CHAPTER 4

THEORY OF RISK AND GARCH MODEL

4.1 The Theory of Risk

Expected Rate of Return (E(r)): is the return that investors expect from their investment. Expected rate of return, (or required return) depend upon the level of the risk-free rate and the perceived risk in the industry or the economy. Investors would demand higher return to cover some risk. The investors face the inherent risk of not being able to precisely predict the market interest rate. This risk actually is related to fundamental variables, as the underlying company's financial condition, its potential eaming power, and so forth. These factors are unique to each company.

Risk is the opportunity that the investors would not get the return from their expectation. We can separate the risk to be two types. One is call systematic risk and the another is unsystematic risk.

4.1.1 Systematic Risk: is the risk that happens for the changing of political economy and social economy. This risk will be effected to the price of stock and it is nondiversifiable or unavoidable. Systematic risk, for example, purchasing power risk, market risk, political risk.

4.1.1.1 Purchasing Power Risk: is the risk that came from declining of purchasing power as a result of value of money is distorted during the inflation or hyper-inflation period. The risk from this investment is high for fixed income securities.

4.1.1.2 Market Risk: is the changing of economy, political, war and will make effect to the price of stock. Investors sometimes find the market is illiquid, so they lack of the ability to dispose of the investment quickly, at low cost, and without having to give much of price concession.

4.1.1.3 Country risk: which is the risk that a govemment might impose restrictions on the ability of the investor to receive the full benefits of equity ownership. (e.g. the expropriation of company assets, currency restrictions that prevent the receipt of dividend, or capital controls that limit the ability to obtain funds from the sale of the investment).

4.1.1.4 **Currency risk**: (for foreign investors), which is the risk that the currency in which the security is denominated may fall relative to the investor's home currency.

4.1.2 **Unsystematic Risk**: is the avoidable risk since these risk factors are unique to each company. Each company has difference fundamental factors such as financial strength, market power, and so forth. Some risks are described as followed:

4.1.2.1 **Interest Rate Risk**: is the risk that came from the changed of interest rate. Stocks, or index normally decrease when the interest rate increase (Reverse relationship) due mainly to investor would require the higher return as well as the risk premium. Investors demands extra premium above the risk free rate as compensation for assuming the risk of equity ownership.

4.1.2.2 **Financial Risk**: which is related to the way in which the company is financed and the impact of fixed financing costs on the total leverage of the firm. It can be cash flow, opportunity lost, liquidity of financial, low dividend payment and so on. Those will make the price of stock to go down.

4.1.2.3 **Management Risk (or Business Risk)**: is the inherent uncertainty associated with the economics of operating the business in which a particular company is engaged, independent of the way it is financed. Some example of management risk is business operating risk.

4.2 Theory of Generalized Autoregressive Conditional Heteroscedasticity (GARCH)

A generalized autoregressive conditional heteroscedasticity (GARCH) model, returns are assumed to be generated by stochastic process with time-varying volatility. Instead of modeling the data after they have been collapsed into a single unconditional distribution, a GARCH model introduces more detailed assumption about the conditional distributions of returns. These conditional distributions change over time in an autoregressive process.

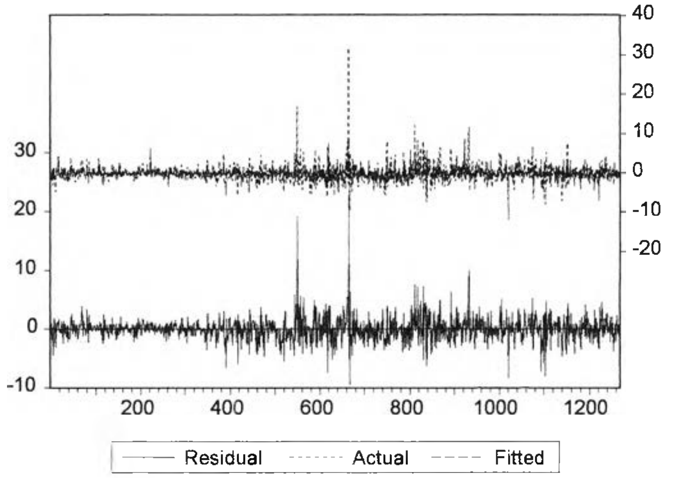
4.2.1 Volatility Clustering:

Many financial time series display volatility clustering, that is, autoregessive conditional heteroscedasticity. Equity, commodity and foreign exchange market often exhibit volatility clustering at the daily, even the weekly, frequency, and volatility clustering becomes very pronounced in intra-day data. Note that two types of news events are apparent. The second volatility clustering shows an anticipated announcement, witch turned out to be good news; the Market was increasingly turbulent before the announcement, but the large positive return at that time shows that punters were pleased, and the volatility soon decreased. The first cluster of volatility indicated that there is turbulence in the market following and unanticipated piece of bad news.

So volatility clustering implies a strong autocorrelation in squared returns, so a simple method for detecting volatility clustering is to calculate the first-order autocorrelation coefficient in squared return:

$$\frac{\sum_{t=2}^{T} r_t^2 r_{t-1}^2}{\sum_{t=2}^{T} r_t^4}$$

Volatility clustering



4.2.2 The Leverage Effect:

In equity markets it is commonly observed that volatility is higher in a falling market than it is in a rising market. The volatility response to a large negative return is often far greater than it is to a large positive return of the same magnitude. The reason for this may be that when the equity price falls the debt remains constant in the short term, so the debt/equity ratio increases. The firm becomes more highly leveraged and so the future of the firm becomes more uncertain. The equity price therefore becomes more volatile. The leverage effect as a secondary cause of the implied volatility skew in equity markets. It also implies an asymmetry in volatility clustering in equity markets; if volatility is higher following a negative return than it is following a positive return then the autocorrelation between yesterday's return and today's squared return will be large and negative.

