Why is the third principal component of the yield curve important:
A point of view from reverse stress test on credit portfolio.

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จุฬาลงกรณ์มหาวิทยาลัย
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ทำไมองค์ประกอบหลักที่สามของเส้นอัตราผลตอบแทนถึงมีความสำคัญ
ในการทดสอบสภาวะวิกฤตย้อนหลังบนพอร์ตโฟลิโอเครดิต

นายภานุวัฒน์ ฤทธิ์พรณรงค์

จุฬาลงกรณ์มหาวิทยาลัย
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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต
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ภานุวัฒน์ ฤทธิ์พรณรงค์ : ทำไมองค์ประกอบหลักที่สามของเส้นอัตราผลตอบแทนถึงมีความสำคัญ ในการทดสอบสภาวะวิกฤตย้อนหลังบนพอร์ตโฟลิโอเครดิต. (Why is the third principal component of the yield curve important: A point of view from reverse stress test on credit portfolio.) อ.ที่ปรึกษาหลัก : รศ. ดร.สิระ สุจินตะบัณฑิต

งานวิจัยนี้ ศึกษาถึงรูปร่างเส้นอัตราผลตอบแทนของพอร์ตโฟลิโอเครดิตในสภาวะวิกฤต โดยเราใช้การทดสอบสภาวะวิกฤตย้อนหลังบนพอร์ตโฟลิโอเครดิตที่มีมูลค่าเกี่ยวข้องกับความเสี่ยงอัตราผลตอบแทนและความเสี่ยงเครดิต เราใช้ตัวบ่งชี้เศรษฐกิจมหภาค ระหว่างปี ๒๕๒๔ ถึง ๒๕๕๗ สำหรับการประมาณค่าทางสถิติให้แบบจำลองทางคณิตศาสตร์และใช้วิธีมอนติคาร์โลในการค้นหาสภาวะวิกฤตที่มีความน่าจะเป็นสูงที่สุดเมื่อมูลค่าของพอร์ตโฟลิโอลดลงจนถึงค่าที่กำหนด

รูปร่างของเส้นอัตราผลตอบแทนจะมีความแตกต่างกันเมื่อเราเพิ่มหรือลดองค์ประกอบหลักที่สามของเส้นอัตราผลตอบแทน เราค้นพบว่าพอร์ตโฟลิโอเครดิตที่มีวันครบกำหนดอายุระหว่าง ๖ เดือนถึง ๓ ปี จะมีรูปร่างของเส้นอัตราผลตอบแทนในระยะ ๖ เดือนถึง ๓ ปีมีลักษณะโก่งโค้งในการทดสอบที่เพิ่มองค์ประกอบหลักที่สามเท่านั้น เมื่อเราเพิ่มระดับความวิกฤตของพอร์ตโฟลิโอจะพบว่ารูปร่างโก่งโค้งจะสูงมากขึ้น บ่งบอกถึงมูลค่าของพอร์ตโฟลิโอถูกกระทบจากการที่อัตราผลตอบแทนเพิ่มขึ้น

ผลจากงานวิจัยนี้เป็นประโยชน์สำหรับสถาบันการเงินในการบริหารสินทรัพย์และหนี้สินที่มีพอร์ตโฟลิโอที่มีวันครบกำหนดอายุระหว่าง ๖ เดือนถึง ๓ ปี ในการเข้าใจถึงผลกระทบและพิจารณาใช้องค์ประกอบหลักที่สามของอัตราผลตอบแทน

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# # 6284053926 : MAJOR FINANCIAL ENGINEERING
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Phanuwat Ritpornnarong : Why is the third principal component of the yield curve important: A point of view from reverse stress test on credit portfolio.. Advisor: Assoc. Prof. SIRA SUCHINTABANDID, Ph.D.

The focus of this study is to analyze the shape of the yield curve on credit portfolios during crises, specifically those that are exposed to both interest rate and credit risks. To achieve this, We utilized a reverse stress test (RST) and macroeconomic measures (such as GDP and U.S risk-free yields) from the period of 1981 to 2014 to estimate mathematical models. We then utilized a Monte Carlo simulation to determine the most likely scenario for the measures if the portfolio value reaches a pre-specified threshold.

The researchers discovered that the shape of the stressed yield curves varied depending on whether or not the third principal component of the yield curve was included in the analysis. They found that stressed credit portfolios with maturities ranging from 0.5 to 3 years exhibited a humped shape on yields within that range when the third principal component was included. As stress levels increased, this humped shape became more pronounced, indicating that the stressed credit portfolio's value was impacted by rising yields.

This study's findings have significant implications for financial institutions as they consider the third principal component in their asset-liability management for portfolios with maturities between 0.5 and 3 years.

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1. INTRODUCTION

The pursuit of crisis-fighting tools has been a hot topic among financial institutions to stand resilient during adverse events. Financial crises occur so rarely in human lifetime that they are usually underestimated. Financial institutions must develop tools that identify, quantify potential risks during crises to their financial portfolios. Once the risks are identified, the risk management teams then can come up with actions that might involves increasing risk capitals or mitigating risks through various instruments. As if that were not enough, the team must be prepared not only for expected risks but also unexpected risks which may have never happened during human lifetime.

Risk management teams usually have insufficient historical data of crises which rarely happen. Statistical tools must be carefully chosen so that the tools could detect rare events (tail events) as well as forecast what would likely happen to financial portfolios during the events. Also, the tools must not be too complicated – the tools should not include too many risk up to the point of being too redundant.

