Chapter 3

Theoretical framework

In this chapter, the NATREX model in small country case will be explained in more detail. There are four sections. In the first section we introduce the concept of NATREX. The market structure for the case of small country will be considered in the second section. The third section shows the relationship of non-tradable relative price and real exchange rate. Finally, in the fourth, the mechanism of adjustment of non-tradable relative price and real exchange rate when there are changes in fundamental variables will be explained.

3.1 Concept of NATREX

The concept of NATREX is the equilibrium real exchange rate that clears the balance of payments in the absence of cyclical factors, speculative capital flows, and movements in international reserves.

NATREX is based on the national-income-accounts equation,

$$I - S + CA = 0, \tag{3.1}$$

where I is desired national investment,

S is desired national saving,

CA is the desired current account.

By the word "desired", the above three variables are compatible with the state that the economy is at capacity output and expectations about inflation are met. The real exchange rate appreciates in response to excess demand for goods, insuring that equilibrium is maintained.

Desired investment minus desired saving, I-S, always discribes the sum of the country's excess flow supplies of financial assets. Under these condition, I - S describes desired net long-term capital inflows, and condition (3.1) is sum of current and capital accounts, or balance-of-payment equilibrium. The equation (3.1) can be interpreted as the equilibrium condition for balance of payment, as well as for goods market.

Desired investment and saving depend on the existing stocks of capital, wealth, and net debt to foreigners. When these stocks are changing, an equilibrium exchange rate becomes a moving equilibrium. NATREX models focus on the results of investment, saving and net capital flows over time.

3.2 The market structure of the model in a small country case

The market structure of the general NATREX model for small country can be summarized as follows.

(a) There are two countries; one small country (in terms of supply of its export goods), taking as the home country trading with one big country (the rest of the world), taking as the foreign country.

(b) There are two types of goods; one produced for domestic consumption only, the other can be as well traded internationally when domestic demand and supply are unbalanced. For short, the goods produced for domestic consumption will be called "non-tradable goods", and the other will be called "tradable goods". Further more, tradable goods can be classified into import able good and exportable goods. Aggregately, for distinction, non-tradable goods will be denoted as good n, exportable goods as good 1, and importable goods as good 2. In other word, the home country produces good n for domestic consumption, export good 1 to and import good 2 from the rest of the world.

Denote excess demand of good n by X_n, of good 1 by X_1, and of good 2 by X_2. Distinguish the excess demands in country by having X_i and X_i' as domestic and foreign excess demands for good i respectively. The national-income-accounts equation can be expressed in terms of excess demands as

$$I - S + CA \equiv (X_n + X_1 + X_2) + (X_1' + X_2'), \qquad (3.2)$$

For the home country, the prices of tradable goods are given by the world market prices, and the excess demands for good 1 and good 2 are perfectly elastic at world prices by the small country assumption. That is to say, the home country can export and import all as much as desired at the fixed prices. Then export of the home country will always be equal to import of the foreign country, or $X'_1 = - X_1 > 0$. Similar result holds for the import good, $X'_2 = -X_2 < 0$. Therefore, the excess demand for all goods become the excess demand for non-tradable,

$$I - S + CA \equiv (X_n + X_1 + X_2) + (- X_1 - X_2) \equiv X_n \qquad (3.3)$$

The equilibrium condition of the home country (3.1) can be determined by the equilibrium condition of the non-tradable market.

3.3 The real exchange rate and non-tradable relative price

In section 3.2 we use the national income account through the consideration of the market structure. In this section we look at relationships of the price in each market both from domestic and international perspectives, and see how nominal and real exchange rates are determined.

As in the previous section, denote the corresponding price for non-tradable good by p_n, for export good 1 by p_1, for import good 2 by p_2, p_i and p'_i distinguish the home and foreign countries' prices. Generally, the excess demands in the three markets can be determined by the two relative prices. One is p_1 in term of p_2 for the tradable market. The other is p_n in term of p_1 for the non-tradable market. Since p_1 and p_2 are given at the world market prices, the good market excess supplies in the home country is determined solely by the relative price p_n in term of p_1. The equilibrium condition for (3.3) is, therefore, the relative price p_n in term of p_1 such that $X_n = 0$.

