cost and sophisticated manufacturing process required for the construction. Let us briefly review some of the well known robotic hands. A very simple hand, yet widely used, is a parallel jaw gripper. The gripper comes with two parallel plates which can squeeze toward each other. A parallel jaw gripper allows a simple form of grasping. Nevertheless, it lacks capability to grasp a complex object. Another robotic hand that is also widely available is a Barrett Hand (Townsend, 2000), commercially made by Barrett Technology Inc. Barrett Hand comes with three fingers, two of which can be spread synchronously by $180°$ around the palm. More sophisticated hand are usually custom made by each researching unit. For example, the university of Utah and AI Lab at M.I.T. introduce Utah/M.I.T. dexterous hand (Jacobsen et al., 1984) which has four finger and looks much like human hand. Another well-known hand is the DLR-Hand (Butterfaß et al., 1998; Liu et al., 1998) and its successor DLR-HAND II (Butterfaß et al., 2001) designed and developed at German Aerospace Center (DLR). The hand consists of four fingers that resembles the human hand but the size is noticeably larger. The Robonaut Hand (Lovchik and Diftler, 1999) designed by NASA's Johnson Space Center consists of five fingers and the size is the same as a very big male hand. There also exist several other hands in the literature, such as HRP2 (Harada et al., 2005), ARMAR (Morales et al., 2006), Domo (Edsinger-Gonzales and Weber, 2004) and STAIR (Saxena et al., 2007).

### 1.1.2 Grasp Analysis

Theoretical works on grasping can be categorized into two major groups: grasp analysis and grasp synthesis. Early works in grasp analysis focus on the definition of grasp property, i.e., it focuses on the question *how do we define a secure grasp*. After a consensus on such question is achieved, the literature then pays attention to deriving the conditions of such property together with the quantitative measurement of a grasp.

#### 1.1.2.1 Definition of Grasps

The concept of a firm grasp is formalized in various ways. The concept is considered in both geometrical and mechanical points of view. Grasping properties that are commonly used are *equilibrium*, *form closure* and *force closure*. A grasp is in equilibrium when the resultant of applied forces and torques are zero. Force closure indicates that the grasp can exert a resisting force and torque that balance any external disturbance on the object. Form closure is closely related to the force closure property and, in many cases, they can be used interchangeably. Usually, form closure speaks of the immobility of an object in the presence of fixed contact points. The motion of the object is prevented by the positioning of contact points. Form closure considers the problem in geometrical point of view (Bicchi, 1995). It does not incorporate the force exerting capability of contact point. This is different from force closure which considers how contact points can exert force and torque on an object. Directions of forces exerted by contact points on an object determines the force closure property. Form closure is shown to be equivalent to force closure when no friction is taken into account (Nguyen, 1986, 1988; Mishra and Silver, 1989). Form closure is considered as a stronger variation of force closure, i.e., every form closure grasp is also a force closure. That is the reason why form and force closure is used interchangeably in the literature, especially in the frictionless case. However, in their essence, the concept is different in the perspective from which the problem is analyzed. An evident distinction between form and force closure is the presence of friction (Bicchi, 1995). Friction effect is considered in force closure while it is neglected in form closure analysis.

Equilibrium, force closure and form closure are the properties determined by a configuration of contact points, i.e., they do not consider other properties of a grasped object except the position and the normal direction of the contact points. Without contact, a hand cannot interact with an object and thus these properties cannot be achieved. [2]

Recently, Rimon and Burdick presented another definition of firm grasp called second order immobility (Rimon and Burdick, 1998a,b, 1996). Second order immobility indicates that a grasp can prevent finite motion but not on infinitesimal one. There exists a non force closure grasp that achieves second order immobility. Second order immobility, in a sense, ensures a firm grasp of an object since the object is confined in an infinitesimal region of configuration space. Unlike the discussed grasping properties, second order immobility relies on curvature information at contact positions. Second order immobility is not applicable in our setting since no knowledge on curvature is assumed.