Knowledge about correlation between various risks during crises is also very important. Risk factors that seem to not correlate during normal time can highly correlate during crises. We give an example of default rates and interest rates that are hardly correlated during normal time but become highly correlated during crises – hiking interest rates pressure floating rate mortgage payers to default because of inability to pay.

Popular tools to deal with crises are ST (Stress test) and RST (Reverse stress test). ST is a scenario analysis tool that test financial portfolios with extreme stressed scenario. Such as,

- What happen if interest rate rise goes up by at least 10%
- What happen if GDP falls by at least 5%

ST give results of stressed portfolio. If the stressed value fails the allowable threshold, actions must be taken. Meanwhile, RST gives us the appropriate stressed scenario that will be used to test the portfolio.

In this paper, we would like to contribute to RST. Our RST has two risks (Credit and interest rate risk) and two macroeconomic variables (GDP and risk-free interest rate).

Firstly, We explored RST on credit portfolio from (Grundke and Pliszka 2015). The RST combined interest rate and credit risk to stress the credit portfolios. The paper also introduces us to PCA which reduces dimensionality of yield curve data. Yield curve data can be up to ten variables depending on how much maturity we would like to select (1-y, 3-y, 5-y,…). Yield curve data with so many variables are very tedious. This is when PCA is introduced – PCA breakdown into two parts PC (Principal component) and PC score (Principal component score).

The yield curve data can be reduced to only three most relevant components called PC. Practitioners usually perceive the first three PCs of yield curve as level, slope, and curvature changes of the yield curve. PC scores determine scalar value of how much the changes.

Literatures about credit portfolio RSTs so far do not focus much on how different PCs can result in different stressful scenarios. We also collect some literature about the PCs of yield curve and their economic interpretation.

The research questions focus on the 3rd PC of yield curve – the curvature component. We study how the effect of excluding/including on our RST can affect the shape of stressed yield curve.

Positive 3rd PC score creates hump shape (elevated medium-term yields) that could lead to price drop in medium-term credit portfolios. We also study how the hump shape related to investors' perception on credit events such as credit downgrading.

The hump shape can also indicate rising default rates since elevated yields pressure borrowers. Therefore, we will see how the inclusion/exclusion of the 3rd PC could make the shape of stressed yield curves different.

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We also find out when including the 3rd PC does not matter. We believe that the 3rd PC does not matter for bad initial credit portfolios. Credit defaults and downgrades should stress bad initial credit portfolios rather than elevated yields.

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The remainders of this paper are constructed as follows.

- In section 2, some brief backgrounds that it could help readers before reading our research questions.
- In section 3, literatures review and our research questions.
- In section 4, our models, and mathematical tools for this paper.
- In section 5, historical data for this paper.
- In section 6, results that answer our research questions.
- Lastly section 7, conclusion on the results.

2. BACKGROUND

2.1 WHAT IS REVERSE STRESS TEST ON CREDIT PORTFOLIO?

Let Bank A owns a credit portfolio with a known market price. Interest rate and GDP (Gross domestic product) can influence the market value of the portfolio. Given next year's expected market portfolio value from the analyst team, Bank A will only allow the portfolio's market price to fall to not over half of the expected market value in the next year. RST (reverse stress test) is a tool to answer the following question: "How much change in interest rate and GDP which is most likely to happen and cut the expected portfolio value by half?"

The value of credit portfolio V is affected by change of interest rate R and GDP X. Knowing only one scenario $\omega_i = \{R = r_i, X = x_i\}$ will give conditional expected value $\mathbb{E}[V|\omega_i]$. Knowing all plausible value of $\mathbb{E}[V|\omega]$, we then can calculate expected port value $\mathbb{E}[V]$.

We are interested in finding scenario ω that stress the portfolio value (conditional expected value is far less than expected port value $\mathbb{E}[V|\omega] << \mathbb{E}[V]$).

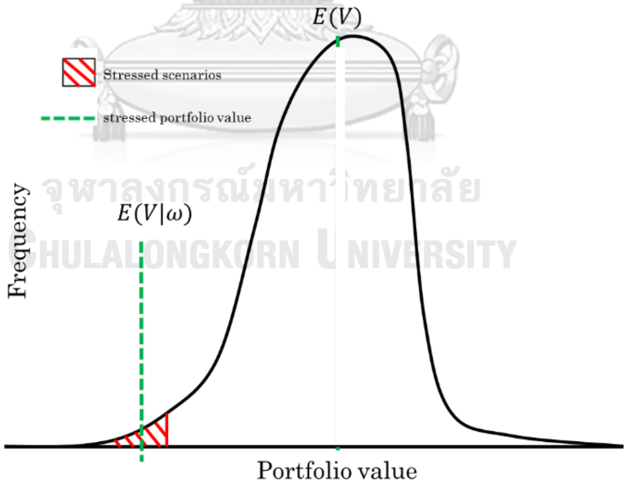

Figure 1
Illustration of how a scenario ω stresses the portfolio value $\mathbb{E}[V|\omega]$ to the point of being much less than the value of expected portfolio $\mathbb{E}[V]$. ($\mathbb{E}[V|\omega] << \mathbb{E}[V]$)

Let B be an allowable threshold the portfolio value can be, e.g., half of the expected value ($B = 0.5\mathbb{E}[V]$). RST finds a scenario $\omega = \{r, x\}$ that stresses the port value $\mathbb{E}[V|\omega]$ to half ($\mathbb{E}[V|\omega] = B$). If there are other

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scenarios $\omega_1, \omega_2, \omega_3$ that give the same threshold $\mathbb{E}[V|\omega_i]_{i=1,2,3} = B$. The only scenario with highest likelihood $L(\omega)$ will be chosen.

