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The study of special ordered sets for the column generation technique to
solve a workload balancing crew rostering problem

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นางสาวสาวิณี เมืองทรัพย์

The study of special ordered sets for the column generation technique to solve a

workload balancing crew rostering problem

Miss Savinee Mueangsap

A Project Submitted in Partial Fulfillment of the Requirements

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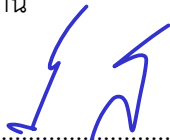
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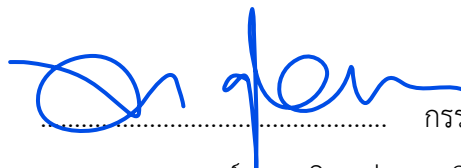
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ปัญหาการมอบหมายงานให้พนักงานสายการบินเป็นปัญหาที่ซับซ้อนและยากจากจำนวนพนักงานสายการบินและเที่ยวบินที่มากนอกจากนี้การจัดตารางปฏิบัติงานของพนักงานต้องสอดคล้องกับเงื่อนไขและกฎ-ระเบียบระหว่างบริษัทและพนักงาน จากวิทยานิพนธ์ของศุภกร เขาแก้ปัญหาการมอบหมายงานโดยใช้เทคนิคการสร้างคอลัมน์โดยมีวัตถุประสงค์เพื่อปรับดุลปริมาณงานและรายได้ของลูกเรือ โครงงานนี้มุ่งเน้นไปที่การศึกษาวิธีการช่วยเพิ่มความเร็วในกระบวนการหาผลเฉลยโดยใช้เซตอันดับพิเศษ (SOS)

ภาควิชา คณิตศาสตร์และวิทยาการคอมพิวเตอร์   ลายมือชื่อนิสิต สาวิณี เมืองทรัพย์

สาขาวิชา   คณิตศาสตร์   ลายมือชื่อ อ.ที่ปรึกษาโครงงานหลัก
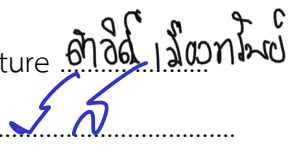
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KEYWORDS : CREW ROSTERING/ COLUMN GENERATION/ SPECIAL ORDERED SETS

SAVINEE MUEANGSAP: THE STUDY OF SPECIAL ORDERED SETS FOR THE COLUMN GENERATION TECHNIQUE TO SOLVE A WORKLOAD BALANCING CREW ROSTERING CREW ROSTERING PROBLEM. ADVISOR : ASSISTANT PROFESSOR KRUNG SINAPIROMSARAN, Ph.D., 32 pp.

A crew rostering problem is complex and difficult due to a large number of crews and flights. Moreover, this crew rostering problem must satisfy constraints and regulations between company and employee. From Suphakorn's thesis, he solved the crew rostering problem using the column generation technique whose objective is to balance workload and earnings among crew members. This project focuses on the way to help speed up the procedure optimization model via using Special Ordered Sets (SOS).

Department	Mathematics and Computer Science	Student's Signature ....
Field of Study	Mathematics	Advisor's Signature ....
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Chapter I

Introduction

1.1 Background and Rationale

Airline industry has an important impact on economic development of a country[1]. It increases world trade activities by enabling faster and easier movement of passengers and goods. Additionally, it also provides jobs to millions of people. An airline receives revenue from passenger tickets and transporting goods which it uses to pay for aircraft, fuel, crew and airport usages. The largest expense comes from the fuel cost while the crew cost constitutes the second-largest expense for the airline[2]. One important management issue is the crew scheduling which is the cost minimization satisfying government regulations between company and its employees. The crew rostering problem is complex and difficult due to a large number of crews and flights.

From Supphakorn's thesis[3], he solved the crew rostering problem using the column generation technique which is a technique used to solve linear programming problems consisting of a large number of variables via the concept from the decomposition principle by Dantzig and Wolfe 1960. Then, In 1984 Desrosiers, Soumis and Desrochers was the first group of researchers to use this idea of the column generation technique with the branch and bound method[4]. In this project, we study the column generation technique for solving the crew rostering problem and study of SOS (special ordered sets) used in the branch and bound method for branching on sets of variables. We focus on the computational speed via the SOS technique.
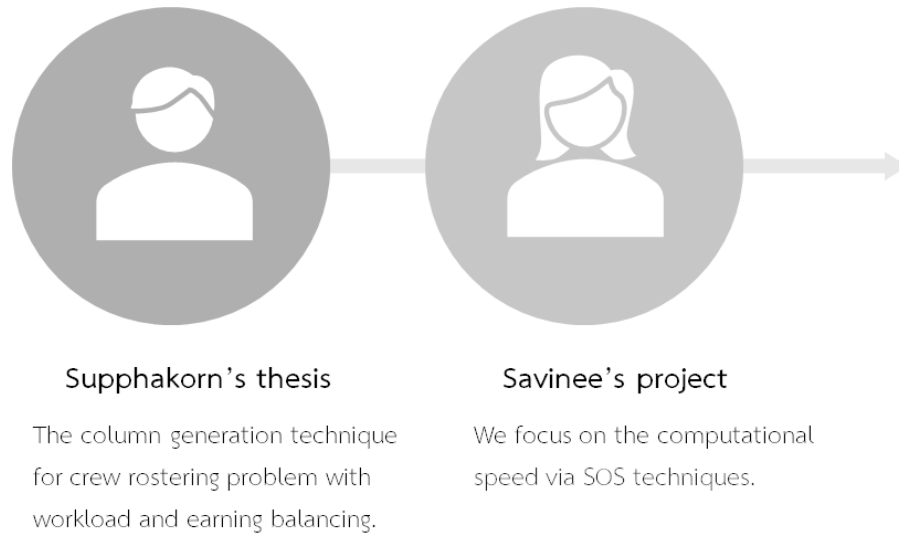
Figure 1.1: Motivation

1.2 Objectives

1. To study the column generation technique for solving an airline crew rostering problem by assigning appropriate balance workload for each crew member using CPLEX.