To see how the equilibrium real exchange rate is related with the non-tradable market equilibrium price, we consider the followings. For the simplicity of the analysis, we assume that the home country produces only non-tradable good n and the tradable good 1 while the foreign country produces her non-tradable good n and the tradable good 2. If price deflator is assumed to be a geometric average of the prices of the goods produced, equation (3.4) expresses the price

deflator for the home country and equation (3.5) expresses the corresponding relation for the foreign country.

$$P = p_n^{a}\, p_1^{(1-a)}, \tag{3.4}$$

$$P' = (p'_n)^{b}\,(p'_2)^{(1-b)}, \tag{3.5}$$

Where a is the weight given to the importance of the good n in the home country, and b is the corresponding figure in the case of the foreign country.

The terms of trade T is the ratio of the price of export good 1 to import good 2 measured in a common currency as given in equation (3.6). Non-tradable relative price is the ratio of non-tradable price to export good 1 in equation (3.7).

$$T = Np_1/p'_2 \tag{3.6}$$

$$R_n = p_n/p_1 \tag{3.7}$$

The real exchange rate can be defined as in equation (3.8). The rise in either N or R signifies an appreciation of home currency.

$$R = N\,(P/P') \tag{3.8}$$

when N = nominal exchange rate ($US/BATH)

R = real exchange rate

P = Thailand GDP deflator

P' = the foreign (in our study, the US) GDP deflator.

The real exchange rate R is arithmetically related to both the exogenous terms of trade T and to $R_n = p_n/p_1$, as stated in (3.8). Since the terms of trade T and

the relative prices (p'_n/p'_2) in foreign country are exogenous for home country, endogenous variations in the real exchange rate correspond to endogenous variations in the ratio of the price of non-tradable to export goods. Normalize the mean of the exogenous term $c = (p'_2/p'_n)^b$ in the foreign to unity, we obtain the last term in (3.9). The equation (3.10) express the real exchange rate in logarithms; and e is the log of the random exogenous term, $c = (p'_2/p'_n)^b$.

$$R = T(R_n)^a(p'_2/p'_n)^b = T(R_n)^a c \sim T(R_n)^a \tag{3.9}$$

$$\log R = \log T + a \log R_n + e \tag{3.10}$$

The weight given to the non-tradable sector in the price deflator is exponent a. The smaller is a, the closer are the real exchange rate and the term of trade. It is arithmetically true that real exchange rate is determined by exogenous T and the non-tradable relative price.

3.4 The relationship of non-tradable market and real exchange rate

In analysis, the non-tradable balance can be present as (3.11) and the basic assumption are presented above.

$$C_n(R_n, k - F; s, T) + (1 - m)\, I(k, T, R_n, F; r', y) = Q_n(R_n, k; y) \tag{3.11}$$

The left hand side of (3.11) the demand for non-trade good consists of consumption C_n and investment (1- m)I. The consumption function[1] depends on non-tradable relative price, wealth equal to capital less foreign debt (k-F), parameter s is an index of thrift, and terms of trade. Most of the variables relate to consumption in negative direction except the wealth.

Fraction (1-m) of investment consists of the non-traded goods. Hence (1-m)I is the demand for non-tradable good used for investment. This equation derive from optimal control and optimize dynamic programming (see appendix A). The variables that determine investment in non-traded production are non-tradable relative price, term of trade, productivity, capital stock, debt, and foreign interest rate. Most of the variables relate to investment in negative direction, except the productivity and capital stock.

On the right hand side is the supply of the non-traded good Q_n which depends on non-tradable relative price, capital stock and productivity. All variables related to the supply of non-traded goods in positive direction.

Box 3.1 is summary of the equations that are used to derive the non-tradable relative price and real exchange rate model.

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[1]This equation is derived from optimal control and optimization dynamic programming. For details see Jerome L Stein, "The Natural Real Exchange Rate of the United States Dollar, and Determinants of Capital Flows," In Fundamental determinants of exchange rates, ed. Jerome L Stein, Polly Reynolds Allen, and associates, (New York: Oxford University Press, 1995), pp. 50-55.