In this work, we emphasize on force closure grasps. Though force closure is equivalent to form closure, their analysis is different. In our work, we consider a grasp especially on its exerting forces and torques and hence the goal is to identify force closure grasp which, obviously, includes form closure grasp.

### 1.1.2.2 Contact Number Requirement

The earliest works on form closure is by Reuleaux (1876) (reprinted in (Reuleaux, 1963)) who shows that at least four contact points are required for form closure in 2D. Lakshminarayana (1978), citing the work of Somov (1900), reported that at least seven contact points are needed in 3D case. The result is confirmed by Markenscoff, Ni and Papadimitriou (1990). It is proved that there is no set of $n$ vectors that positively span $\mathbb{R}^n$. In frictionless setting, this indicates that we need at least four (seven) contact points to achieve form closure in 2D (respectively 3D).

Upper bounds on the number of end effectors are investigated by Mishra, Schwartz and Sharir (1987). It is shown, by using Carathéodory's theorem and Steinitz's theorem, that there exists upper bounds of the number of contact points that can always achieve form closure for piecewise smooth objects, with some exception to a particular class of objects (circular or rotational symmetric object) which cannot be grasped by any number of contact point. In 2D, an object can be grasped with equilibrium and with form closure by four and six contact points, respectively. The required number of contact points increases to seven and twelve in 3D case. This marks a loose bound on the number of required contact points.

The bound is tighten by Markenscoff, Ni and Papadimitriou (1990). They show that, without considering the exception class of the problematic objects in (Mishra et al., 1987), form clo-

[2] This is different from caging which does not require contact between a hand and an object. Caging is not considered in this works. Interesting reader should confer, for example, to (Rimon and Blake, 1996; Sudsang et al., 2000; Sudsang, 2002) for more information about caging.

sure can always be achieved by four wrenches in 2D and can be achieved by seven wrenches in 3D. This bridges the gap between the upper bounds and the lower bounds presented in (Lakshminarayana, 1978).

For the frictional case, Markenscoff, Ni and Papadimitriou (1990) also show that force closure can always be achieved by three and four contact points for 2D and 3D case, respectively. Under non-zero friction, force closure always exists for any piecewise smooth objects without any exception. These include circles and rotational symmetric objects which are problematic in the form closure case. Unlike the frictionless case, the lower bounds for frictional case are not the same with the upper bound. Nguyen (1989) shows that it is possible to construct a force closure grasp using two contact points in 2D.

#### 1.1.2.3 Force Closure Testing

Several conditions and algorithms for force closure assertion exist in the literature. The earliest work is also by Reuleaux (1876) who devised a simple method to consider set of possible rotations according to relative position and direction of contact points in 2D. The space considered by the Reuleaux's method is called *oriented plane* which is studied in (Stolfi, 1988; Guibas et al., 1983). Brost and Mason (1989; 1991) proposes a very efficient test using oriented plane representation.

Several works also consider the problem from the placement of contact points. Force closure requires that lines of action of the forces must be linearly dependent. Grassmann (1988) categorizes the possibilities that lines are linearly dependent into various groups: coplanar lines, intersecting lines, a regulus and two flat pencils having a line in common. Of all these groups, the grasp with intersecting lines of contact forces is the most common in the literature. This is popularized by Ponce and his colleagues as follows. Ponce and Faverjon (1995) present a condition for 2D frictional force closure for two and three fingers, also with different group of colleagues, Ponce et al. (1997) present a geometrical condition for four fingers in 3D. Informally, it is shown that force closure is achieved when there exist forces lying inside friction cone that positively span and intersect at the same point. They also present a weaker condition for force closure that can be described by a set of linear inequalities for which a linear programming can be employed to solve the problem. Recently, Li et al. (2003) present a very simple geometrical condition for force closure in intersecting cases. The condition can be applied only for three finger grasps in both 2D and 3D cases.