2. To compare and analyze the results from simulated problems using the SOS technique.

1.3 Scope

1. Eight datasets from Thai Airways International to schedule crews were extracted from the Master thesis "Column generation technique for crew rostering problem with workload and earnings balancing" written by Acting Sub Lt. Supphakorn Sumetthapiwat, 2012.

2. Crews for the crew rostering problem will be considered only employees' in-flight manager.

3. IBM ILOG CPLEX Optimization will be used to solve an airline crew rostering problem.

4. Simulated integer programming problems will be solved via IBM ILOG CPLEX Optimization with the SOS technique.

1.4 Project Activities

1. Study background for tools to solve the crew rostering problem.

2. Collected and transformed data into appropriate format.

3. Study the concept of SOS related to the branch and bound method.

4. Write the new model and solve via the CPLEX Optimization.

5. Compare the performance of 8 datasets from Thai Airway.

6. Write the senior project report.

1.5 Benefit

1. Comprehend the concept of SOS (special ordered sets) and apply to the column generation technique to solve the airline crew rostering problem.

2. Experience solving optimization using the CPLEX Optimization.

Chapter II

Background knowledge and Literature review

In this part, we review the background knowledge of an airline crew scheduling problem and the column generation technique to solve a crew rostering problem.

2.1 Airline crew scheduling problem

An airline job scheduling problem is a very important issue for airlines. Currently, the costs for hiring staff is the second largest expense after the cost of fuel used in aircraft refueling Therefore, the company always effectively optimizes a work plan for employees such as the number of employees per flight, the number of working hours of the employees and the number of resting hours for the employees. Effective human resource management in the organization will help reduce the company cost and maximize the cost effectiveness. The problem of scheduling the airline staff is complex and complicated because each airline generally has many employees and many flights to be scheduled and planned. In general, the airline crew scheduling process can be divided into two subprocesses [2]: a crew pairing and a crew assignment as shown in Figure 2.1.
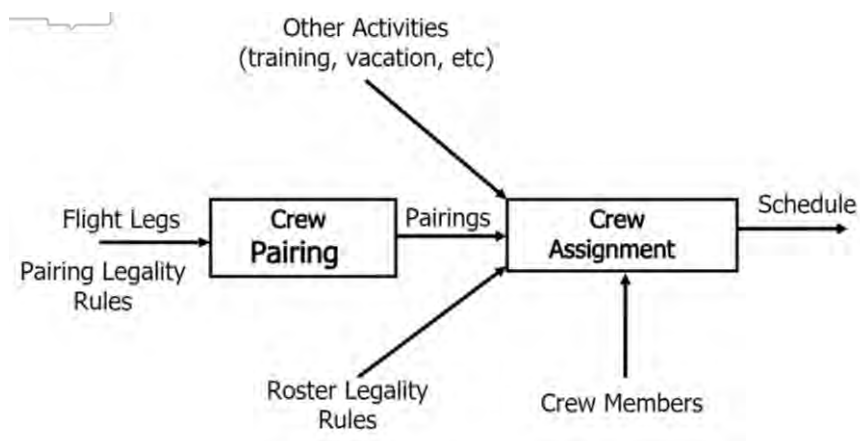
Figure 2.1: Diagram of the airline crew scheduling process[2].

The crew pairing is to assign enough crews to flights on both the outbound and the return flights for customers on each flight. The objective of the crew pairing is to determine the minimum cost of a set of trips that covers a set of flights.

The crew assignment is to schedule work for each employee. It must also consider work efficiency and employee satisfaction with workload and income. Therefore, the company needs to assign the rostering of the employees so that the distribution of income and workload of all employees are approximately equal.

The crew rostering assignment is a complex and huge task due to the labor laws and crew constraints as shown in Figure 2.2. Moreover, the size of this problem is usually huge since there are many flights, pairings and crew. Currently, fatigue constraint is becoming more of a hot topic in aviation because a duty assignment is legal and within company policy, but it does not necessarily mean the crew can perform efficiently [3] so airlines must consider balancing workload and per-diem for the crew satisfaction.
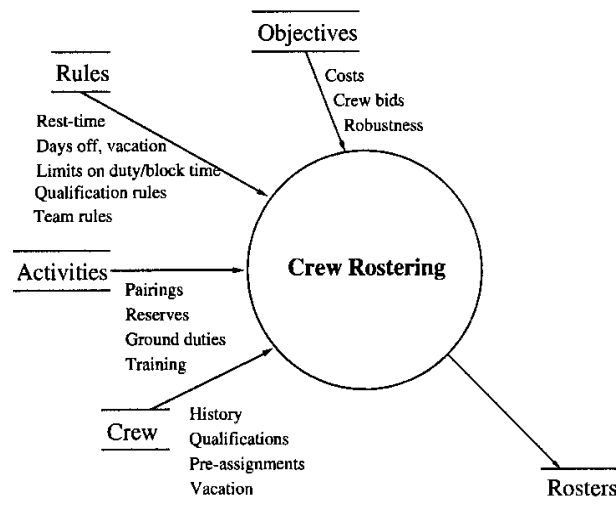
Figure 2.2: Representation of the airline crew rostering problem [3].

