A MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL FOR HEAT EXCHANGER NETWORKS RETROFIT

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ABSTRACT

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Today all industries must deal with volatile crude oil prices and additional investments are required to meet more stringent environmental regulations. Therefore, optimization of energy usage through more efficient heat exchanger network retrofit and heat integration with the cost reduction are playing an extensive attraction. The grass-root and retrofit designs of heat exchanger network for process industries are addressed in this paper. A strategy is proposed as the MILP formulation based on the special transshipment structure concept. This methodology can generate networks where utility cost, heat exchanger areas and selection of matches are optimized simultaneously. In addition, the simplicity in model assumption, non-isothermal mixing, comes with handling constraints such as stream splitting and allowed/forbidden matches which bring the model structure more convenient to use.

The application examples presented here show that the automatic rigorous MILP model can be successfully applied to the complex hot and cold process streams which provides a profitable network for both grass-root design and retrofit, 24.06% total cost saving in the case study of crude distillation unit. This approach also gives a clear advantage over Hypertargets method (Briones, 1999) with more saving about 118 k$/yr of total cost. Finally, in special scenario, relocation topology can be used for further reduction in total cost. The MILP also gives the highest annual cost saving for retrofit HEN by comparing with the approaches of Ciric et al. (1989) and Kin-Lung et al. (2000).

บทคัดย่อ

นฤมล วิภาณุรัตน์: แบบจำลอง MILP เพื่อการออกแบบและปรับปรุงเครือข่ายแลกเปลี่ยนความร้อน (A Mixed Integer Linear Programming (MILP) Model for Heat Exchanger Networks Retrofit) อ. ที่ปรึกษา: ผศ. ดร. กิติพัฒน์ สีมานนท์ และ ศ. ดร. มิเกล บากาเฮวิซ 193 หน้า ISBN 974-9651-83-9

เนื่องจากภาวะราคาน้ำมันที่ผันผวนในสถานการณ์ปัจจุบันนี้ ทุกอุตสาหกรรมมักประสบปัญหาภาวะต้นทุนการผลิตที่เพิ่มสูงขึ้น อีกทั้งยังมีค่าใช้จ่ายในการติดตั้งอุปกรณ์เพื่อความปลอดภัยทางสิ่งแวดล้อม ดังนั้น การออกแบบและปรับปรุงกระบวนการที่เหมาะสมเพื่อก่อให้ประสิทธิภาพในการใช้พลังงานสูงสุด และลดภาระค่าใช้จ่ายต่างๆภายในโรงงานอุตสาหกรรมจึงเป็นสิ่งสำคัญ โดยเฉพาะอย่างยิ่ง การออกแบบเครือข่ายแลกเปลี่ยนความร้อนซึ่งถือเป็นกระบวนการที่ใช้พลังงานปริมาณมาก กระบวนการหนึ่งภายในโรงงานอุตสาหกรรม วิทยานิพนธ์นี้ นำเสนอวิธีการออกแบบและปรับปรุงเครือข่ายแลกเปลี่ยนความร้อนโดยอาศัยแบบจำลองทางคณิตศาสตร์และคอมพิวเตอร์โปรแกรม โดยวิธีการที่ใช้ จะสามารถคำนวณค่าใช้จ่ายทั้งหมดในเครือข่ายการแลกเปลี่ยนความร้อน ซึ่งประกอบด้วย ค่าใช้จ่ายของสาธารณูปโภค พื้นที่ และจำนวนของเครื่องแลกเปลี่ยนความร้อน ในเวลาเดียวกัน แบบจำลองทางคณิตศาสตร์นี้ยังสามารถประยุกต์ใช้กับกระบวนการที่มีข้อจำกัดต่างๆ เช่น ข้อจำกัดในการจับคู่ของสายสตรีมร้อนและสายสตรีมเย็น การแบ่งสายสตรีมเพื่อเพิ่มความสามารถในการแลกเปลี่ยนความร้อน เป็นต้น ในวิทยานิพนธ์ฉบับนี้ ยกตัวอย่าง การปรับปรุงกระบวนการเครือข่ายการแลกเปลี่ยนความร้อน ของหน่วยการกลั่นน้ำมัน ซึ่งเมื่อประยุกต์ใช้แบบจำลองนี้แล้วพบว่าสามารถประหยัดค่าใช้จ่ายได้ถึง 1.65 ล้านเหรียญสหรัฐต่อปี แบบจำลองนี้ยังสามารถปรับปรุงให้ลดค่าใช้จ่ายได้มากขึ้นอีก โดยอาศัยหลักการย้ายตำแหน่งของเครื่องแลกเปลี่ยนความร้อนจากคู่สายสตรีมเดิมไปอยู่ในตำแหน่งคู่สายสตรีมใหม่ ซึ่งผลที่ได้พบว่าสามารถลดค่าใช้จ่ายได้เพิ่มขึ้น และเครือข่ายแลกเปลี่ยนความร้อนที่ออกแบบโดยหลักการการย้ายตำแหน่งของเครื่องแลกเปลี่ยนความร้อนนี้ ยังมีค่าใช้จ่ายที่น้อยกว่าวิธีการออกแบบเครือข่ายแลกเปลี่ยนความร้อน ที่เสนอโดย Ciric et al., (1989) และ Kin-Lung et al., (2000)

