Chapter 7

Estimating the Elasticity of Labour Demand

Having estimated the production function parameters, we then proceed to estimate the firm's responsiveness to an increase in the price of labour hours.

The response comprises two elements. Those are the substitution toward a more capital-intensive production (substitution effect) and a change in employment due a change in production output (scale effect).

That the substitution effect and scale effect determine the elasticity of demand was discussed earlier in section 2.2.

We defined the "own price elasticity of factor input demand" (γ) as:

$$\gamma = \tau + \eta \qquad (2.26)$$

τ: is the own-price elasticity for constant output (being derived from the production function).

η: is the own-price elasticity for variable output (being derived by examining the output price elasticity and the elasticity of production cost on increase in total cost of operation).

In chapter 1 and 2 the expressions were derived for the general factor demand without considering the types of input demands. This chapter is concerned with labour hours, therefore; x_i changes to Lh and v_i changes to C_{Lh}.

7.1 Own Price Elasticity for Constant Output

In section 2.4 we found that for the Cobb-Douglas function the own-price elasticity for constant output is defined as

$$\tau_{x_i} = \frac{\alpha - s}{s} \qquad (2.44)$$

s: is the sum of the factor input exponents ($s= \alpha + \beta$) and, as we saw in *equation 2.33*, equal to the returns to scale of the Cobb-Douglas production (ε). α was estimated to be 0.92 and β to be 0.625 in section 6.4.

The labour hour's own-price elasticity for constant output is estimated as:

$$\tau_{Lh} = \frac{0.92-(0.92+0.625)}{(092+0.625)} = -0.405 \qquad (7.1)$$

7.2 Own Price Elasticity for Variable Output

It was discussed in section 2.4 that since the Cobb-Douglas function is homogeneous (elasticity of scale does not vary with output or factor proportions) there is no profit maximising point for increasing returns to scale and perfect output market. That is exactly the case in this study. Therefore, the output scale elasticity needs an additional assumption incorporated: Since the factor share income is constant over time the measured scale elasticity shall be interpreted as the output change the firm will response with in order to keep the same income share. That is, that the company wants as a minimum to keep the same contribution margin.

$$\eta = e_{x_i,q} \cdot e_{q,P} \cdot e_{P,MC} \cdot e_{MC,v_i} \qquad (2.29)$$

In the case where all firms, in competitive markets, experience the price: the increase in labour-hours cost is directly reflected in prices. Since, in a competitive market the firms are assumed price takers ($P=MC$) and therefore operates on "a flat cost curve".

However, as discussed earlier, this study assumes that only the Danish Steelworks experiences the change in cost of labour hours. Therefore, the price of output will not change, and hence, the last three terms in *equation 2.29* requires some reinterpretation[1].

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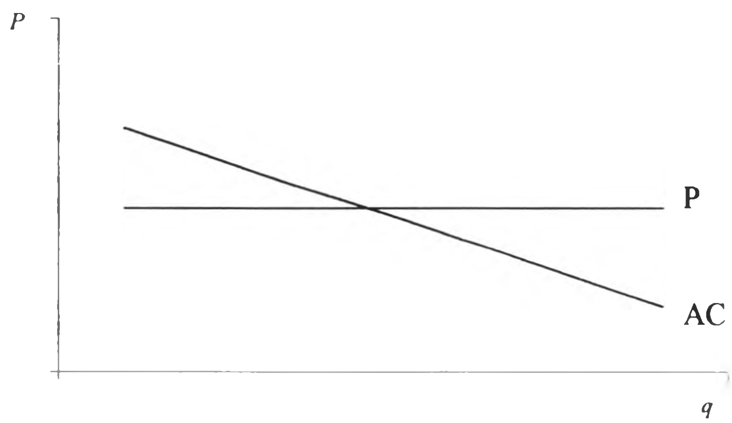
[1] Walter Nicholson. Microeconomic Theory. (The Dryden Press. Harcourt Brace College Publishers. 1998) p. 649.

Output elasticity of labour-hours demand (the first term in *equation 2.29*) is equal to the inverse of output elasticity of labour hours (α), since the K-Lh ratio is constant for movements along the expansion path:

$$e_{Lh,q} = \frac{\partial Lh}{\partial q}\frac{q}{Lh} = \frac{\partial \ln(Lh)}{\partial \ln(q)} = \frac{1}{\alpha} = \frac{1}{092} = 1.09 \qquad (7.2)$$

Reinterpretation of the last three terms. The elasticity of scale of the production was, in section 6.3, found to be 1.545. The firms cost is then characterised by decreasing average-cost with increasing output. Moreover, the cost function is linear due to the honmogenity of production function. Since the firm prices its products equally the market price, the firm's relationship between price and demand is as depicted in *figure 7.1*.

