CHAPTER V
RESULTS AND DISCUSSION

The modeling of shooting method with Euler's Method has been used to analyze the capillary rise in annular geometry following the Young-Laplace theory. While the results from the force-balance analysis and experiment are useful comparisons as well as assertions to the modeling mode.

These phenomena, capillary rise in annular geometry, were investigated in a system of water-air-glass at 25°C where,

Density of water, ρ, is 1,000 kg/m^3 (Holman, 1997)

Surface tension of water, γ, is 70.63 mN/m (Experimental result)

The contact angle between water and glass is 10° (Middleman, 1998)

Gravitational acceleration, g, is 9.81 m/s^2 (Wilkes, 1999)

5.1 Vertical Annular Tube Alignment

The meniscus shapes in different gap width of annular tubes are simulated by the model with circular annulus assumption presented in Figure 5.2. This prediction here is suitable and rational to identify the meniscus shape by the assumption of $R_2 >> R_1$ as shown in Appendix D. The results showed that the meniscus shape was varied when the gap width was changed, while the results obtained from the model together with the outcomes from the force-balance analysis, by Equation (C.1), and the experiment are presented to draw a comparison in Table 5.1.

Comparing the meniscus shapes in inner/outer tube size of annular tubes at 10/15 mm and 15/20 mm, the results show that meniscus configurations are the same no matter how big or small the annular tube is as long as its gap width is constant. These incidents result from the pressure difference across an interface, hydrostatic pressure and the pressure due to the curvature at that point, which can be described by Equation (3.2). If the meniscus heights are equal, so their meniscus shapes are identical because the meniscus height is directly related to the meniscus shapes.

Table 5.1 Results of circular annulus surface analysis for vertical annular tube alignment

Inner/Outer Tube Diameter (mm)	Gap (mm)	Meniscus Height (mm)		
		Young-Laplace	Force-Balance	Experiment
7/10	0.5	28.93733	28.91178	27.96 ± 1.93
10/15	1.5	9.640112	9.63726	9.58 ± 0.64
15/20	1.5	9.640112	9.63726	9.57 ± 0.03

In addition, the smaller gap width gives superior competency for water rising in the gap tube. As the results, the meniscus height that is from inner/outer tube at 7/10 mm is higher than that for both the inner/outer tube of 10/15 mm and 15/20 mm. These results agree with the force-balance analysis and experiment.

5.2 Inclined Annular Tube Alignment

The meniscus height in inclined annular tube was derived from the circular annulus model with the assigned dimension and inclination of annular tube. The results from the model of different gap width and inclination of annular tubes together with the results from the force-balance analysis, by Equation (B.2), and experiments are presented in Table 5.2, meanwhile the meniscus shapes in inclined annular tube are shown in Figures 5.3, 5.4, 5.5, 5.6, 5.7 and 5.8. The meniscus shapes resulted from the model exhibit other shapes throughout gap size as well as annular tube inclination.

The characteristic of water rising in inclined annular tube shows the same manner with water rising in vertical annular tube. The meniscus height is varied when the gap width is changed. The smaller gap width, the higher ability for water rising up into the gap tube.

Table 5.2 Results of circular annulus surface analysis for inclined annular tube alignment

Inner/Outer Tube Size (mm)	Angle	Gap (mm)	Meniscus Height (mm)		
			Young-Laplace	Force-Balance	Experiment
7/10	30°	0.5	13.484	7.228	7.41 ± 1.06
	45°	0.5	18.923	14.456	14.40 ± 1.53
	60°	0.5	23.323	21.684	21.30 ± 1.52
10/15	30°	1.5	4.560	2.409	2.41 ± 0.88
	45°	1.5	6.270	4.819	4.72 ± 1.30
	60°	1.5	7.772	7.228	7.14 ± 1.44
15/20	30°	1.5	4.560	2.409	2.36 ± 0.49
	45°	1.5	6.270	4.819	4.76 ± 1.06
	60°	1.5	7.772	7.228	7.18 ± 1.07

Besides, the larger slope of annular tube inclined present higher water rising up into the gap tube as long as gap width is constant. As shown by the meniscus height of 60° inclination, the result is higher than the others (30° and 45° inclination) for unchanging tube size. This behavior was taken place by the surface tension force pulling the water up in the vertical direction, in which the inclined annular tube of 60° inclination can compensate the vertical force more than the others. The relationship between inclined annular tube position and the surface tension force can be easily expressed in the Figure 5.1.