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ABBREVIATIONS

Sets

B	= { (i,j) | more than one heat exchanger unit is permitted between hot stream i and cold stream j }
C^z	= { j | j is a cold stream present in zone z }
C_n^z	= { j | j is a cold stream present in temperature interval n in zone z }
CU^z	= { j | j is a heating utility present in zone z } ($CU^z \subset C^z$)
H^z	= { i | i is a hot stream present in zone z }
H_m^z	= { i | i is a hot stream present in temperature interval m in zone z }
HU^z	= { i | i is a heating utility present in zone z } ($HU^z \subset H^z$)
M^z	= { m | m is a temperature interval in zone z }
M_i^z	= { m | m is a temperature interval belonging to zone z, in which hot stream i is presented }
m_i^0	= { m | m is the starting temperature interval for hot stream i }
m_i^f	= { m | m is the final temperature interval for hot stream i }
N_j^z	= { n | n is a temperature interval belonging to zone z, in which cold stream j is presented }
NI^H	= { i | non-isothermal mixing is permitted for hot stream i }
NI^C	= { j | non-isothermal mixing is permitted for cold stream j }
n_i^0	= { n | n is the starting temperature interval for cold stream j }
n_i^f	= { n | n is the final temperature interval for cold stream j }
P	= { (i,j) | heat exchange match between hot stream i and cold stream j is permitted }
P_{im}^H	= { j | heat transfer from hot stream i at interval m to cold stream j is permitted }

P^{C}_{jn}	= { i | heat transfer from hot stream i to cold stream j at interval n is permitted }
S^{H}	= { i | splits are allowed for hot stream i }
S^{C}	= { j | splits are allowed for cold stream j }
Z	= {z| z is a heat transfer zone}

Parameters

$A^{z^0}_{ij}$	Area of an existing exchanger between streams i and j in zone z prior to retrofit
A^{z,k^0}_{ij}	Area of the k-th existing exchanger between streams i and j in zone z prior to retrofit
$A^{z}_{ij\max}$	Maximum shell area for an exchanger matching hot stream i and cold stream j in zone z
$A^{z^N}_{ij\max}$	Maximum area for a new heat exchanger matching hot stream i and cold stream j in zone z
$\Delta A^{z^0}_{ij\max}$	Maximum area addition for an existing heat exchanger matching hot stream i and cold stream j in zone z
$\Delta A^{z,k^0}_{ij\max}$	Maximum area addition for the k-th existing heat exchanger matching hot stream i and cold stream j in zone z
$\hat{C}_{im}$	Heat capacity of hot stream i at temperature interval m
$\hat{C}_{jn}$	Heat capacity of cold stream j at temperature interval n
c^{H}_{i}	Cost of heating utility i
c^{C}_{j}	Cost of cooling utility j
c^{F}_{ij}	Fixed charge cost for a heat exchanger matching hot stream i and cold stream j

$c_{ij}^{A^N}$	Variable cost for a new heat exchanger matching hot stream i and cold stream j
$c_{ij}^{A^0}$	Area addition cost for an existing heat exchanger matching hot stream i and cold stream j
$CHEAD_{im,jn}$	Temperature difference between interval m of hot stream i and interval n of cold stream j at cold end
F_i	Flow rate of hot process stream i
F_j	Flow rate of cold process stream j
F_i^U	Upper bound for the flow rate of heating utility i
F_j^U	Upper bound for the flow rate of cooling utility j
h_{im}	Film heat transfer coefficient for hot stream i in interval m
h_{jn}	Film heat transfer coefficient for cold stream j in interval n
$\Delta H_{im}^{z,H}$	Enthalpy change for hot stream i at interval m of zone z
$\Delta H_{jn}^{z,C}$	Enthalpy change for cold stream j at interval n of zone z
$HHEAD_{im,jn}$	Temperature difference between interval m of hot stream i and interval n of cold stream j at hot end
$k_{\max}$	Maximum number of heat exchangers allowed between hot stream i and cold stream j in zone z when $(i,j) \in B$
k_e	Number of existing heat exchangers between hot stream i and cold stream j in zone z when $(i,j) \in B$
k_e'	Number of existing heat exchangers in the original network
q_{ijm}^L	Lower bound for heat transfer from hot stream i at interval m to cold stream j
q_{ijn}^L	Lower bound for heat transfer from hot stream i to cold stream j at interval n