Figure 7.1 Cost-price determined output

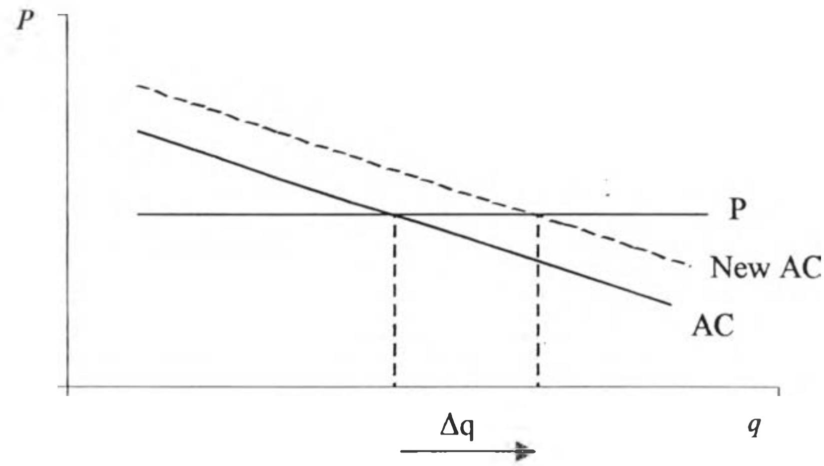


An increase in cost of labour hours will shift the cost curve upward and create a new break-even on a higher output. Therefore, an increase in cost of labour hours will in the long run, *ceteris paribus*, force the firm to increase output (see *figure 7.2*) and it is therefore interesting to estimate the elasticity of output with respect to cost of labour hours ($e_{q,C_{Lh}}$), which is equivalent to the last three terms in *equation 2.29*

Elasticity of output with respect to cost of labour hours is defined:

$$e_{q,C_{Lh}} = \frac{\textit{percent increase in } q}{\textit{percent increase in } C_{Lh}} \quad (7.3)$$

Figure 7.2 New cost-price determined output



Elasticity of cost is defined as:

$$\varepsilon_c = \frac{\partial TC}{\partial q} \cdot \frac{q}{TC} = \frac{\textit{percent change in } TC}{\textit{percent change in } q} \quad (7.4)$$

The labour-hours cost's share of total cost (cs_{Lh}) can tell us how much an increase in C_{Lh} will lead to an increase in TC (variable total cost in the short run). That is:

$$\textit{percent change in } TC = cs_{Lh} \cdot \textit{percent change in } C_{Lh} \quad (7.5)$$

Therefore, rewriting *equation 7.4* with respect to *percent increase in q*:

$$\textit{percent change in } q = \frac{cs_{Lh} \cdot \textit{percent increase in } C_{Lh}}{\varepsilon_C} \quad (7.6)$$

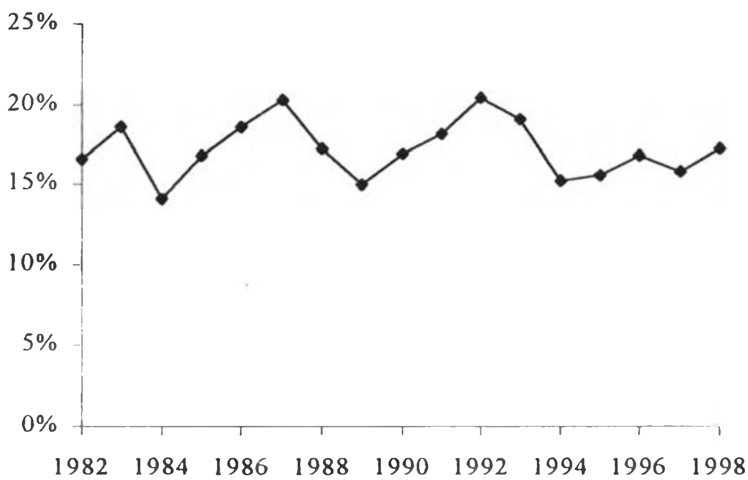
Substituting *equation 7.6* into *7.3* yields the final expression for the output elasticity with respect to cost of labour hours:

$$e_{q,C_{Lh}} = \frac{cs_{Lh}}{\varepsilon_C} \qquad (7.7)$$

The determinant (ε_C) of *equation 7.7* is determined as follows: Since total cost (*TC*) is a linear function of *q* and the production function is homogeneous of degree 1.545, and hence, the elasticity of scale (ε) is constant and equal to the degree of homogeneity (ε=1.545). Therefore, the cost elasticity with respect to output (ε_C) is the inverse of ε ($\varepsilon_C=1/\varepsilon=0.647 \approx 0.65$).

The numerator of *equation 7.7* is determined as follows: The share of "cost of labour hours" on total cost of operation has stayed constant over the observed period[2]. The share is depicted below in *figure 7.3*.