The deviation of Young-Laplace equation model from experimental results presented in a large magnitude at the small slope of inclination, since the Young-Laplace equation model has a limitation of investigation, which can only predict the meniscus surface in circular body. Meanwhile, one part of meniscus shape, which is turning back curve (as shown in Figure 5.1) and is not circular shape, can not be investigated by Young-Laplace equation model.

Moreover, the meniscus which was predicted by the Young-Laplace equation model shows clearly a circular shape. Whereas, actually, the meniscus should not be exactly formed the circular surface inside the inclined annular tube. Hence, this is another reason for the deviation of Young-Laplace equation model from experimental results.

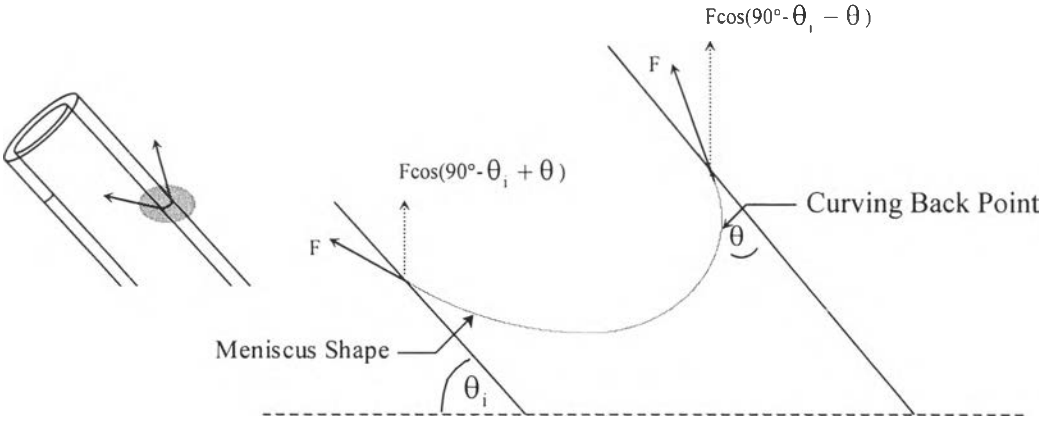

Figure 5.1 Enlarged diagram of the surface tension force pulling water in the vertical direction in inclined annular tube.

5.3 Annular Cone

The meniscus height in annular cone which was derived from inclined circular annulus with the assigned dimension and opening angle of annular cone. The result from the model in accompany with the results from the force-balance analysis, by Equation (C.5), and experiment are presented in Table 5.3, meanwhile the meniscus shape which is derived by the model is presented in Figure 5.9.

Table 5.3 Results of circular annulus surface analysis for annular cone

Inner/Outer Cone Size (mm)	Angle	Gap (mm)	Meniscus Height (mm)		
			Young-Laplace	Force-Balance	Experiment
7/10	45°	3.0	3.214	2.409	2.543 ± 0.10

The deviation of Young-Laplace equation model from experimental result can be likely explained as the water rising in inclined annular tube. Since the Young-Laplace equation model can predict the meniscus surface until it get to the beginning of turning back curve or curving back point in circular body only. Meanwhile, the part of turning back curve, which is not circular shape, can not be investigated by Young-Laplace equation model. Furthermore, the meniscus should not be exactly formed the circular surface inside the gap of annular cone, while the meniscus surface predicted by the Young-Laplace equation model shows a clearly circular shape.