A geometrical approach usually does not consider all groups of Grassmannian geometry since they are so different that a unified condition would be too complicated. Many other works consider the problem in the conjoined space of force and torque called *wrench space*. Mishra et al. shows that a force closure problem is related to origin-inside-convex hull problem. Since the dimensions of wrenches may be as high as six, a multidimensional algorithm for constructing a convex hull is needed. A very popular implementation of convex hull is Qhull (Barber et al., 1996). However, naively constructing a convex hull and check for the origin is not efficient since we do not actually need the convex hull. Liu (1999) transforms the problem of the origin inside a convex hull into a ray shooting problem. The new problem can be solved more efficiently by linear programming. The method is improved in (Zheng and Qian, 2006). Recently, Zhu et al. (2004) discussed that the problem can also be transformed into the problem of calculation of distance between convex objects. They propose the use of pseudodistance function, such as GJK algorithm (Gilbert et al., 1988; Gilbert and Foo, 1990), to solve the problem.

There also exists other conditions for a convex hull containing the origin. Liu (1998) considered the problem, specifically in 2D grasping (which requires 3D wrench space), in a geometrical manner and derived a condition that reduces the wrench dimensions of the problem by one. Ding et al. (2001b) also presented a condition that, for a given fixed set of wrenches, identifies whether an additional wrench positively span the wrench space when combined with the given set.

#### 1.1.2.4 Grasp Quality

Although we aim to treat all grasps without quantitative measurement, it is worth considering some existing performance indices of a grasp. Most performance measurement can be categorized into two broad groups: those that value stability of a grasp and those that value grasping accuracy. The stability of a grasp is, informally, how well a grasp can withstand external disturbance. For accuracy, two issues are considered. The first one is how severe the error of contact point positioning affects configuration of the object. The second one is how well a grasp tolerate positioning error.

The most general stability measurement does not take a priori knowledge of disturbance, i.e., it assumes that an external wrench is uniformly distributed in every direction. Intuitively speaking, it measures the minimum magnitude of a particular external wrench that breaks the force closure property, given a fixed grasping force. In the wrench space, this is equivalent to the radius of a maximal ball that can fit inside the convex hull of primitive contact wrenches. This measurement is introduced by Kirkpatric et al. (1990) and popularized by Ferrari and Canny (1992). However, this measurement is not invariant to the choice of the origin since a wrench contains torque which depends on the choice of the origin.

The frame dependence of wrench is pointed out by Li and Sastry (1988). They instead suggest the use of the volume of the primitive contact wrench space, referred as Task Ellipsoid. Still, the radius of maximal ball is used in many works, such as (Mirtich and Canny, 1993; Borst et al., 2003; Jia, 1995). Recently, Zhu and Wang (2003) present the concept of *Q distance* which generalizes the concept of Kirkpatric et al. (1990). *Q* Distance replace a ball with a pre-specified arbitrary convex shape and thus the measurement can be adjusted to be suitable to the choice of the origin. Later, the improved version of the condition is proposed in (Zheng and Qian, 2006).

Pollard (Pollard, 1994) proposes object dependents measurement called Object Wrench Space (OWS). The Object Wrench Space which describes the best grasp that can be achieved on a specific object. Wrench space also possesses non-uniformity since the unit of torque and force are different. Interestingly, scaling of objects or contact positions dramatically affects the wrenches. Pollard (Pollard, 1994) also suggests that to reduce such effect, the torque component should be scaled to the length of the longest object axis. Borst, Fischer and Hirzinger (2004) combine the idea of Task Ellipsoid (Li and Sastry, 1988) and the idea of OWS (Pollard, 1994).

Another stability measurement is the magnitude of wrenches that achieve a firm grasp. Trinkle (1992) proposes a measurement that measures how far the grasp is from losing the form closure property. He uses the maximum of minimum value of the wrench magnitude that achieves equilibrium. Kerr and Roth (1986) minimize the equilibrium forces applied by the contact point. Markenscoff and Papadimitriou (1989) minimize the magnitude of force, under the worst case scenario that a grasp still achieves form closure under a unit external disturbing wrench.