4.2.3 The Conditional Mean and Conditional Variance Equations:

A simple linear regression can provide a model for the conditional mean of a return process. For example, in a factor model regression the expected value of a stock return will change over time, as specified by its relationship with the market return and any other explanatory variables. This expectation is the conditional mean. The classical linear regression model assumes that the unexpected return ε_t , that is, the error process in the model, is homoscedastic. In other words, the error process has a constant variance $V(\varepsilon_t) = \sigma^2$ whatever the value of the dependent variable. The fundamental idea in GARCH is to add a second equation to the standard regression model; the conditional variance equation. This equation will describe the evolution of the conditional variance of the unexpected return process, $V(\varepsilon_t) = \sigma^2_t$.

The dependent variable, the input to GARCH volatility model, is always a return series. Then a GARCH model consists of two equations. The first is the conditional mean equation. This can be anything, but since the focus of GARCH is on the conditional variance equation. It is usual to have a very simple conditional mean equation. Many of the GARCH models used in practice take the simplest possible conditional mean equation $r_t = c + \varepsilon_t$. In this case the unexpected return ε_t is just the mean deviation return, because the constant will be the average of returns over the data period. In some circumstances it is better to use a time-varying conditional mean, but must be very careful not to use many parameters in the conditional mean equation otherwise convergence problems are likely. If there is a significant autocorrelation in returns we should use an autoregressive conditional mean. If there is a structural break, where the mean return jumps to new level although the volatility characteristics of the market are unchanged, then a dummy variable can be included in the conditional mean. The second equation in a GARCH model is the conditional variance equation.

4.2.4 A Survey of Univariate GARCH Models:

There are many different types of GARCH models have been proposed in the academic literature, but not all have found good practical applications. The first autoregressive conditional heteroscedasticity (ARCH) model, introduced by Engle[1] (1982), was applied to economic data. For financial data it is more appropriate to use a generalization of this model, the symmetric GARCH introduced by Bolllerslev[2] (1986). Following this very many different GARCH models have been

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[1] R. Engle, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," Econometrica 50 (1982), pp. 987-1008.

[2] Tim Bollerslev, "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics 31 (1986), pp. 307-327.

developed, notably the exponential IGARCH model, one of the first asymmetric GARCH models to be introduced.

4.2.5 ARCH:

The ARCH (p) process captures the conditional heteroscedasticity of financial returns by assuming that today's conditional variance is a weighted average of past squared unexpected return:

$$\sigma^2_t = \alpha_0 + \alpha_1 + \varepsilon^2_{t-1} + \ldots + \alpha_p + \varepsilon^2_{t-p}$$

If a major market movement occurred yesterday, the day before or up to p days ago, the effect will be to increase today's condition variance because all parameters are constrained to be non-negative (and α_0 is constrained to be strictly positive). It makes no difference whether the market movement is positive or negative, since all unexpected returns are squared on the right-hand side.

ARCH models are not often used in financial market because the simple GARCH models perform so much better. In fact the ARCH model with exponentially declining lag is equivalent to a GARCH (1,1) model, so the GARCH process actually models an infinite ARCH process, with sensible constraints on ARCH (p) models to GARCH (1,1) as increases is illustrated.

As the lag increases in an ARCH model it becomes more difficult to estimate parameters because the likelihood function becomes very flat. Since we need very many lags to get close to a GARCH (1,1) model, which has only three parameters, the use of standard ARCH models for financial volatility estimation is not recommended.

4.2.6 Symmetric GARCH:

The full GARCH (p,q) model adds q autoregressive terms to the ARCH (p) specification, and the conditional variance equation takes the form

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \lambda_j \sigma_{t-1}^2$$

σ^2_t is the variance at time t. α_0 is the coefficient of variance at constant. α_1 is the coefficient of variance at time 1. And ε^2_{t-1} is the square of error term of one lag time. λ_j is the coefficient of variance at one lag of variance itself. σ^2_{t-1} is the one lag of variance at time t. sometime GARCH (1,1) can write in the form of GARCH (p,q).

The variance today depends upon three factors: a constant, yesterday's forecast variance (the GARCH term), and yesterday's news about volatility which is taken to be the squared residual from yesterday (the ARCH term), The (1,1) in GARCH (1,1) refers to one GARCH term and one ARCH term.

This specification makes sense in financial settings where an agent or trader infers today's variance by forming a weighted average of a long-term average or constant variance, the forecast from yesterday, and what was learned yesterday. If the asset return was large in either the upward or the downward direction, then the trader will increase the estimate of the variance for the next day. This specification of the variance equation incorporates the familiar phenomenon of volatility clustering, which is evident in financial return data. Large returns are more likely to be followed by large returns of either sign than by small returns.

If equation is lagged by one period and substituted for the lagged variance on the right hand side, then an expression with two lagged squared returns and a two period lagged variance is obtained. By successively for the lagged conditional variance, an illuminating expression is found:

$$\sigma_t^2 = \frac{\alpha_0}{1 - \lambda_1} + \alpha_1 \sum_{j=1}^{\infty} \lambda_1^{j-1} \varepsilon_{t-j}^2$$

An ordinary sample variance would give each of the past squares an equal weight rather than declining weights. Thus the GARCH variance is like a sample variance but it emphasizes the most recent observations. Since is the forecast variance one day ahead based on past information, it is called the conditional variance.