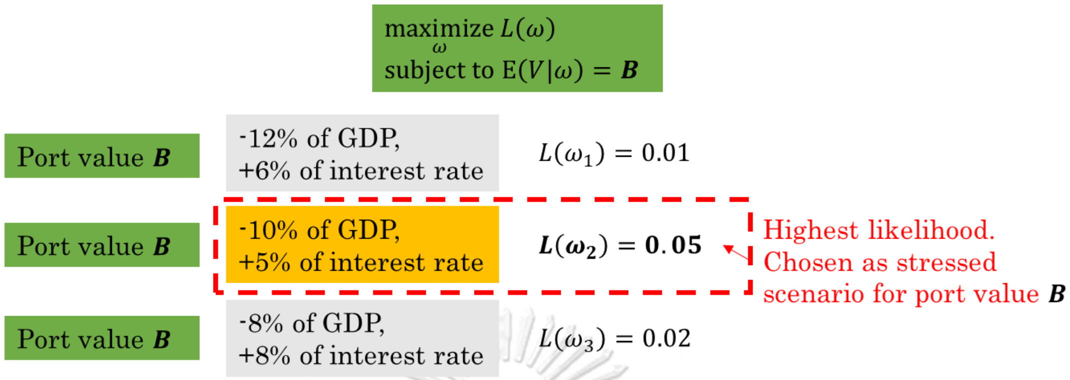

Figure 2

In RST, the scenario with highest likelihood will be chosen given a value of stressed portfolio B.

2.2 RISK FACTORS SELECTION FOR REVERSE STRESS TEST

Relevant risk factors must be carefully chosen for the RST. The cost of computation will be high if too many risk factors ($\omega = \{R, X, \dots\}$) are included to calculate the portfolio value $\mathbb{E}[V|\omega]$.

This paper will explore the approach of portfolio valuation from (Grundke and Pliszka 2015). A credit portfolio consists of multiple defaultable zero-coupon bonds with given maturities and initial credit ratings from the best to the worst (e.g., credit ratings $AAA, AA, .., CCC/C$). At valuation date in the future, the bonds can change the credit ratings. The value of individual credits can be calculated from its credit spread and risk-free interest rates (treasury yield). The main risk factors are as follows.

- Z = latent systematic risk factor (hidden risk)
- X = change in GDP (Gross domestic product)
- $C^{j=1,\dots,k}$= the treasury yield PC score (Principal component score)

The treasury yields and credit spreads are used to calculate the value of individual credits. In this paper, we assume that treasury yields change are influenced by the PC score. The changes in credit ratings of individual credits are influenced by hidden risk factor, GDP and PC scores $Z, X, C^j, j = \{1, \dots, k\}$. Recovery rate is independent.

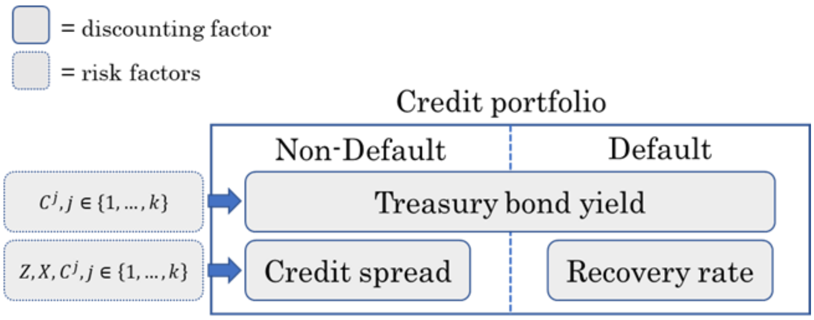

Figure 3

Discounting factors of the portfolio valuation in this paper. Individual credits in the portfolio will be discounted by treasury bond yields, credit spreads, recovery rates.

2.3 THE THREE PRINCIPAL COMPONENT OF YIELD CURVE

To value a portfolio consists of several maturities in the future, the movement of the yield curves must be simulated. The linear equation of changes in yield curve is as follows.

$$\overbrace{\begin{bmatrix} r^{3m} \\ r^{6m} \\ \vdots \\ r^{30y} \end{bmatrix}}^{\tilde{R}\ =\ \text{Changes in yield curve}} = C^1\boldsymbol{u}^1 + \cdots + C^k\boldsymbol{u}^k = C^1 \overbrace{\begin{bmatrix} {u_1}^{3m} \\ {u_1}^{6m} \\ \vdots \\ {u_1}^{30y} \end{bmatrix}}^{PC1} + \cdots C^k \overbrace{\begin{bmatrix} {u_k}^{3m} \\ {u_k}^{6m} \\ \vdots \\ {u_k}^{30y} \end{bmatrix}}^{PCk} \tag{1}$$

1. PC scores $C^1, \ldots, C^k$ which are dynamic scalar value over time
2. PCs $\boldsymbol{u}^1, \ldots, \boldsymbol{u}^k$ which are static vector over time

PC scores and PCs can generate the changes of yield curve $\tilde{R}$ with maturities $\{3m, 6m, \ldots, 30y\}$. The changes of yield curve can be used on initial yields to calculate simulated yield curves.

Practitioners usually choose only the first 3 principal components as the three components detect most of the movement of yield curves. The principal components also reduce dimensionality of the yield curve data (Patel, Mohamed et al. 2018).

- The first PC is parallel shift (level).
- The second PC is tilt (slope).
- The third PC is bend (curvature).