2.2 Literature review

An airline scheduling problem is a popular problem as it is a large and complex problem which creates challenges for researchers. From our review, there are many solutions to this problem such as:

Wariya [5] solves the crew rostering problem by 0-1 multi-commodity flow problem where the objective function is to minimize the sum of the per-diems and workload among the crew members. The optimizer cannot solve the problem of large size within 60 hours.

Supphakorn [4] also solves the crew rostering problem and presented a mathematical model and procedure for solving this problem using the column generation technique. The result exhibits the balance distribution of income and workload of all crews. In the Supphakorn's model, he tested the model with eight realistic datasets from Thai Airway (for in-flight managers only) dealing with at most 392 pairings.

The regulations for cabin attendants of Thai Airways are shown in Table 2.1 and Table 2.2.

Table 2.1: Block time constraints for each cabin crew.

Number of Days	Total Block Time
Every 7 consecutive days	should not exceed 34 hours
Every 28 consecutive days	should not exceed 110 hours
Every 365 consecutive days	should not exceed 1000 hours

Table 2.2: Rest period constraints for each cabin crew.

Flight Duty Period	Rest Period
< 8 hrs	≥ 8 hrs
8 – 10 hrs	≥ 10 hrs
10 – 12 hrs	≥ 12 hrs
12 – 14 hrs	≥ 14 hrs
14 – 16 hrs	≥ 16 hrs
16 – 20 hrs	≥ 24 hrs

2.3 The column generation technique

The column generation technique or the delayed column generation technique is an efficient algorithm for solving a large linear programming problem. One technique from this group is the Dantzig–Wolfe decomposition algorithm. Additionally, the column generation technique has been applied to many problems such as the crew scheduling problem, the vehicle routing problem, and the capacitated p-median problem [6]. The problem being solved is split into two problems: the master problem (MP) and the sub-problem (SP) where MP ensures covering all tasks. MP gives primal and dual solutions. SP creates a new problem with respect to the dual solution from MP. The solutions from MP and SP are used to calculate the negative reduced cost in order to improve the solution. The procedure is then repeated until no further improvement can be made.

In this section, we review Supphakorn's thesis which uses the column generation technique to solve the crew rostering problem. MP is formulated as a set-partitioning model while SP is formulated as a multi-commodity flow model.
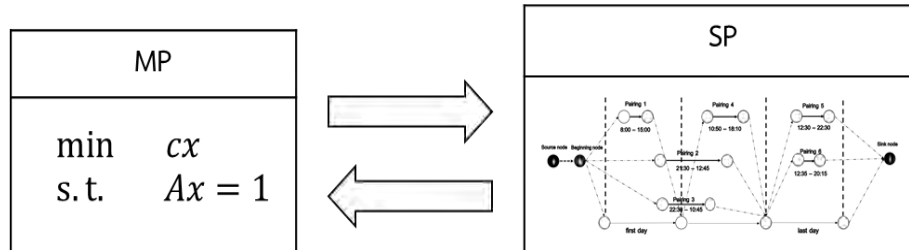
Figure 2.3: An overview of the column generation technique.

The MP model is a set partitioning model using the following parameters.

$Crew$ is a set of crew members,

cw is the index of $Crew$ $(cw \in Crew)$,

R^{cw} is a set of all possible work schedules for crew member cw in $Crew$,

P is a set of pairings,

p is the indices of P $(p \in P)$,

x_j is the binary decision variable where $x_j = 1$ if the crew was assigned in schedule j and $x_j = 0$ otherwise,

w_j is a total workload of schedule j,

I_j is a total earnings of schedule j,

$\overline{w}$ is the average of workload,

$\overline{I}$ is the average of earnings,

W is a weighted average earnings per workload which $W = \dfrac{\overline{I}}{\overline{w}}$,

a_{pj} is an assignment coefficient where $a_{pj} = 1$ if pairing p in schedule j and 0 otherwise.

The mathematical formulation of the set-partitioning model, as shown in below:

minimize $\quad\sum_{cw\epsilon Crew}\sum_{j\epsilon R^{cw}}(|\bar{I}-I_j|+W|\bar{w}-w_j|)x_j$ (1)

subject to $\quad\sum_{j\epsilon R^{cw}}x_j=1, cw\epsilon Crew$ (2)

$\quad\sum_{cw\epsilon Crew}\sum_{j\epsilon R^{cw}}a_{pj}x_jj=1, p\epsilon P$ (3)

$\quad x_j\epsilon\{0,1\}, j\epsilon\bigcup_{cw\epsilon Crew}R^{cw}$ (4)

The set of constraints can be explained as follows.

Constraints (2) and (3) form set-partitioning constraints since a crew must be assign to a single roster and he/she cannot be assigned to another roster.

Constraint (2) represents that the crew receive only 1 roster per person.

Constraint (3) represents that for each pairing must be assigned only one crew because we considered for in-flight managers only.

Constraint (4) limits the domains of the variables.

From this step, a nonbasic variable which can improve the solution will be identified and its reduced cost is

$$\text{reduced cost} = (|\bar{I}-I_j|+W|\bar{w}-w_j|)-\sum_{i=1}^{n}\pi_i a_{pj},$$

where π_i is the dual variable of constraint i.

The formulation of the restricted master problem (RMP) is similar to MP, but it only considers subset J' of J. We do not solve the RMP directly, but we solve the LP relaxation with $x_j\in[0,1]$.