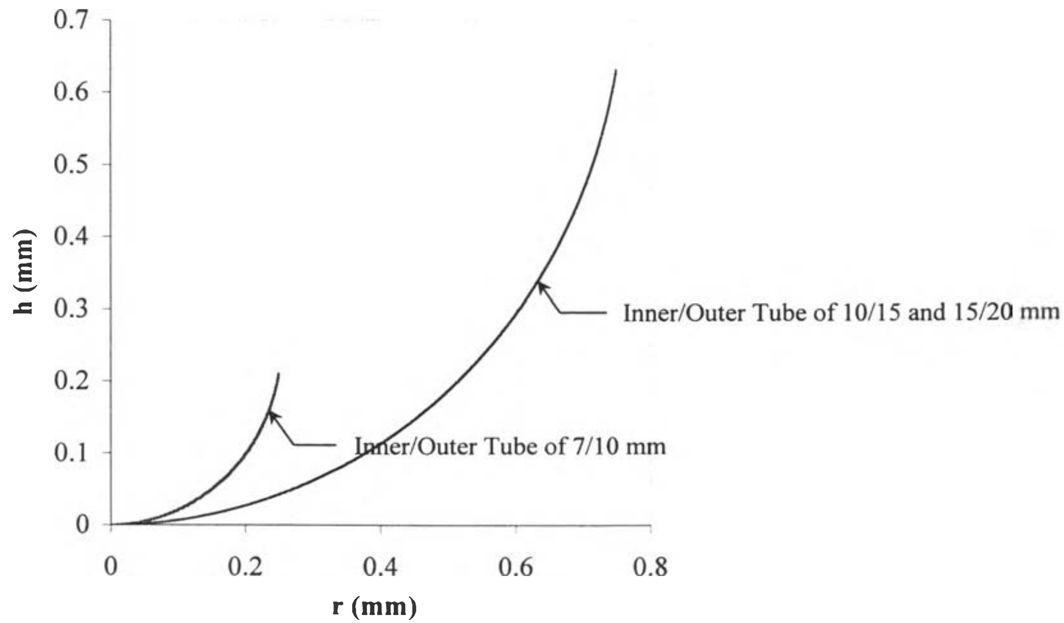

Figure 5.2 Meniscus shapes in the vertical annular tube with circular annulus surface analysis.

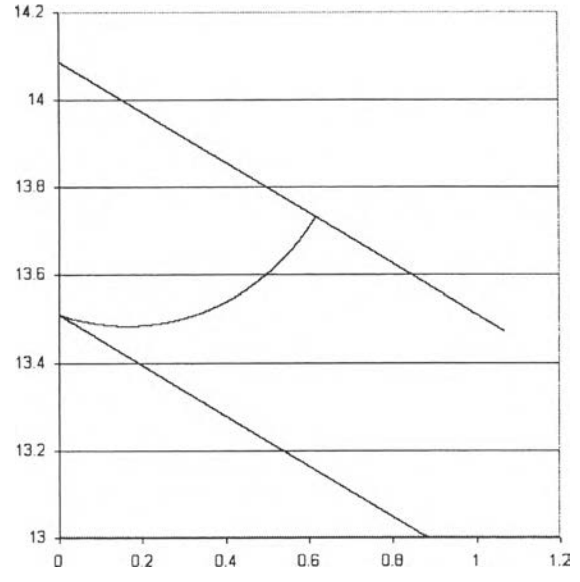

Figure 5.3 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 7/10 mm with 30° inclination by circular annulus surface analysis.

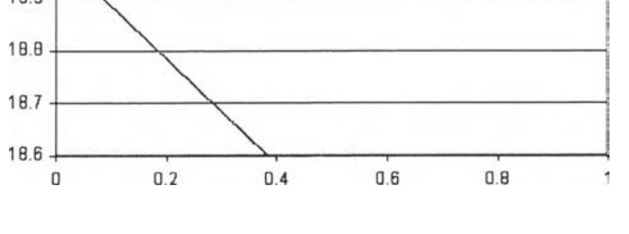

Figure 5.4 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 7/10 mm with 45° inclination by circular annulus surface analysis.