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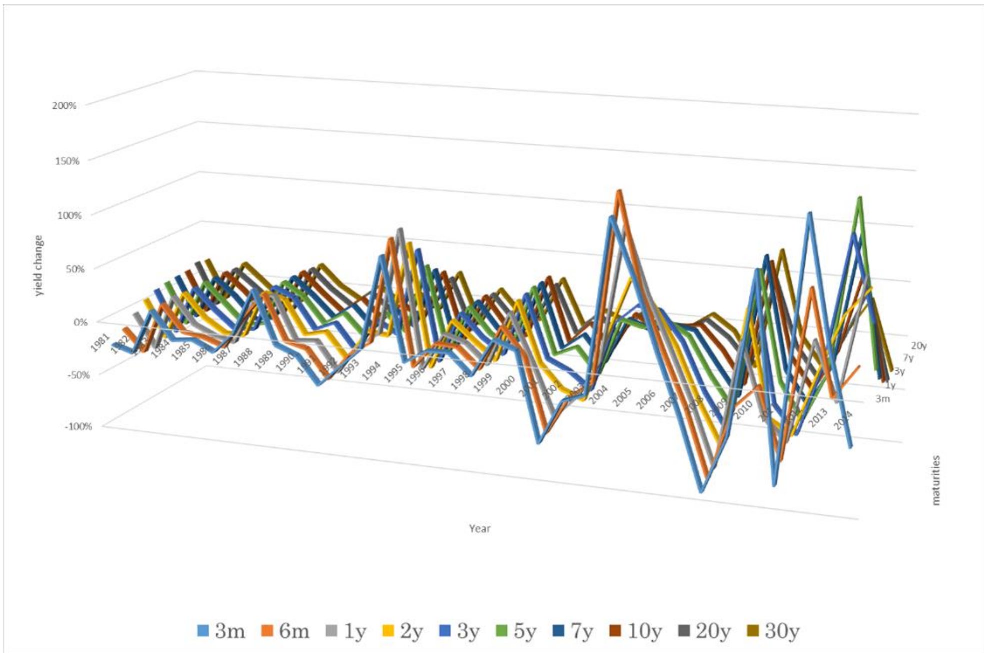
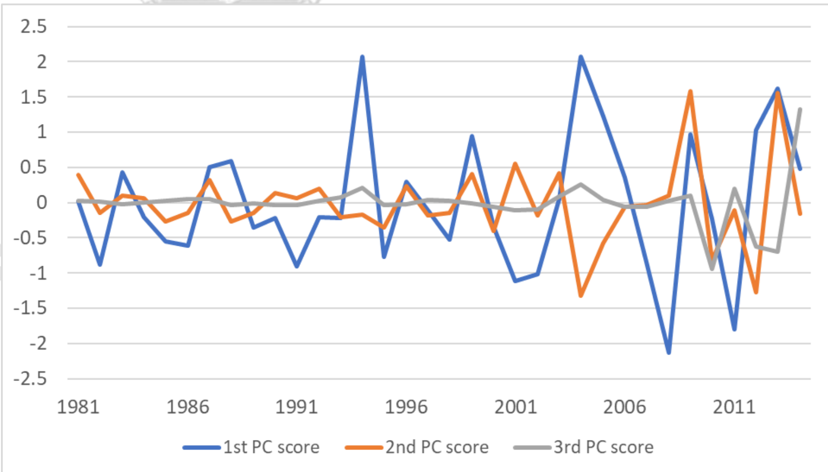


Figure 4
Annual changes of U.S. treasury yield curves (secondary market) from year 1981-2014. The vertical axis represents yield changes in percentage. The horizontal axis represents times in year, The depth axis represents maturities of yield curves (6-m to 30-y).

(Figure 5u) Principal component score (PC score)



(Figure 5l) Principal component (PC)

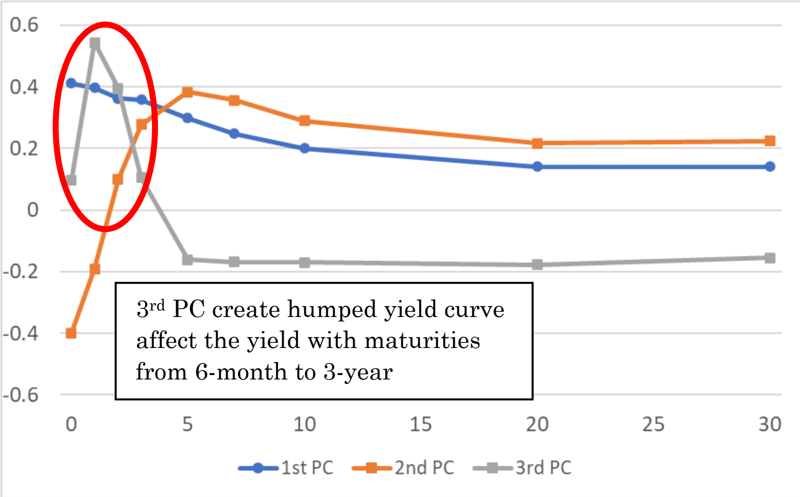


Figure 5

(upper) PC scores C^1, C^2, C^3 of U.S. treasury yield curves (secondary market) from year 1981-2014. The vertical axis represents PC scores in percentage. The horizontal axis represents times of year.

(lower) PCs $\boldsymbol{u}^1, \boldsymbol{u}^2, \boldsymbol{u}^3$ of U.S. treasury yield curves (secondary market) from year 1981-2014. The vertical axis represents the principal components (a.k.a. factor loadings in some literatures). The horizontal axis represents maturities of yield curves in year.