The RMP model is

minimize $\quad\sum_{cw\epsilon Crew}\sum_{j\epsilon R^{cw}}(|\bar{I}-I_j|+W|\bar{w}-w_j|)x_j,$

subject to $\quad\sum_{j\epsilon R^{cw}}x_j=1, cw\epsilon Crew,$
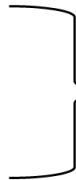
$$\sum_{cw \epsilon Crew} \sum_{j \epsilon R^{cw}} a_{pj} x_j j \; = \; 1, p \epsilon P,$$

$$x_j \epsilon \{0,1\}, j \epsilon \bigcup_{cw \epsilon Crew} R^{cw},$$
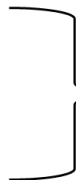
$$x_j \geq 0 \, , j \in J'.$$

The columns characteristic in MP can be shown below:

	Possible schedules				
	R_1	R_2	...	R_n	
Crew 1	1	0	...	0	
Crew 2	0	1	...	0	In accordance with constraint (2).
⋮	⋮	⋮	⋱	⋮	
Crew C	0	0	...	1	
Pairing 1	0	1	...	0	
Pairing 2	1	0	...	0	In accordance with constraint (3).
⋮	⋮	⋮	⋱	⋮	
Pairing P	0	0	...	1	

Figure 2.4: The column of the crew rostering from a set-partitioning model.

The optimal solution of RMP may be not optimal for MP so we need to generate the sub-problem (SP) to re-optimize RMP. The minimum reduced cost column is tested with the reduced cost of the nonbasic variable as

$$\overline{\left| \bar{I} - I_j \right| + W \left| \bar{w} - w_j \right|}^* := Min \left\{ \left(\left| \bar{I} - I_j \right| + W \left| \bar{w} - w_j \right| \right)(\hat{a}) - \hat{\pi}^T \hat{a} \right\}, \hat{a} \, \epsilon \, A.$$

Where A is the set of all columns of the problem,

$\left| \bar{I} - I_j \right| + W \left| \bar{w} - w_j \right| (\hat{a})$ is the coefficient which is calculated from column $\hat{a}$,

$\hat{\pi}$ is a vector of dual variables of the problem.

When $\overline{\left| \bar{I} - I_j \right| + W \left| \bar{w} - w_j \right|}^* < 0$, variable x_j and its coefficient column $\left(\left| \bar{I} - I_j \right| + W \left| \bar{w} - w_j \right|, a_j \right)$ corresponding to a minimizer j are added to RMP.

The sub-problem (SP) is formulated as a network flow model. The network flow is a directed graph with its capacity and each edge receives a unit of flow.

Th SP model using the following parameters.

e_{st} is cost for arc from node s to node t,

y_{st} is decision variable for flowing from node s to node t,

f_{st} is number of hours in node s to node t,

A_k is a set of beginning arc at date k,

d_{max} is the last day of the roster,

D is a set of date in roster $= \{1,2,3,\ldots,d_{max}\}$.

The mathematical formulation of the network flow model, as shown in below:

Define the network flow is a directed graph $G = (N, A)$,

where N is nodes are composite activities,

A is arcs connect activities that are legal to be connected in pairs.

minimize $\sum_{(s,t)\epsilon A} e_{st} y_{st}$ (5)

subject to $\sum_{t:(s,t)\epsilon A} y_{st} - \sum_{t:(t,s)\epsilon A} y_{ts} = \begin{cases} 1 \; ; \; s \text{ is source node} \\ -1 \; ; \; s \text{ is sink node}, s\epsilon N \\ 0 \; ; \; otherwise \end{cases}$ (6)

$\sum_{k=d}^{min\,(d+6,d_{max})} \sum_{(s,t)\epsilon A_k} f_{st} y_{st} \le 34 \, , d\epsilon D$ (7)

$\sum_{k=d}^{min\,(d+27,d_{max})} \sum_{(s,t)\epsilon A_k} f_{st} y_{st} \le 110 \, , d\epsilon D$ (8)

$\sum_{k=d}^{min\,(d+364,d_{max})} \sum_{(s,t)\epsilon A_k} f_{st} y_{st} \le 1000 \, , d\epsilon D$ (9)

$y_{st}\epsilon\{0,1\}$

The set of constraints can be explained as follows.

Constraints (5) is the objective function to minimize the cost of the flow path.

Constraint (6) is the flow conservation at the source where a flow must exit the source node only.

Constraint (7) is the block time constraint to ensure that each crew member cannot work more than 34 hours a week on any 7 consecutive days.

Constraint (8) is the block time constraint to ensure that for each crew member cannot work more than 110 hours a week on any 28 consecutive days.

Constraint (9) is the block time constraint to ensure that each crew member cannot work more than 1,000 hours a week on any 365 consecutive days.