Ponce and Faverjon (1995) propose an indirect measurement of stability. They minimize the distance between the centroid of an object and the center of mass of contact points. In fact, they use $L_\infty$ distance so that the problem can be casted as a linear programming problem. The work is extended into 3D in (Ponce et al., 1997). Ding, Lui and Wang (2001b) use quadratic programming (QP) to optimize the grasp under the same measurement. By utilizing a QP, they can use Euclidian distance, instead of $L_\infty$ distance. Lowering the distance between the centroid of an object and the center of mass of contact points decreases the effect of gravitational and inertial force on the object.

Ding et al. (2001a) choose contact points that minimize positioning error of an object when the contact points are misplaced. The objective of this minimization follows D-optimality criteria used by Wang (2000). Uncertainty in contact positioning and friction are also considered in (Zheng and Qian, 2005).

Nguyen (1988) suggests that solution of grasp planning should be a set of regions rather than a set of precise contact positions. As long as each contact point is placed on each region, force closure is guaranteed regardless of the exact position of the contact point. The regions, referred to as *independent contact regions*, copes with the error of contact point placement. Larger regions give more tolerance on erroneous of contact point placement. Ponce and his colleagues derived algorithms that use the size of independent contact regions as a measurement (Ponce et al., 1993; Ponce and Faverjon, 1995).

It can be seen that there are a large number of quality measurements. These measurements are not only distinct in implementation detail but also in their underlying idea of good grasp. Plethora of such indices is also a good indicator that there is no consensus on the quality of a grasp thus supporting our claim that there is no generally good grasp but only a suitable grasp for a particular task.

### 1.1.3 Grasp Synthesis

Another dominant are in grasping research is grasp synthesis. With established works on grasp analysis, the next natural step is to synthesize a grasp (see (Shimoga, 1996) for a past survey).

Several works tackle the grasp synthesis problem under a wide range of assumptions on representation of the grasped object, models of the grasping hands and grasping quality measurement. For example, earlier object modeling is predominated by linear representation such as a polygon or a polyhedron. Nguyen (1986) presents an early work on constructing a force closure grasp for a 2D polygonal object. Tung and Kak (1996) propose a geometrical approach to compute two finger force closure grasp of a polygonal object. The linear structure of the object representation allows application of efficient optimization tools, e.g., linear programming or quadratic programming. Ponce et al. (1995; 1997) simplify force closure into a sufficient condition called $\theta$-positively span which is linear in nature and solve the problem using linear programming. Given predetermined positions of some contact points not achieving force closure, Ding, Liu and Wang (2001b) proposes a method, utilizing sequential quadratic programming, to synthesize position of the remaining contacts that form force closure grasp. Later, the same group of authors and Wang also propose a grasp synthesis method that does not assume predetermined contact positions (Ding et al., 2001a). They employ quadratic programming to solve the problem.

Although linear representation allows efficient algorithm to be used, it cannot represent a complex shape such as a curve figure. Under a non-linear representation, a traditional approach is to apply a gradient descent method which is usually affected by the local optimal problem. Much fewer works consider curved objects. See (Faverjon and Ponce, 1991; Gatla et al., 2004; Cheong and van der Stappen, 2005) for some examples. Zhu and Wang (2003) propose a differentiable quantitative test of force closure together with a gradient search algorithm to construct a grasp on a curved object. They also propose a better method in (Zhu and Ding, 2006). Jia proposes a series of work concerning localization and grasping of a curved object (Jia, 2000, 2001, 2002a,b, 2004.).

Recently, researchers began to consider a discrete representation, such as point cloud or discretized curves. This representation naturally suits the real-world grasping application where the object model is not provided in advance and has to be acquired through some sensory devices. For this representation, the techniques of choice usually are systematic search methods such as hill climbing or branch and bound. Other optimizers also find some interest such as genetic algorithm or other evolutionary computations. Work of Brost and Goldberg (1996), also Wallack and Canny (1994), considers a fixturing problem where the fixturing device is discrete while the object is not. A truly discretized object is considered in the work of Wang where he presented a greedy algorithm for grasp synthesis on discrete point set (Wang, 2000). Later, Liu, Lam and Ding (2004) proposes a complete algorithm for the same problem. Recently, Cornella and Suarez (2006) propose an efficient algorithm for grasp synthesis in 2D.