Box 3.1 The NATREX model for a small open economy

Goods market clearing = balance in non-tradable:

$$I - S + CA = 0 \qquad (3.8)$$

$$C_n(R_n, k\text{-}F; s,T) + (1\text{-}m)I(k,T, R_n,F;r', y) = Q_n(R_n, k;y) \qquad (3.11)$$

Real exchange rate:

$$R = T(R_n)^a \qquad (3.6)$$

Investment equation:

$$dk/dt = I = I(q); \qquad I(1) = 0,\ I'>0 \qquad (A.1)$$

$$I = I(q) + nk = I(k, T, R_n, y, r)$$

$$I_k < 0,\ I_T > 0,\ I_r < 0,\ I_y > 0,\ I_{Rn} < 0$$

Capital inflow:

$$dF/dt = I - S - nF$$

Saving equation:

$$S = Y(k; y) - r'F - C(k\text{-}F; 1\text{-}s) = S(k, F; Z) \qquad S_k > 0\ \ S_F > 0 \qquad (A.3)$$

Portfolio balance:

$$r = r' + h(F, t) \qquad (A.4)$$

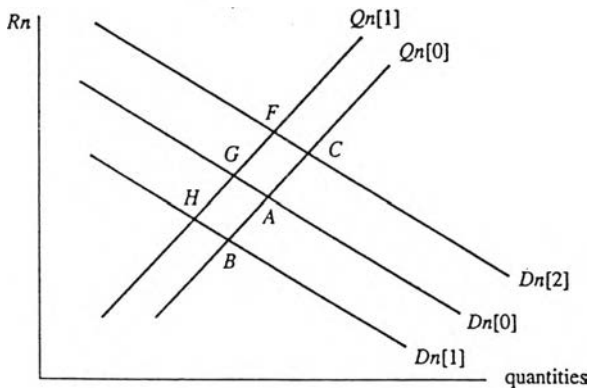
Symbols: all quantity variables are measured per unit of effective labor in the entire economy; k capital per worker; Y(k;y) real GDP per worker; F real foreign debt (+) per worker; R real exchange rate (rise is appreciation); $R_n = p_n/p_1$ = relative price of non-tradable to export good, I = investment per worker; S = saving per worker; r(r') real domestic (world) interest rate; B real trade balance per worker; T terms of trade = p_1/p_2 ;s thrift parameter; y productivity parameter; Z = (T, s, y, r').

Source: Guay C Lim and Jerome L Stein, (1995). pp101.

By the assumption perfect elastic in export and import good, to find the equilibrium real exchange rate we need only to solve for equilibrium in the non-tradable market. The relative price of non-tradable $R_n = p_n / p_1$ equilibrates the market for non-tradable in equation (3.11). The relative price R_n is a function of both the exogenous variables Z, and capital and debt which are endogenous over a longer period. Change in the parameters $Z = (T, s, y, r')$ have both direct impacts and indirect impacts upon the real exchange rate because they effect the evolution of capital and debt. The evolving values of capital and debt produce new solution for R_n in equation (3.11).

In a small open economy, the terms of trade are exogenous. Since the market for tradable is always in equilibrium at the exogenous terms of trade, equilibrium in the goods market, $I - S + CA = 0$, follows when the market for non-tradable is in equilibrium. Using equations (3.11), the equilibrium in the market for non-tradable (3.12) is graphed in Figure 3.1.

Figure 3.1 The relative price of non-tradable



Source: Guay C Lim and Jerome L Stein, (1995). p.107.

The demand for non-tradable D_n , on the left-hand side of (3.11) is negatively related to R_n, because a rise in the relative price of non-tradable decreases the quantity demanded both of non-tradable consumption and investment.

The supply of non-tradable Q_n , on the right hand side of (3.11) is positively related to R_n, because a rise in the relative price of non-tradable increase the quantity supplied. Both the demand and the supply of non-tradable (Figure 3.1) depend upon the exogenous variables Z and the evolving endogenous variables capital and debt.

Solve (3.11) for the relative price of non-tradable we obtain (3.12). The relative price, R_n is a function of capital, debt, and the exogenous variables Z.