Outline of the process which is a hybrid of the branch and bound method and the column generation method are shown in Figure 2.5.
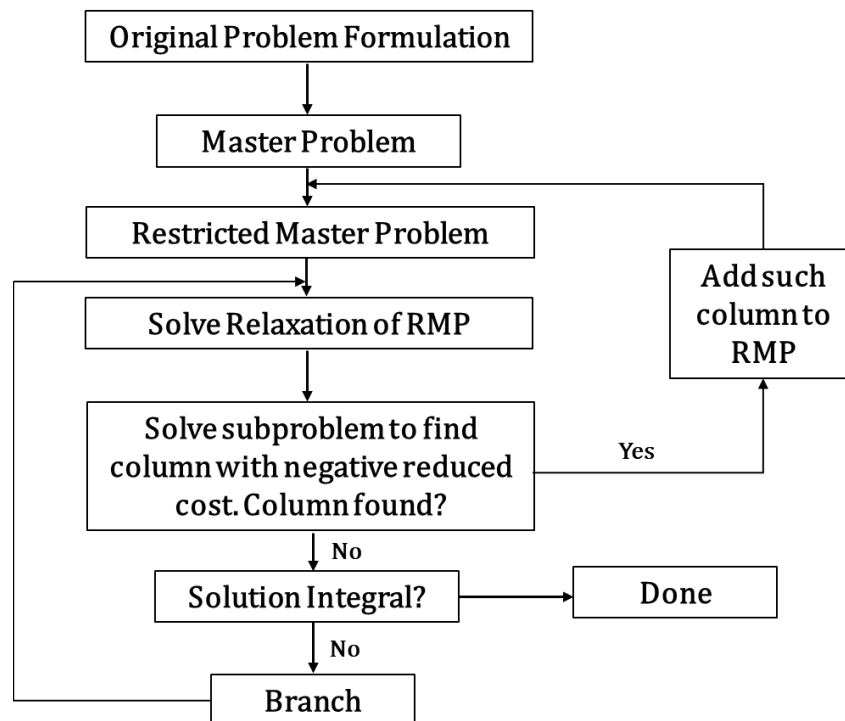
Figure 2.5: The diagram to find the answer [6].

2.4 Software

CPLEX is named for the simplex method implemented in the C programming language although today it supports interfaces other than C. It was originally developed by Robert E. Bixby and sold commercially from 1988 by CPLEX Optimization Inc. This was acquired by ILOG in 1997 and ILOG was subsequently acquired by IBM in January 2009. CPLEX continues to be actively developed by IBM [11].

The IBM ILOG CPLEX Optimization Studio program consists of various parts which can be described as follows Figure 2.6.
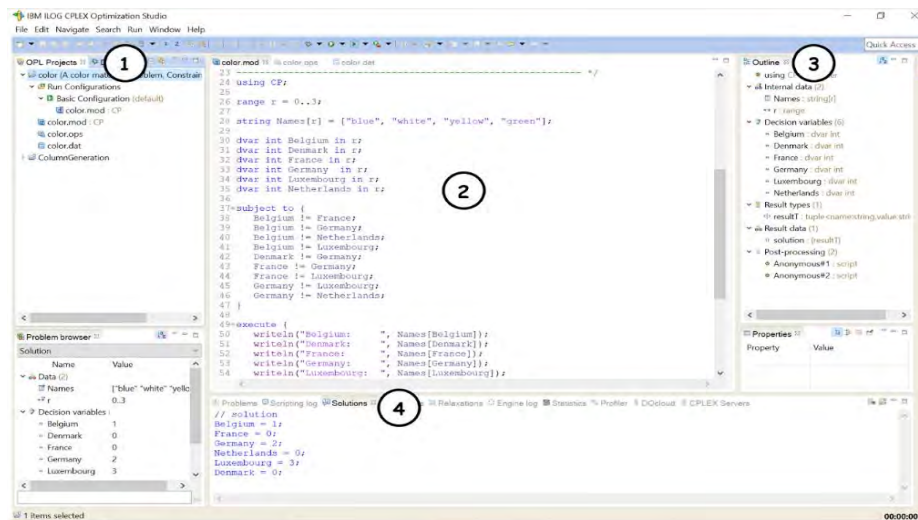
Figure 2.6: The main window of IBM ILOG CPLEX Optimization Studio program.

The circled 1 in Figure 2.6 is the OPL Project displayed projects in a file. It can contain multiple projects where each project consists of 3 files which are the model file (* .mod), the data file (* .dat) and the setting file (* .ops).

The circled 2 in Figure 2.6 is the editing area that can displayed the contents of the model file, the data file and the setting file.

The circled 3 in Figure 2.6 is the object window to display model objects in the project.

The circled 4 in Figure 2.6 is the processing window. It uses for displaying texts during the calculation of the solution.

Chapter III

Special Ordered Sets

In discrete optimization, the Special Ordered Set (SOS) technique is to assign binary decision variables according to a user defined order. It uses as an additional heuristic to specify integrality conditions in an optimization model. SOS is usually used in the branch and bound method for branching on a constrained set of variables [8].

One of the most important binary integer programming problems is the one where the binary variables are partitioned based on SOS. They were introduced by Beale and Tomlin [8]. The SOS technique can reduce the computational time of the branch and bound method.

3.1 Special Ordered Set type 1

Special Ordered Set type 1 (SOS1 or S1) is defined to be a set of decision variables that only one among these variables can only be set to 1 and other variables are zeros. This situation arises where the decision variable is a yes - no decision variable. Consider function $f(x)$ as shown in Figure 3.1.

Figure 3.1: Function $f(x)$ using SOS1.

Given the tabulated coordinates $(x_i, f(x_i)), i = 1, 2, \ldots, n.$

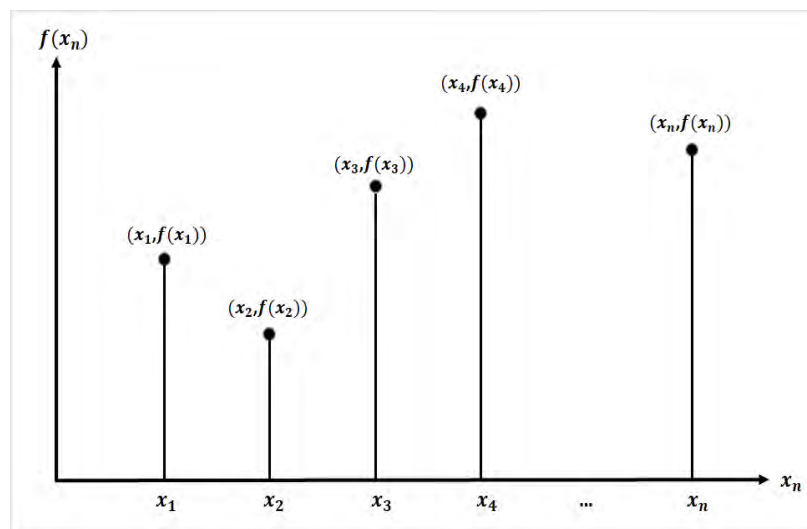A group of binary variables for SOS1 to be set for one of these function values are