Noted that the 3rd PC affects maturities from 6-month to 3-year. The maturities have the highest proportion of corporate bonds traded by amount compared to other maturities during 2000 to 2020.

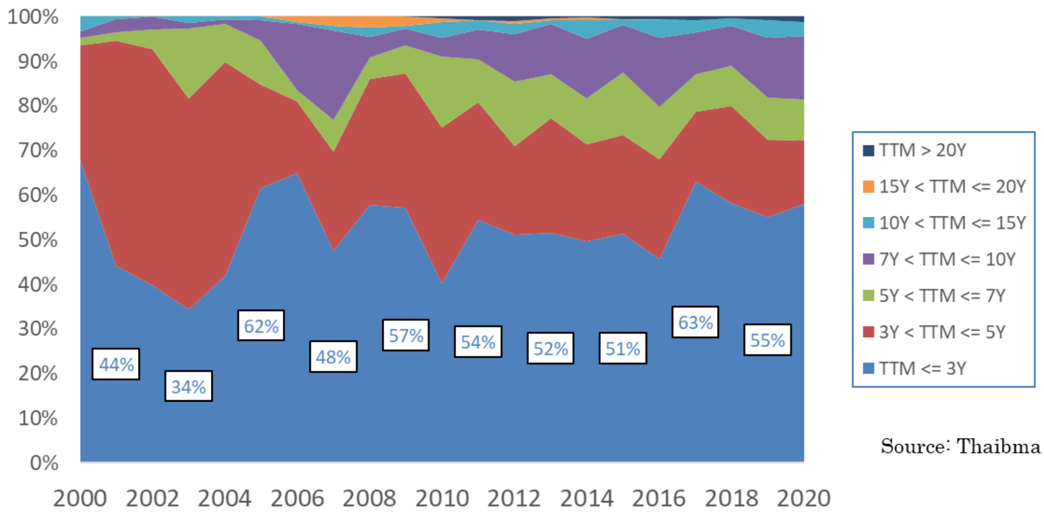


Figure 6

The proportion of traded corporate bonds by group of time to maturities (TTM): Bonds with TTMs between 6-month to 3-year are the most traded by amount compared to other TTMs in the corporate bond market during the year 2000 to 2020.

3. LITERATURE REVIEW AND RESEARCH QUESTIONS

This section will explore what studies have been done regarding the development of credit portfolio RST (reverse stress test). We then produce research questions with our hypotheses regarding the 3rd PC of treasury yield curves used on our RST of credit portfolios.

3.1 LITERATURE REVIEW

(Grundke 2005) first mentioned of integrating interest rate risk factors and credit risk factors on valuation of credit portfolio. The portfolio consisted of defaultable zero-coupon bonds. The prices of the bonds are discounted by stochastic risk-free interest rates and credit spreads. If a bond defaults, the value of the bond will be determined by recovery rate.

(Grundke 2011) RST (Reverse stress test) on credit portfolios was developed based on valuation model by (Grundke 2005). Their RST found the scenarios that are most likely to happen given threshold of portfolio losses. (Grundke and Pliszka 2015) then used the same RST framework with historical macroeconomic data and let the RST gave stressed scenarios of it – U.S. GDP (gross domestic product) and treasury yield.

The RST framework by (Grundke and Pliszka 2015) use PCA (Principal component analysis) to identify the three most relevant movements of U.S. treasury curve from historical data. It was concluded that only the first two PCs (Principal components) are sufficient for their credit portfolio reverse stress test – Level and slope movements. The first two PCs cover about 95% of the yield curve movement variability while PC scores are scalar value telling how much the movements.

Many literatures on the PCs of U.S treasury yield curves suggested that the 3rd PC can be a crucial risk factor. The 3rd PC explains bending (curvature) movement of treasury yield curves (Patel, Mohamed et al. 2018). The bending movement creates humped yield curve that elevate medium maturity rates (less than 5-year) and can be harmful to medium maturity portfolios.

The 3rd PC also indirectly affects the credit events as well. There were positive correlations between occurrence of the humped treasury yield curve and default rates (Moench 2012). As the burden of borrower increases as interest rates rise.

In this paper, we seek the differences between including and excluding the 3rd PC from the RST framework by (Grundke and Pliszka 2015). The shapes of the stressed yield curves are especially important for risk management teams to assess financial portfolios.

3.2 RESEARCH QUESTIONS

> *Research question 1: Does the importance of the 3rd PC (3rd principal component) depend on the initial rating of the portfolio, provided that the maturity of the portfolio is medium-term? Specifically, between a bond portfolio with good initial rating and one with bad initial rating, which portfolio has a stressed yield curve shape that is more sensitive to the inclusion/exclusion of the 3rd PC? We will try with several stress levels of portfolio value.*

(Hypothesis 1) For **medium maturity portfolio (2-year)**, we expect that the shapes of stressed yield curves show humped shape that raises the treasury yields between 0.5-year to 3-year when the 3rd PC risk factor is included.

1. **2-year, good credit rating portfolio -** the shapes of stressed yield curves should be different when the 3rd PC is included (compared to when it is not). Stressed yield curves should show hump shape when the 3rd PC is included. For higher stress level (expected portfolio value worsen), the hump should be higher as it elevated the yields between 0.5-year to 3-year more.
2. **2-year, bad credit rating portfolio -** the shapes of stressed yield curves should not be different whether the 3rd PC is included or not. Stressed yield curves should not show hump shape when the 3rd PC is included. For higher stress level, the shape of stressed yield curves shape should be the similar between including or excluding the 3rd PC

(Hypothesis development 1) Credit portfolios in this paper are affected by credit risk and interest rate risk. Credit risk consists of credit rating downgrades and defaults. Interest risk is when credit value falls as interest rate rises.