The evolving values of capital and debt with investment and saving, thus producing a trajectories of NATREX and current account deficit or surplus, produces new solution for R_n in equation (3.12).

$$R_n(t) = R_n[\, k(t), F(t); Z(t)]; \quad Z = (\, T, s, y, r' \,) \tag{3.12}$$

The real exchange rate R(t) is (3.13), base upon (3.12) and (3.6).

$$R(t) = T[R_n(k(t),F(t);Z(t))]^a = R[\, k(t), F(t); Z(t)] \tag{3.13}$$

The evolution of capital and debt shift the demand and supply of non-tradable curves and thereby generate a trajectory of $R_n(t)$ and R(t). The evolution of the real exchange rate is (3.14). There are two basic determinants of the real exchange rate. The direct effect are the last term and the indirect effects are the terms in square brackets.

$$dR/dt = [R_k dk/dt + R_F dF/dt] + R_Z dZ/dt \tag{3.14}$$

where R_k, R_F, and R_Z are partial derivatives with respect to capital , debt, and fundamental variables respectively.

As capital and debt evolve[2], the demand D_n and supply Q_n curves for non-tradable in figure 3.1 shift. The value of the relative price of non-tradable which

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[2]The detail can see Guay C Lim and Jerome L Stein, "The Natural Real Exchange Rate of the United States Dollar, and Determinants of Capital Flows," In Fundamental determinants of exchange rates. ed. Jerome L Stein, Polly Reynolds Allen, and associates, (New York: Oxford University Press, 1995), pp. 108-110.

produces equilibrium in the market for non-traded goods changes. The evolution of capital dk/dt is equation (3.15), obtianed by using investment function , portfolio balance function, and relative price of non-tradable function(3.12) .

$$dk/dt = J(k, F ; Z) \qquad J_k < 0 \quad J_F < 0 \qquad (3.15)$$

The evolution of foreign debt dF/dt, which is the capital inflow, is equation (3.16) obtianed by using the evolution of capital(3.15) and saving function. Function L =J -S, is investment less saving.

$$dF/dt = J - S = L(k, F ; Z) \qquad L_k < 0 \quad L_F < 0 \qquad (3.16)$$

$$\text{when} \qquad S = S(k, F ; Z) \qquad S_k > 0 \quad S_F > 0$$

$$G = (J_k L_F - J_F L_k) > 0.$$

Hence the evolution of real exchange rate (3.14) is (3.17).

$$dR/dt = [R_k J(k, F ; Z) + R_F L(k, F ; Z)] + R_Z dZ/dt \qquad (3.17)$$

Equation (3.17) is the complete dynamical system concerning the evolution of the real exchange rate, capital, and the foreign debt. The current-account deficit is dF/dt + nF. In the steady state dk/dt = J = 0 and dF/dt = L = 0, as described by (3.18) and (3.19).

The simultaneous solution of (3.18) and (3.19) implies (3.20) and (3.21), that the steady-state value are function of Z. The steady state relative price of non-tradable is (3.22) and the real exchange rate is (3.23), vector Z = (T, s, y, r') of the fundamental is exogenous.

$$J(k^{\bullet}, F^{\bullet}; Z) = 0 \quad J_k < 0 \; J_F < 0 \tag{3.18}$$

$$L(k^{\bullet}, F^{\bullet}; Z) = 0 \quad L_k < 0 \; L_F < 0 \tag{3.19}$$

$$G = (J_k L_F - J_F L_k) > 0$$

$$k^{\bullet} = k(Z) \tag{3.20}$$

$$F^{\bullet} = F(Z) \tag{3.21}$$

$$R^{\bullet}_n = R_n(k(Z), F(Z); Z) = R^{\bullet}_n(Z) \tag{3.22}$$

$$R^{\bullet} = T(R^{\bullet}_n)^a = R^{\bullet}(Z) \tag{3.23}$$

Figure 3.2 the trajectories of capital and debt to steady state

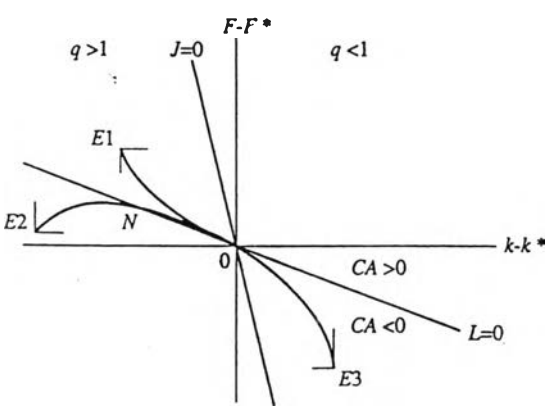

Figure 3.2 Trajectories of capital and debt to steady state, the origin. Productivity produces trajectory E2 -N-0; Saving produces E1-0; term of trade produces E1- 0 or E2-N-O; world rate of interest produces E3-0.