Figure 7.3 Labour-hours' income share of total revenue



source: the Danish Steel Works financial statements

The average share (cs_{Lh}) for the observed period is 17 percent. The estimated output elasticity with respect to cost of labour hours is then:

[2] The numbers are taken from the Danish Steelworks' financial statements, where it is assume that the cost of labour hours is equal to other wages and that the labour hours count for 80 percent of total hours.

$$e_{q,C_{Lh}} = \frac{cs_{Lh}}{\varepsilon_C} = \frac{0.17}{0.642} = 0.265 \qquad (7.7)$$

The **"own price elasticity of factor input demand for variable output"** (γ) can then be estimated:

$$\eta = 1.09 \cdot 0.265 = 0.29 \qquad (7.6)$$

7.3 The Elasticity of Labour Demand

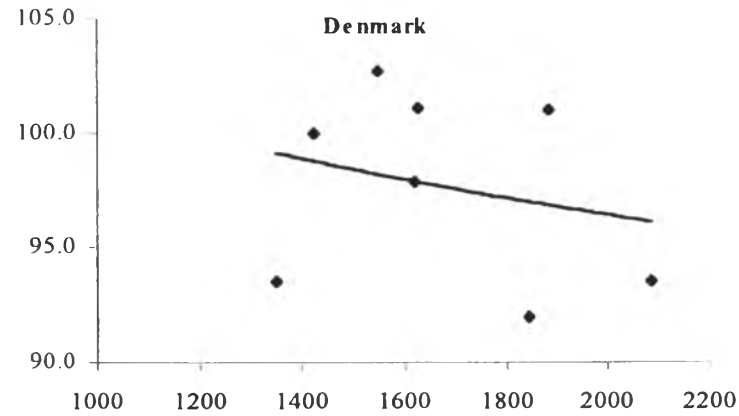
The "own price elasticity of factor input demand" (γ) or just "the elasticity of labour hours demand" is found from *equation 2.26*:

$$\gamma = \tau + \eta = -0.405 + 0.29 = -0.115 \approx -0.12 \qquad (7.7)$$

The interesting question now is whether this rather surprising result suggests that the scale effect have a significant positive effect on the elasticity of demand for labour hours. is in conformity with the Danish Steelworks' behaviour? The findings are discussed below.

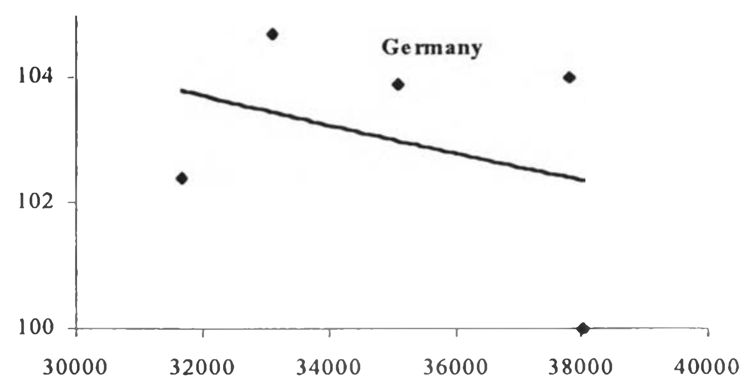
Regarding the output-price relationship: Examining the output (the horizontal axis) and steel price-index (the vertical axis) data from 1987 to 1996: suggests a lag of 2 years from prices drop to the consumption follows up (se *figure 7.4* and *7.5*).

Figure 7.4 Output-price relationship Denmark



source: EUROSTAT

Figure 7.5 Output-price relationship Germany



source: EUROSTAT

However, the firm's financial statements explain: That cyclical movements in USA and the East Asian markets, and the exchange rate fluctuations historically influence the European steel market. In addition, there is a historically problem of excess capacity in Europe because of heavy state subsidies as well as price dumping from third countries.

For that reason, it is likely that prices are determined by consumption and not *vice versa* and the assumption that the Danish Steelworks act as price takers is not so heroic[3].

[3] The 1989 financial statement reports a price increases of 16 percent with a 3 percent output increase as a direct follow and the 1995 financial statement tells about 10 percent price increase and a 5 percent output increase.

Regarding the positive sign of the scale elasticity: the firm's observed response to decreasing output prices and increasing labour hours cost is down-adjustments of its output, and hence, also labour hours. However, those adjustments are only short run adjustments.

The financial statements report that the company craves for an expansion of its production from the present 700,000 tonnes per year to 1,000,000 tonnes, but has so far not been able to achieve approval. This is because the European steelworks operate under a quota system and under regulation of their respective country's environmental laws.

7.4 Interpreting the result

Due to above discussion the estimated responsiveness: τ= -0.405 is interpreted as the *observed* substitution behaviour to increases in cost of labour hours. In contrast to τ the estimated η= 0.29 does not catch the actual behaviour but is interpreted as (under assumptions of no output and capital restrictions): the reaction the firm will do, in order to keep as a minimum the same contribution margin, in the long run.