Other approaches also find some uses in grasp synthesis. Borst, Fischer and Hirzinger (2003) speculates that force closure grasp is relatively common in the grasp space and suggests that a few randomly generated grasps should yield solutions with adequate quality. Obviously, a fast force closure test is desirable for this generate-and-test scheme. Some authors rely on evolutionary computation to solve grasping problem (Hasegawa et al., 2000; Katada et al., 2001). Machine learning approach also receives noticeable attention in grasping example (Hsiao and Lozano-Perez, 2006; Saxena et al., 2007). Pollard and her colleagues also publish several works in generating a grasp from examples (Pollard, 1996, 2004; Pollard and Zordan, 2005; Li et al., 2007).

Several authors also notice a task-dependency issue of quality metrics and pursue on computing all force closure grasps, rather than an optimized one. Liu (1998) propose a method to compute all force closure grasp on selected facets of a polygon. The same problem is tackled by Zhu, Ding and Wang (2003) for 2D polygonal and 3D polyhedral objects. Cheong and van der Stappen (2005) computes all force closure grasps on a curved object using segment-arc intersection algorithms. The method is improved, in terms of efficiency in (Cheong et al., 2006). Our previous works (Niparnan and Sudsang, 2004) also aims to compute several force closure grasps on 3D discrete contact points. We also propose a complete solution for 2D discrete contact point in (Niparnan and Sudsang, 2006a) and a faster solution with heuristic in (Niparnan and Sudsang, 2006b).

### 1.2 Problem Statement

Given a set of contact points each of which is described by its contact position and its inward normal direction, we wish to identify all force closure grasps formed by some contact points in the set.

#### 1.2.1 Contribution

With the aforementioned goal, this dissertation proposes several algorithms to compute every grasp that satisfies the required criteria in several settings. Each setting specifies two properties: the dimensions of the workspace and the presence of friction in the contact model. Specifically, there are four settings: 2D frictionless grasps, 2D frictional grasps, 3D frictionless grasps, and finally, 3D frictional grasps.

As mentioned in Section 1.1.2.2, 2D frictionless grasp requires at least four contact points. This means that the number of solutions itself is in $O(n^4)$. However, we provide an output sensitive algorithm which runs in $O(n^3 \lg^2 n + K)$ where $K$ is the number of output. Similarly, the case of 2D frictional grasp has the number of solutions in $O(n^3)$. An output sensitive algorithm which runs in $O(n^2 \lg^2 n + K)$ is provided for this case. Both of these algorithms exploit a priori knowledge that the grasps to be identified is for the same object. This fact allows us to use a special data structure that holds specific information about the object allowing the computation of force closure to be done efficiently. In a sense, the whole algorithms are designed specifically to compute all force closure grasps of the same objects. Additionally, we provide a separate algorithm for single query force closure test, i.e., given three frictional contact points, the algorithm identifies whether the grasp achieves force closure. To the best of our knowledge, this algorithm is the fastest algorithm for force closure testing in 2D frictional setting.

The cases of 3D grasp are different. We provide a necessary condition for force closure along with an efficient implementation. The implementation takes minuscule running time relative to the existing test of force closure. Hence, it can be used as a filtering criteria. When used in conjunction with a test of force closure, the whole algorithm takes less time than using only the existing test.

### 1.3 Dissertation Outline

In the next chapter, we provide a theoretical preliminaries on grasping which is used subsequently in the remaining of the dissertation. The remaining chapters describe algorithms to solve the problem in each setting. In each chapter, a brief introduction, literature reviews and conclusion are also given. Chapter 3 proposes a single query test of force closure in 2D frictional grasp. Chapter 4 and Chapter 5 describe algorithms to compute all force closure grasps of 2D object where contact is frictionless and frictional, respectively. The filtering criteria for 3D frictionless grasp and 3D frictional grasp are given respectively in Chapter 6 and Chapter 7. Finally, Chapter 8 concludes our work and describes future extension of our work.