Source: Lim and Stein (1995) pp109.

The curve J = 0 in Figure 3.2 describes (3.20): it is the set of capital and debt at which the rate of net investment is zero. It is negatively sloped because a rise in investment lowers the marginal product of capital and tends to make net investment negative. A decline in debt F reduces the risk premium h(F) in the portfolio balance equation, and lowers the real rate of interest to restore the rate of net investment to zero. To the left (right) of J = 0, net investment is positive (negative), as indicated by the vectors. The L = 0 curve in the equation (3.21) is the set of capital and debt at which the capital flow dF/dt = I - S = - CA is zero. It is also negatively sloped if $(S - J)_k > 0$. A rise in capital raises saving less investment, produces a capital outflow, and decreases the debt. Above (below) the L = 0 curve, there are current-account surpluses (deficits) and the debt declines (rises).

For stability, several conditions must be satisfied.

(1) A rise in the capital intensity lowers the marginal product of capital and adversely affects investment.

(2) A rise in the debt, given capital, must rise saving relative to investment, produces a capital outflow, and reduces the debt back to the L = 0 curve.

(3) The L = 0 curve must have an algebraically greater slope than the J = 0 curve.

Table 3.1: Steady- state effects upon capital and debt of changes in Z

Z	$dk^{\bullet}/dZ$	$dF^{\bullet}/dZ$	J_Z	$L_Z = (J - S)_Z$	$\partial R_n/\partial Z$
T	+	-	+	+	?
y	+	-	+	+	$y_n < 0, y_t > 0$
s	+	-	0	-	-
r'	-	+	-	-	-

Note $G = (J_k L_F - J_F L_k) > 0$, $dk^{\bullet}/dZ = (J_Z S_F - J_F S_Z)/G$, $dF^{\bullet}/dZ = (J_k S_Z - J_Z S_k)/G$.

The effect J_Z and L_Z refer to the effect of a rise in Z upon the investment function and upon the capital inflow, investment less saving, respectively. The effect $\partial R_n/\partial Z$ is the direct effect of a rise in Z upon the relative price of non-tradable, given capital and debt.

Each disturbance Z has a direct effect upon the relative price of non-tradable, $\partial R_n/\partial Z$ which is the first term in (3.24) and the last column in Table 3.1. There is also an indirect effect operating upon the subsequent changes in capital and debt, the first two columns in Table 3.1, which correspond to the $dk^{\bullet}/dZ$ and $dF^{\bullet}/dZ$ term in (3.24).

$$dR_n/dZ = \partial R_n/\partial Z + [(\partial R_n/\partial k)\, dk^{\bullet}/dZ + (\partial R_n/\partial F)\, dF^{\bullet}/dZ] \qquad (3.24)$$

Exogenous fundamental disturbances (Z)-such as changes in thrift and productivity, changes in the terms of trade and the world real interest rate - influence the non-tradable relative price and NATREX in two ways. They first

affect desired investment, saving, or the current account, inducing a change in the non-tradable relative price and NATREX in the medium run(or direct effect). Second, by changing the rates of accumulation of k, w, and F, the exogenous fundamentals alter the trajectory of the non-tradable relative price and NATREX as it moves toward its new long-run equilibrium. The impact of fundamental variables on the non-tradable relative price and real exchange rate can be shown in the followings.

(a) **Thrift**. A rise in saving ratio decreases the demand for consumption of non-tradable and the relative price of non-tradable declines. From figure 3.1, A is the starting point. A rising in saving will shift down the demand curve to point B. The real exchange rate depreciates. Since an increase in saving is reflected by a decrease in real interest rate.