We first focus on a 2-year, good credit rating portfolio. Good credit borrowers are unlikely to be exposed to credit risk. They should be immune to defaults or credit downgrades. Instead, their credit market value should be more likely to fall from rises in interest rates. The 3rd PC will create a hump shape that raises treasury yields between 0.5-year to 3-year affecting the 2-year maturity of the portfolio. We expect that the stressed yield curves will show humped shape when 3rd PC is included. We should see the difference of stressed yield curves between including and excluding the 3rd PC where we see hump shape only from including the 3rd PC.

2-year, bad credit rating portfolio should be more likely stressed by credit risk. The shapes of stressed yield curve should matter less. The shapes of stressed yield curves should be similar whether the 3rd PC is included or not.

We will also experiment on several stress levels on the credit portfolios. The expected portfolio value will worsen when stress level is higher. When the good credit portfolio is stressed, the 3rd PC should raise the yields between 0.5-year to 3-year higher, making the stressed yield curve humpier. The shape of stressed yield curve on bad credit portfolio should stay the same when the stress level is higher.

Research question 2: Does the importance of the 3rd PC (3rd principal component) depend on the initial rating of the portfolio, provided that the maturity of the portfolio is long-term? Specifically, between a bond portfolio with good initial rating and one with bad initial rating, which portfolio has a stressed yield curve shape that is more sensitive to the inclusion/exclusion of the 3rd principal component? We will try with several stress levels of portfolio value.

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(Hypothesis 2) The shapes stressed yield curves on **long maturity portfolio (10-year)** should not show hump shape whether the 3rd PC is included or not.

1. **10-year, good credit rating portfolio -** the shapes of stressed yield curves should be similar whether the 3rd PC is included or not. The hump shape should not appear at any stress level.
2. **10-year, bad credit rating portfolio -** the shapes of stressed yield curves should not be similar whether the 3rd PC is included or not. The hump shape should not appear at any stress level.

(Hypothesis development 2) The 3rd PC highly affects medium maturity yields between 0.5-year to 3-year. Therefore, the shapes of stressed yield curves on longer maturity portfolio (more than 3-year)

should be less affected by the 3rd PC – The stressed yield curves should not show hump shape by the 3rd PC.

The portfolios should be more likely stressed by rising interest rate from the 1st PC and 2nd PC. Therefore, including the 3rd PC to the RST makes no difference to the stressed yield curve.

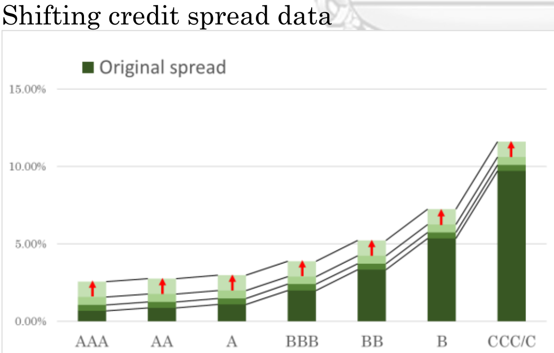
Research question 3: How does the spread scale (the credit spread of bonds across different credit ratings) affect the extent to which the inclusion/exclusion of the 3rd principal component alter the stressed yield curve?

(Hypothesis 3) With **2-year good credit ratings**, we will try to **perturb the credit spread scales** (collection of credit spreads for each credit rating) by raising it up in two ways.

1. Shift the credit spread: all credit rating spreads get the same basis point increase.

2. Tilt the credit spread: bad credit ratings spreads get more basis point increase compared to good credit ratings.

Shifting credit spread data



Tilting credit spread data

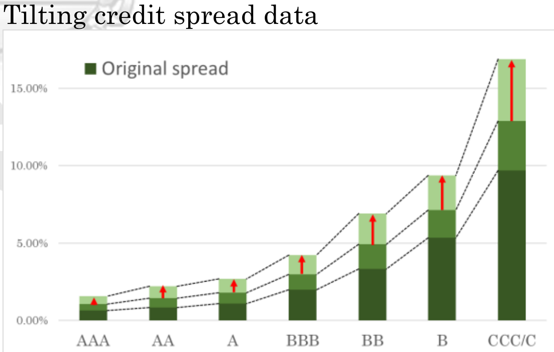


Figure 7

Shifting credit spread data, we add basis points to the original credit spread data equally.

Tilting credit spread data, we add more basis points to the bad credit rating (BBB to CCC/C).

Leaving other environments constantly. We expect that both shifting and tilting credit spread scales will diminish the hump shape on stressed yield created by the 3rd PC. Furthermore, we expect that tilting the spread scales diminishes the hump shape faster than shifting the spread scales.

(Hypothesis development 3) Given recent event of GFC (global financial crisis) in 2008, we had observed surging credit spreads that severely stress credit market value. We would like to see how the hump shape of stressed yield curves changes as we manually change the credit spread data.

In the 3rd research question, we would like to see how perturbing the credit spread scales would change the hump shape of stressed yield curve created by the 3rd PC on 2-year, good credit portfolio.

We will raise the credit spread scale data (originally used in research question 1) by shifting and tilting with small steps. In doing so, we expect the hump shape of stressed yield curves to gradually diminish as credit portfolio will be stressed by higher spreads rather than interest rates.

Comparing between shifting and tilting, we expect tilting the credit spread should make credit risk even more likely at stress level and diminish the hump shape faster than shifting.