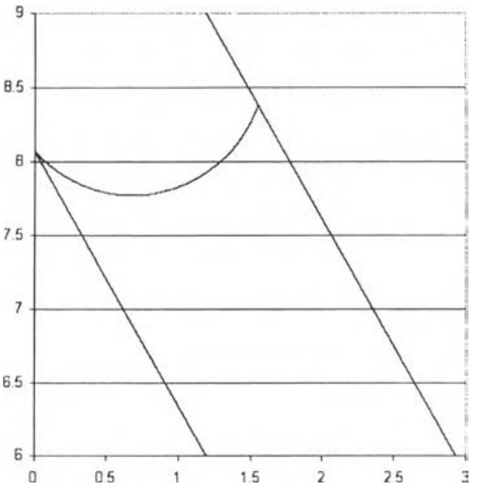

Figure 5.5 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 7/10 mm with 60° inclination by circular annulus surface analysis.

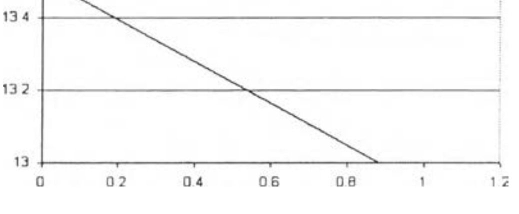

Figure 5.6 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 10/15 and 15/20 mm with 30° inclination by circular annulus surface analysis.

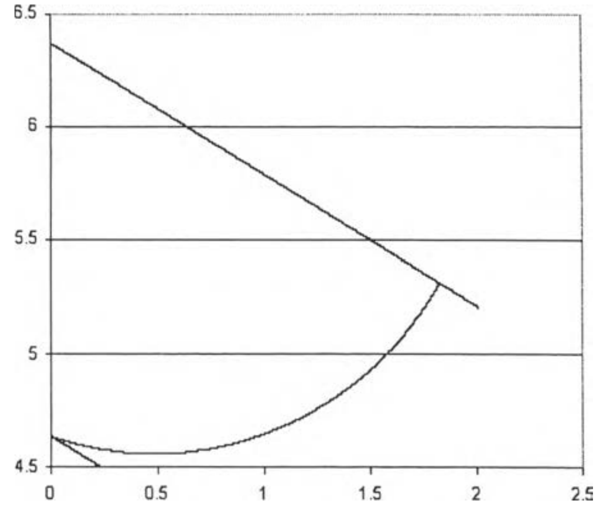

Figure 5.7 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 10/15 and 15/20 mm with 45° inclination by circular annulus surface analysis.

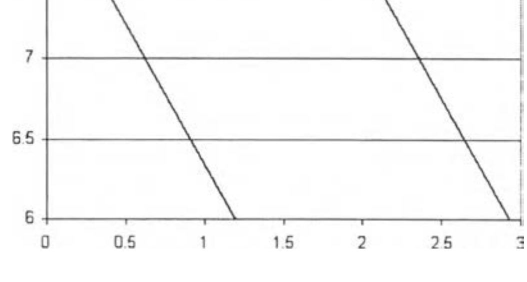

Figure 5.8 Meniscus shape predicted in the inclined annular tube at inner/outer tube of 10/15 and 15/20 mm with 60° inclination by circular annulus surface analysis.