The long run effect (Table 3.1) is to decrease debt and increase capital. A rise in the saving less investment results in exchange rate depreciation, current account surplus, and capital outflow. The initial rise in saving which lowers the real interest rate may stimulate some capital formation. The economy is initially at the point E1 in the Figure 3.2 with trade surplus and capital outflow. Capital outflow reduces the debt which, in turn, reduces the risk premium and stimulates capital formation. The decline in the debt lowers saving (this is a stability condition) which offsets the initial rise in the saving ratio. The endogenous steady state debt stabilizes at a lower level. The decline in debt and rise in capital further affect the market for non-tradable. Wealth rises the demand for non-tradable, and make the relative price of non-tradable and the real exchange rate appreciate. The rise in capital has ambiguous effect upon the supply of non-tradable, depending

upon which sector is capital intensive. If the non-tradable sector is labor intensive, the rise in capital decreases the supply of non-tradable. Thus the relative price rises. In that case, both of the indirect effect raise the relative price of non-tradable. If the indirect effect is larger than the direct effect, the relative price of non-tradable rises. But if the direct effect is larger than the indirect effect, the non-tradable relative price will decrease and real exchange rate will depreciate

(b) **Productivity**. . If we assume that the productivity parameter y_n in non-tradable sector is larger relative to that in the tradable sector, an increase in the productivity of the non-tradable sector will result in a rise in the capital intensity k_n and fraction of labor force L_n/L in non-tradable goods. The supply of non-tradable goods rises and the relative price of non-tradable goods decreases. Therefore, the effect of a rise in productivity is to depreciate the relative price of non-tradable and also to depreciate the real exchange rate. This is the direct effect.

The rise in y_n raises the marginal product of capital in the non-tradable goods sector. This raises investment relative to saving through the rise in q-ratio. The rise in y_n shifts resources from the tradable sector to the non-tradable sector, and decreases the supply of export good. Hence the trade balance declines. The current account deficits as a result of worsening trade balance raise the debt which has been financing capital formation. As capital accumulates, the q-ratio declines and output gradually rises. The latter tends to raise saving relative to investment. When saving exceeds investment, there are capital outflows. Along the trajectory capital rises and debt declines. The rise in wealth increases the demand for non-tradable, but the rise in y_n was the cause of the rise in capital. Hence the capital invested in the non-tradable sector rises and further increase the supply for non-

tradable. The indirect effects are to increase the demand for non-tradable, but the supply increases by even less. If the indirect effects are more than direct effect, the total effect dR_n/dy_n is to appreciate the relative price of non-tradable. Hence the rising in the relative price of non-tradable also produces a appreciation in the real exchange. But if the indirect effects are less than direct effect, the non-tradable relative price and real exchange rate are depreciation.

(c) **Terms of trade**. A rise in terms of trade $T= p_1/p_2$ means a rise in $p_n/p_2=TR_n$, the relative price of non-tradable to the import good. The rise in p_n/p_2 decreases the consumption of non-tradable good. At the same time, the rise in the terms of trade raises the q-ratio and increases investment demand. The non-traded component of investment increases. The net medium-run effect of a rise in the terms of trade upon the demand curve and hence the relative price of non-tradable is ambiguous. However, we assume that the consumption effect dominates. Then, the rise in the terms of trade reduces the demand and reduces the relative price of non-tradable.

The price effect is to increase the current account. There is capital accumulation and the current account surpluses. It is possible that the rise in investment generated by the terms of trade exceeds saving and produces a current account deficit. In either case, capital accumulation leading to a higher capital stock. The steady state debt also declines. In one case, the movement is monotonically convergent to the origin. In the other case, the debt first rises and then declines. Thus the economy will have a higher capital and debt less. The medium-run effect shifted the equilibrium in non-tradable. The rise in capital will affect both the supply of and demand for non-tradable. Since the non-tradable

sector is labor intensive, the supply will decrease. The decline in debt raises wealth and increases the demand for non-tradable. Therefore indirect effect will produce non-tradable relative price depreciate. The net total effect of a rise in the terms of trade is a negative effect upon the relative price of non-tradable if the indirect effect is larger than the direct effect. Then, real exchange rate will depreciate. But if the direct effect is larger than the indirect effect, the non-tradable relative price and real exchange rate both appreciate.

(d) **World interest rate**. A rise in the rate of interest r' reduces investment demand, since some of the capital goods are non-tradable. The demand for non-tradable declines as a result of the decline in investment, and the relative price of non-tradable declines. The capital intensity is declining due to the fall in q-ratio. The rise in the rate of interest produces current account deficits and the debt rises. Since capital has diminished and non-tradable sectors is labor intensive, the supply of non-tradable good increases, the relative price of non-tradable good depreciates. Similarly, the decline in wealth k-F further reduces the demand for non-tradable. The sum of all these effects is to reduce the long run relative price of non-tradable and the real exchange rate.