$$f(x) = f(x_1)\lambda_1 + f(x_2)\lambda_2 + \ldots + f(x_n)\lambda_n \qquad (10)$$

$$x = x_1\lambda_1 + x_2\lambda_2 + \cdots + x_n\lambda_n \qquad (11)$$

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = 1 \qquad (12)$$

$$\lambda_1, \lambda_2, \ldots, \lambda_n \in \{0,1\}. \qquad (13)$$

Constraints (12) specifies that at most one binary variable is set to 1 and other binary variables are set to zero.

Constraints (13) specifies that each variable can only be set to 0 or 1 only.

The set of variables $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ is called a Special Ordered Set.

The weighting variables λ_n is called Special Ordered Set type one variables and the rows (10), (11) and (12) are called function rows, reference rows and convexity rows respectively.

3.2 Declaring using MPS

One way to declare SOS in ILOG CPLEX is to use the SOS declaration section within an MPS file [12].

MPS (Mathematical Programming System) is a file format for presenting and archiving linear programming (LP) and mixed integer programming problems [13]. It is a widely accepted standard for defining LP problems. MPS format is a column-oriented format where an LP problem is specified by column (variable) rather than by row (constraint) [14].

MPS data file consists of two types which are indicator records and data records. The indicator records separate the individual sections of the MPS file. Each indicator record contains a single word that begins in the first column.

The first section in an MPS file is the NAME section to specify the problem name. The second section is the ROWS section to specify names and senses for each constraint. These lines indicate the constraint type using a single character where E for equality, L for less-than-or-equal, or G for greater-than-or-equal, and the constraint name.

The third and largest section of an MPS file is the COLUMNS section to specify the name assigned to each variable (column) and the nonzero constraint coefficients corresponding to that variable. The beginning of an integer section is marked by an INTORG marker:

    MARK0000 'MARKER'   'INTORG'.

The end of the section is marked by an INTEND marker:

    MARK0000 'MARKER'   'INTEND'.

The fourth section is the RHS section which specifies the names of right-hand side vectors and values for each constraint (row). If no RHS section is present, CPLEX generates a warning message and sets all right-hand side values to 0. The fifth section is the BOUNDS section to specify the limits within which each variable (column) must remain. If no BOUNDS section is present, all variables have their bounds set from zero to positive infinity. The BOUNDS section is mandatory. The next section is the optional SOS section. The representation for a single SOS constraint contains one line that provides the type of the SOS set. Members of SOS should be given unique weights that in turn define the order of the variables in the set. The last line is ENDATA which signals the end of the MPS file.

Here's an example of declaring SOS1 in an MPS CPLEX format:

    In LP format:

$$\text{Min} \quad -x_1 - x_2 - 3\,x_3 - 2\,x_4;$$

$$\text{Subject to} \quad c_1: -x_1 - x_2 + x_3 + x_4 \le 30;$$

$$c_2: +x_1 + x_3 - 3\,x_4 \le 30;$$

$$x_1 \le 40;$$

$$x_2 \le 1;$$

In MPS format:

NAME	Example		
ROWS			
N	obj		
L	cons01		
L	cons02		
L	cons03		
L	cons04		
COLUMNS			
	MARK0000	'MARKER'	'INTORG'
	x1	obj	-1
	x1	cons01	-1
	x1	cons02	1
	x1	cons03	1
	x2	obj	-1
	x2	cons01	-1
	x2	cons04	1
	x3	obj	-3
	x3	cons01	1
	x3	cons02	1
	x4	obj	-2
	x4	cons01	1
	x4	cons02	-3
	MARK0001	'MARKER'	'INTEND'
RHS			
	rhs	cons01	30
	rhs	cons02	30
	rhs	cons03	40
	rhs	cons04	1
BOUNDS			

LI	bnd	x1	0
LI	bnd	x2	0
LI	bnd	x3	0
LI	bnd	x4	0
SOS			
S1	set1		
	x1	1	
	x2	2	
	x3	3	
	x4	4	
ENDATA			

Chapter IV

Experimental results

In this project, we reimplement the CPLEX model using the column generation technique for solving the crew rostering from Supphakorn's thesis [4] by focusing on only the in-flight manager (IM) crew members. We use the IBM ILOG CPLEX Optimization version 12.6.3 running on Intel(R) Core(TM) i3-6006U CPU B960 @2.20GHz 2.20 GHz, RAM 4.00 GB to solve the Thai Airway datasets.

4.1 Data description

In this project, we follow Supphakorn's thesis [4] and implement the crew rostering model from Thai Airway datasets having different number of pairings and the crew members.

Table 4.1: The details of each data set.