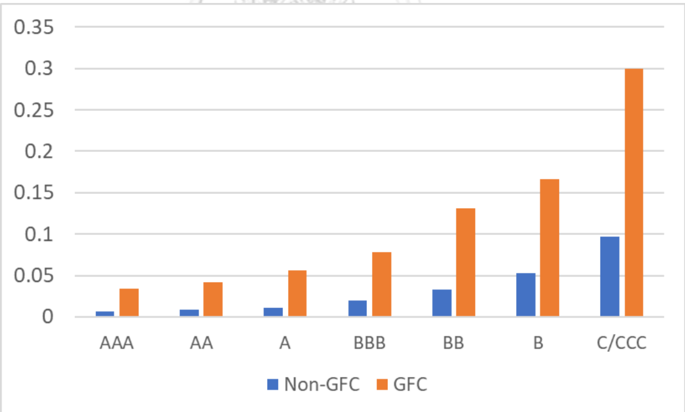


Figure 8
Example of surging credit spreads during GFC compared to non-GFC year (2014). Bad credit ratings (Lower than BB) got much more increase in credit spread compared to good ratings (Higher than BB)

4. METHODOLOGY

We will introduce mathematical tools that build the reverse stress test in this paper. We modify the framework developed by (Grundke and Pliszka 2015) and use it to answer our research questions.

(Section 4.1) Firstly, reverse stress test is a constrained optimization problem where we want to find the most likely set of risk factors ω^* given pre-specified stressed portfolio value B.

(Section 4.2) The set ω contains realization of risk factors $\omega = (Z, X, C^1, C^2, C^3)$ which are jointly distributed by likelihood function $P(\omega)$,

(Section 4.5) For every possible set of risk factors $\omega = (Z, X, C^1, C^2, C^3)$, portfolio value $\mathbb{E}[V|\omega]$ must be calculated. The portfolio value is a sum of individual credit face values discounted by treasury bond yields and credit spreads. The credit spread scales and credit recovery rate historical data will be used in this procedure. However, the portfolio configuration such as maturity and initial credit rating will be modified.

(Section 4.3) The simulated treasury bond yield is calculated by PC score C^1, C^2, C^3. The historical market U.S treasury yield curve data will be used.

(Section 4.4) Credit spread of each individual credit in the portfolio will be determined by **credit quality factor** model. The estimation of the model will use the historical GDP log return, U.S treasury yield curve data, and default rate.

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(Section 4.6) Lastly, we conduct RST twice, including and excluding the 3rd PC then we compare the shape of the stressed yield curves to answer our research questions.

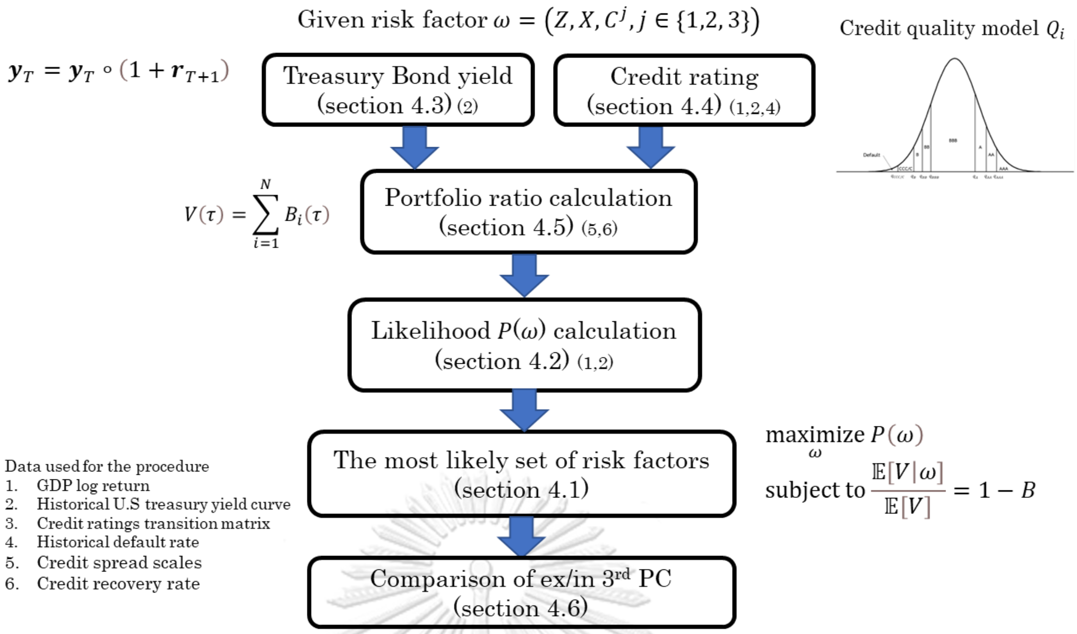

Figure 9
Summary of methodologies on this paper.

4.1 REVERSE STRESS TEST ON CREDIT PORTFOLIO

RST (Reverse stress test) is an optimization problem that find a set of risk factors ω^*. The risk factor must be a maximizer of likelihood function $P(\omega)$ given stressed portfolio value B.

$$\begin{aligned} &\underset{\omega}{\text{maximize}}\, P(\omega) \\ &\text{subject to } \mathbb{E}[V|\omega] = B \end{aligned} \tag{2}$$

The stressed portfolio value B must be given before conducting RST. Since several sets of risk factors ω that satisfy the constrain $\mathbb{E}[V|\omega] = B$. We choose candidate ω that maximize likelihood function $P(\omega)$ as the scenario that stress portfolio to value B.