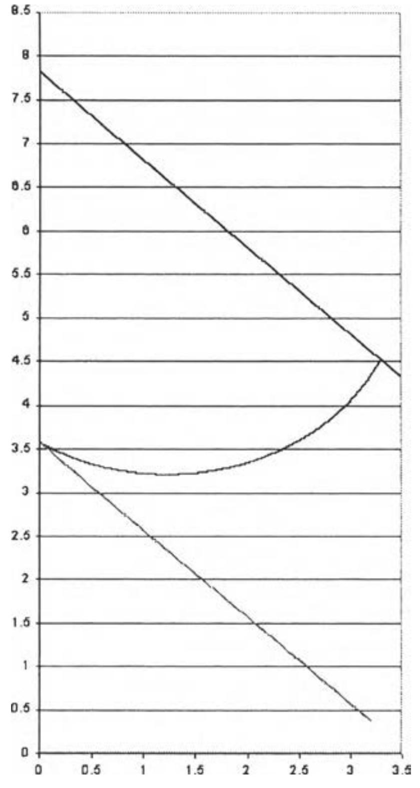

Figure 5.9 Meniscus shape predicted in annular cone at gap width 3.0 mm with 45° opening angle by circular annulus surface analysis.

At this place, it can be summarized to what are the causes of water rise in annular geometry and how do they make an effort to treat its behavior.

The height of rising water can be determined by the curvature of the meniscus, which was described by the mathematics. The hydrostatic pressure at any point on the meniscus surface is balanced by the pressure due to the curvature at that point, as can be shown straightforwardly by Equation (3.2)

$$\rho gh = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \tag{3.2}$$

the pressure changes across a curved interface and is equal to the product of interfacial energy and the curvature of the interface.

The meniscus that forms at the junction between a water surface and the wall of a glass annular tube is an indication of the spreading or wetting tendency of water on annular tube. The water spreads up the annular tube wall and is limited in its ascent only by the gravitational force.

The curved meniscus of water in an annular tube is created by the contact angle of the water against annular tube surface. If the water spreads on the tube surface, the meniscus has the form of a hemisphere (Morrison, 2002), and have the higher competency going up inside the tube. This case is recognized for liquids that wet the inside of the annular tube with a contact angle near zero, in very small annular tubes.

Whenever the gap tube is sufficiently wide, the meniscus along the peripheral contact with the tube does not overlap. The contact angle were exactly 90° would the water surface be perfectly plane and coincident with the cross-section of the tube, no pressure difference is created there.

The meniscus in annular tube turns into other shapes throughout along the tube position and the dimension of space between inner and outer tube, whereas does not depend on its geometry. Meanwhile, a more narrow space of annular tube increases ability of water uptake inside the tube.

For other liquids, however, the pressure difference causes liquid to flow either up or down the tube depending on whether the curvature is concave or convex, which is the result from their wetting properties. For example, mercury does not wet the glass tube, the liquid surface assumes the convex form of sessile drop (Morrison, 2002). The flow of liquid continues until the hydrostatic pressure, ρgh, just balances the Laplace pressure difference, Δp.

The capillary rise in annular tube can be affected by a number of reasons. The experimental results of the height of rising water in annular tube may be inexact due to the unequal of the gap width along the axial direction of tube and imperfectly smooth tube surface. These causes can strongly effect to the displacement of water rising height in annular tube, as elucidated by Equation (3.1)

$$\gamma = \frac{\rho gh(r_o - r_i)}{2\cos\theta} \tag{3.1}$$

The meniscus rising height was significantly varied by unsteady gap width along inner and outer tube, which also affect to the term in the bracket of Equation (3.1). Furthermore, ununiform of gap width along an annular tube make the massive problem for an experimental part. That is, water can not equally rise into annular tube as also illustrated in appendix B, and the height of rising water from the flat water surface to the bottom of the meniscus can be presented in error.

One influence of such gap tube likewise, the incorrectness was perhaps owing to a large gap width of annular tube, because the accuracy in the measurement of rising liquid height is inversely proportional to the gap width.

The temperature variation has been reported to decrease contact angle. Adamson (1990) reported that the temperature derivatives of contact angle is negative with $|\, d\theta/dT \,| \approx 0.1$ deg K^{-1} for many systems at low temperature (5-100°C). Ruijter *et al.* (1998) reported that the relaxation of the contact angle depends on the temperature. Nakamatsu *et al.* (2000) found that the character of the treated surface changes when the temperature changes (i.e., from water to air).