Data set	Number of working days	Number of pairings/days	Total number of pairings	Number of crew members
1	7	7	49	15
2	7	14	98	45
3	7	26	182	56
4	14	7	98	30
5	14	14	196	61
6	14	26	364	113

| 7 | 28 | 7 | 196 | 61 |
| 8 | 28 | 14 | 392 | 122 |

4.2. Computational Time from CPLEX

We did not report the results of the data set number 6 and 8 (364 and 392 pairings with 113 and 122 crews respectively) because they cannot be solved within 24 hours.

Table 4.2: The computational results of each data set.

Data set	Number of working days	Number of pairings/days	Total number of pairings	Number of crew members	Computational time (hh:mm:ss)
1	7	7	49	15	00:00:09
2	7	14	98	45	00:01:24
3	7	26	182	56	00:22:57
4	14	7	98	30	00:27:28
5	14	14	196	61	00:28:31
6	14	26	364	113	-
7	28	7	196	61	00:05:54
8	28	14	392	122	-

4.3 Append SOS to the MPS file from CPLEX

In Supphakorn's model, the column generation technique will generate the master problem and create a new set of columns and re-solved by the sub-problem which will be used with the master problem. After the master problem is optimal, then additional columns are

included in the problem. When we export MPS from this problem, the optimal solution appears in MPS. Adding SOS1 and SOS2 solves the problem in less than a second so we cannot use SOS from MPS from CPLEX based on the Supphakorn's model.

In this section, we demonstrate the advantage of using SOS in MPS via simulated models. Define x_i as a binary variable which $i=0,1,...,n$. Since, we assume that we can choose only one of x_i. The example model consists of 3 sets: n=4, n=9 and n=99. So, those five variables, ten variables and one hundred variables are members of a special ordered set respectively. And we assume that there is an order relationship among the x_i variables as follows in Appendix I,II and III.

Table 4.3: Show the computational time between the original model which solves via the IBM ILOG CPLEX Optimization, and MPS appending SOS1 which solves via the interactive command of CPLEX.

	The computing time	
	Original Model (sec.)	MPS format with SOS1 (sec.)
Example SOS1 with 5 variables.	0.34	0.06
Example SOS1 with 10 variables.	0.46	0.09
Example SOS1 with 100 variables.	0.56	0.16

The Table 4.3 show that the benefit of using Special Ordered Sets type 1 (SOS1) is decrease the computing time to find the result.

Chapter V

Conclusion

We successfully run ILOG CPLEX to solve the Supphakorn's crew rostering problem. Then we use SOS1 to decrease computing time of the integer programming model. We transform the original model (Supphakorn's model) to MPS file format. Nevertheless, the generated MPS is not only included the original constraints but it also includes the columns of the optimal solution. Therefore, adding the SOS section in MPS and execute them just returns the result immediately. Moreover, we could not generate the crew rostering problem as MPS since the column generation technique will add more columns that leads to the optimal solution. All imported data from excel must be done via external programming or generating of the column must be done via script. We can demonstrate only toy world problems using SOS which speeds up the solution time.

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Appendix

Appendix I : Code of the toy world problem using SOS1 with $x_i, i = 0,1,\ldots,4$.

```
//variables
int n = 5;
range I = 1..n;
int c[I] = [2,8,2,6,10];
int j=3;
range J=1..j;
int a[J][I] = [[0,3,0,4,7],[2,0,6,0,8],[0,3,9,2,0]];
int r=3;
range R=1..r;
int Cap[R]= [30,20,10];
 // Decision Variable (Select if = 1 -> selected : if = 0 -> NO selected )
dvar int+  x[I]in 0..1;


//Objective Function
minimize sum(i in I) c[i]*x[i];
subject to {
  con01: sum(i in I , r in R, j in J) a[j][i]*x[i] <=Cap[r];
  sum(i in I) x[i]==1;
}
```

Appendix II : Code of the toy world problem using SOS1 with $x_i, i = 0,1,\ldots,9$.

```
//variables
int n = 10;
range I = 1..n;
int c[I] = [5,8,2,6,10,3,5,7,8,11];
int j=3;
range J=1..j;
int a[J][I] = [[6,3,1,4,7,2,2,3,5,9],[2,5,6,7,8,2,1,4,3,0],[1,3,9,2,4,5,1,6,7,2]];
int
 r=3;
range R=1..r;
int Cap[R]= [30,20,10];
 // Decision Variable (SelectRoster  if = 1 -> selected : if = 0 -> NO selected )
dvar int+  x[I]in 0..1;


//Objective Function
minimize sum(i in I) c[i]*x[i];
subject to {
  con01: sum(i in I , r in R, j in J) a[j][i]*x[i] <=Cap[r];
  sum(i in I) x[i]==1;
}
```