T_m^U Upper temperature of interval m

T_m^L Lower temperature of interval m

T_n^U Upper temperature of interval n

T_n^L Lower temperature of interval n

ΔT_i Temperature range of stream i

ΔT_j Temperature range of stream j

ΔT_{mn}^{ML} Mean logarithmic temperature difference between intervals m and n

$U_{ij}^{z^0}$ Number of existing heat exchangers between hot stream i and cold stream j in zone z

$U_{ij}^{z^N}$ Maximum number of new heat exchangers allowed for the retrofit design

Variables

A_{ij}^{z} Total required area for a match between hot stream i and cold stream j in zone z

A_{ij}^{z,k^0} Area of the k-th existing heat exchanger between hot stream i and cold stream j in zone z after retrofit

$A_{ij}^{z^N}$ Area of a new heat exchanger between hot stream i and cold stream j in zone z

$\Delta A_{ij}^{z^0}$ Area addition for an existing heat exchanger between hot stream i and cold stream j in zone z

$\Delta A_{ij}^{z,k^0}$ Area addition for the k-th existing heat exchanger between hot stream i and cold stream j in zone z

$K_{ijm}^{z,H}$ Determines the beginning of a heat exchanger at interval m of zone z for hot stream I with cold stream j. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$.

$K_{ijn}^{z,C}$ Determines the beginning of a heat exchanger at interval n of zone z for cold stream j with hot stream i. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$.

$\hat{K}_{ijm}^{z,H}$ Determines the end of a heat exchanger at interval m of zone z for hot stream i with cold stream j. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$.

$\hat{K}_{ijn}^{z,C}$ Determines the end of a heat exchanger at interval n of zone z for cold stream j with hot stream i. Defined as binary when $(i,j) \in B$ and as continuous when $(i,j) \notin B$.

k_e Number of existing heat exchangers between hot stream i and cold stream j in zone z when $(i,j) \in B$

$q_{im,jn}^{z}$ Heat transfer from hot stream i at interval m to cold stream j at interval n in zone z

$\overline{q}_{inm}^{z,H}$ Non-isothermal mixing heat transfer for hot stream i between intervals m and n in zone z

$\overline{q}_{jmn}^{z,C}$ Non-isothermal mixing heat transfer for hot stream i between intervals m and n in zone z

$\hat{q}_{ijm}^{z,H}$ Heat transfer from hot stream i at interval m to cold stream j in zone z

$\hat{q}_{ijn}^{z,C}$ Heat transfer to cold stream j at interval n from hot stream j in zone z

$\tilde{q}_{ijm}^{z,H}$ Auxiliary continuous variable utilized to compute the hot side heat load of each heat exchanger when several exchangers exist between hot stream i and cold stream j in zone z

$\tilde{q}_{ijn}^{z,C}$ Auxiliary continuous variable utilized to compute the cold side heat load of each heat exchanger when several exchangers exist between hot stream i and cold stream j in zone z

$\tilde{q}_{im,jn}^{z,H}$ Auxiliary continuous variable utilized to compute the area of individual heat exchangers between hot stream i with cold stream j in zone z when $(i,j) \in B$.

U_{ij}^{z} Number of heat exchangers between hot stream i and cold stream j in zone z

$X_{im,jn}^{z}$ Auxiliary continuous variable equals to zero when an exchanger ends at interval m for hot stream i and at interval n for cold stream j. A value of one corresponds to all other cases.

$\hat{X}_{ijm}^{z,k}$ Auxiliary binary variable that determines whether the k-th between hot stream i with cold stream j in zone z exists at interval m of when $(i,j) \in B$.

$Y_{ijm}^{z,H}$ Determines whether heat is being transferred from hot stream i at interval m to cold stream j. Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$.

$Y_{ijn}^{z,C}$ Determines whether heat is being transferred from hot stream i to cold stream j at interval n. Defined as binary when $(i,j) \notin B$ and as continuous when $(i,j) \in B$.

$\delta_{ij}^{z,k}$ Auxiliary binary variable used for heat exchanger relocation. Auxiliary binary variable that determines whether the k-th original heat exchanger of zone z has is serving the match between hot stream i and cold stream j, when $(i,j) \notin B$.

$\delta_{ij}^{z,hk}$ Auxiliary binary variable used for heat exchanger relocation. This variable determines whether the k-th original heat exchanger of zone z has is serving the h-th exchanger streams i and j, when $(i,j) \in B$.

$\alpha_{ijm}^{z,H}$ Auxiliary continuous variable equal to one when heat transfer from interval m of hot stream i to cold stream j occurs in zone z and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases.

$\alpha_{ijn}^{z,C}$ Auxiliary continuous variable equal to one when heat transfer from hot stream i to interval n of cold stream j occurs in zone z and it does not correspond to the beginning nor the ending of a heat exchanger. A value of zero corresponds to all other cases.