Appendix III : Code of the toy world problem using SOS1 with $x_i, i = 0,1,\ldots,99$.

```
//variables
int n = 100;
range I = 1..n;
int c[I] =
[5,8,2,6,10,3,5,7,8,11,3,2,6,7,8,9,10,11,12,16,13,11,5,9,7,3,15,3,8,9,11,15,14,3,2,1,6,1,7,5,9,10,2,3,6,11,2,1,8,6,17,2,1,3,1,4,2,1,8,11,12
,18,19,14,21,8,7,12,10,2,1,8,11,17,13,1,2,8,9,10,16,2,5,8,2,5,6,12,14,13,11,2,21,16,17,12,13,1,2,3];
int j=3;
range J=1..j;
int a[J][I] =
[[4,5,1,2,3,1,6,1,8,3,7,9,11,12,13,2,10,11,7,5,1,4,0,2,3,2,1,1,8,11,12,7,1,5,8,7,9,1,5,0,2,6,7,9,0,4,8,0,1,11,12,13,15,0,1,0,0,2,3,5,7,6,1,2,
3,4,5,7,8,9,10,10,1,0,2,3,0,1,0,5,7,0,8,0,10,11,7,5,2,3,0,1,1,1,4,0,2,3,1,5],[1,8,11,12,7,0,1,5,2,1,11,3,0,1,2,5,7,1,8,0,10,11,7,5,2,3,0,3,2,1
,4,0,2,3,0,1,0,0,12,13,1,5,0,1,0,0,2,3,5,7,6,1,2,3,4,5,7,8,9,10,15,16,17,15,0,0,0,1,0,2,0,6,4,2,0,2,8,0,5,8,0,4,0,1,2,3,7,5,9,2,3,4,11,13,6,1
1,18,15,0,1],[2,4,8,10,2,7,0,5,3,1,2,7,9,7,11,1,1,12,13,15,3,0,1,0,5,7,0,8,0,10,11,7,5,2,3,0,11,12,7,0,5,8,7,9,1,5,0,2,6,7,9,0,4,8,0,1,11,12,
13,15,1,1,2,0,2,3,5,7,6,1,2,3,4,5,7,8,9,10,3,0,1,0,5,7,11,7,5,2,3,4,5,2,1,4,1,2,3,0,1,3]];
int r=3;
range R=1..r;
int Cap[R]= [30,20,10];
 // Decision Variable (SelectRoster  if = 1 -> selected : if = 0 -> NO selected )
dvar int+  x[I]in 0..1;


//Objective Function
minimize sum(i in I) c[i]*x[i];
subject to {
  con01: sum(i in I , r in R, j in J) a[j][i]*x[i] <=Cap[r];
  sum(i in I) x[i]==1;
}
```

Appendix iv: Project Proposal

กลุ่มที่ ....

เอกสารนี้ได้รับการอนุมัติจากอาจารย์ที่ปรึกษาโครงงานแล้ว

ลงชื่อ ....

(วันที่ 12 Nov. 2019)

The Project Proposal of Course 2301399 Project Proposal
Academic Year 2019

Project Tittle (Thai)	การศึกษาเซตอันดับพิเศษ สำหรับเทคนิคการก่อกำเนิดคอลัมน์เพื่อแก้ปัญหาการ ปรับดุลการะงานของการจัดตารางพนักงานสายการบิน
Project Tittle (English)	The study of special ordered sets for the column generation technique to solve a workload balancing crew rostering problem.
Project Advisor	Assistant Professor Krung Sinapiromsaran
By	Miss Savinee Mueangsap ID 5933552823
	Mathematics Science Program, Department of Mathematics and Computer Science. Faculty of Science, Chulalongkorn University

Background and Rationale

An airline industry has an important impact on economic development of a country. It increases world trade activities by enabling faster and easier movement of passengers and goods, also provides jobs to millions of people. An airline receives revenue from passenger tickets and transporting goods which uses to pay for aircraft, fuel, crew and airport usages. The largest expense comes from the fuel cost while the crew cost constitutes the second-largest expense for the airline. One important management issue is the crew scheduling which consists of flights, pairings and days off for each crew member under constraints which are cost minimization and regulations between company and employee. The crew rostering problem is complex and difficult due to a large number of crews and flights.

From Suphakorn's thesis, he solved the crew rostering problem using the column generation technique which it is a technique used to solve linear programming problems consisting of many variables via the concept from decomposition principle by Dantzig and Wolfe 1960. Then, In 1984 Desrosiers, Soumis and Desrochers was the first group of researchers to use these ideas to create column generation technique with the branch and bound method. In this project we propose to study of SOS (special ordered sets) used in the branch and bound method for branching on sets of variables to help reduce the solution time for solving the crew rostering problem.

Objectives

1. To study SOS (special ordered sets) for solving an airline crew rostering problem by assigning appropriate balance workload for each crew member.

2. To compare and analyze the results of using SOS with an airline crew rostering problem.

Scope

1. Eight datasets from Thai Airways International to schedule crews were extracted from the Master thesis "Column generation technique for crew rostering problem with workload and earnings balancing" written by Acting Sub Lt. Supphakorn Sumetthapiwat, 2012.

2. Crews for the crew rostering problem will be considered only employees in-flight manager.

3. The IBM ILOG CPLEX Optimization will be used to solve an airline crew rostering problem.

Project Activities

1. Study background for tools to solve the crew rostering problem.

2. Collected and transform data into appropriate format.

3. Study the concept of SOS related to the branch and bound method.

4. Write the new model and solve via the CPLEX Optimization.

5. Compare the performance of 8 datasets from Thai Airway.

6. Write the senior project report.

Schedule

Project Activities	August 2019 – April 2020								
	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr
1. Study background for tools to solve the crew rostering problem.	■								
2. Collected and transform data into appropriate format.		■							
3. Study the concept of SOS related to the branch and bound method.			■						
4. Write the new model and solve via the CPLEX Optimization.					■				
5. Compare the performance of 8 datasets from Thai Airway.							■	■	
6. Write the senior project report.								■	■

Benefits

1. Comprehend the concept of SOS (special ordered sets) and apply to the column generation technique to solve the airline crew rostering problem.

2. Experience solving optimization using the CPLEX Optimization.

Equipments

1. Hardware
   - Laptop
     Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz 3.60 GHz
     RAM: 8.00 GB
2. Software
   - IBM ILOG CPLEX Optimization
3. Material
   - A4 paper

Budget

1. Paper	800	Baht
2. Pagemaking for a book	100	Baht
3. Hard Disk	3190	Baht
4. Keyboard and Mouse Wireless	910	Baht

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