4.2 MULTIVARIATE DISTRIBUTION OF RISK FACTORS

We assume that our risk factors $\omega = (Z, X, C^1, C^2, C^3)$ are jointly distributed.

- Z = latent systematic risk factor
- X = change in GDP

- C^j= the jth-principal component score (PC score) of yield curve, j is index number (C^1 = change in the first principal component score, etc.)

Change of GDP (gross domestic product) X represents how well the overall economy of a country performs. Positive X mean growing economy. Negative X mean shrinking economy.

The PC scores C^1, C^2, C^3 represent the movements of treasury yield curve – Level, Slope, and Curvature movements.

There is also a latent hidden risk factor Z which is assumed unaccountable and is assumed to be i.i.d. standard normal distributed.

$$\phi(Z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Z^2}{2}\right) \tag{3}$$

We will use multivariate t-distribution for the likelihood function of accountable risk factors $\boldsymbol{x} = [X \quad C^1 \quad C^2 \quad C^3]^T$ which have tail events of the risk factors.

$$t(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) = \frac{\Gamma[(\nu+d)/2]}{\Gamma(\nu/2)(\nu\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} \left[1 + \frac{1}{\nu}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]^{-(\nu+d)/2} \tag{4}$$

- $\Gamma(\cdot)$ is gamma function
- d is the dimension of $\boldsymbol{x} = [X \quad C^1 \quad C^2 \quad C^3]^T$
- Mean vector $\boldsymbol{\mu}$ covariance matrix $\boldsymbol{\Sigma}$ and degrees of freedom ν

Then the joint likelihood function of the risk factors is the multiplication of equations (3) and (4).

$$P(Z, X, C^1, C^2, C^3) = \phi(Z) \cdot t(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) \tag{5}$$

4.3 PRINCIPAL COMPONENTS OF YIELD CURVE

Interest rate yield curve is a collection of bond yields with same credit quality but different maturities. Investors pay close attention to how the bond yields move within time since bond yields and bonds prices have an inverse relationship: bond yields increase when bond prices decrease and vice versa.

In this paper, our credit portfolios are valued as a portfolio of zero-coupon bonds. The yield curve contains several bonds yields with different maturities ranging from 3-month to 30-year.

We only extract a handful of the most relevant components from the yield curve which can be done by PCA (principal component analysis). Then we use PC scores as risk factors for our RST $\left(C^j, j \in \{1, \dots, 3\}\right)$.

4.3.1 PRINCIPAL COMPONENT ANALYSIS OF YIELD CURVE

We start with a single observation of yield curve in row vector $\boldsymbol{y}$. Each column of the vector contains bond yields with n maturities arranged in order from the first column being the shortest maturity (3-m: 3-month) and the nth column being the longest maturity (30-y: 30-year).

$$\boldsymbol{y} = [y^1 \quad y^2 \quad \cdots \quad y^n] = \begin{array}{c} \begin{array}{cccc} \scriptstyle\text{3-m} & & & \scriptstyle\text{30-y} \end{array} \\ [0.04 \quad 0.12 \quad \cdots \quad 2.75] \end{array} \tag{6}$$

For example, (figure 9) shows the historical U.S. treasury yield curve from year 1980 to 2014. Where $T = 35$ years and $n = 10$ maturities. (Equation 7) show the historical data represented in matrix.

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{y}_0 \\ \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \vdots \\ \boldsymbol{y}_T \end{bmatrix} = \begin{bmatrix} y_0^1 & y_0^2 & \cdots & y_0^n \\ y_1^1 & y_1^2 & \cdots & y_1^n \\ y_2^1 & y_2^2 & \cdots & y_2^n \\ \vdots & \vdots & \ddots & \vdots \\ y_T^1 & y_T^2 & \cdots & y_T^n \end{bmatrix} = \begin{bmatrix} 14.30 & 13.76 & \cdots & 11.98 \\ 11.08 & 11.98 & \cdots & 13.65 \\ 7.92 & 8.00 & \cdots & 10.43 \\ \vdots & \vdots & \ddots & \vdots \\ 0.04 & 0.12 & \cdots & 2.75 \end{bmatrix} \tag{7}$$

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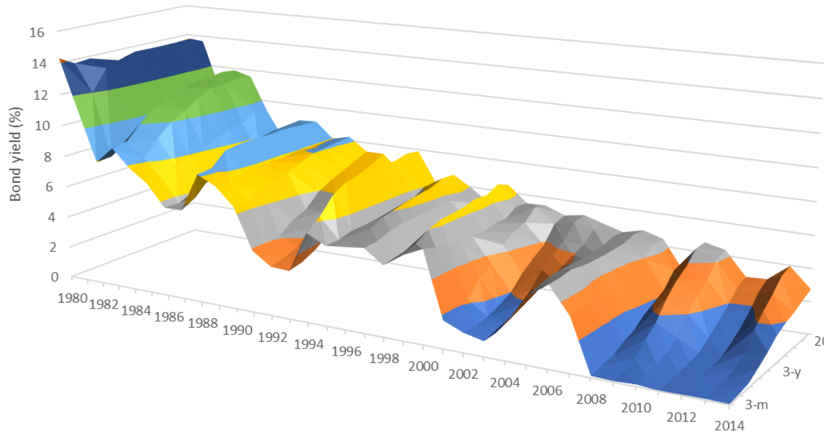

Figure 10
U.S. treasury yield curve from year 1980 to 2014. The vertical axis represents bond yield in percentage. The horizontal axis represents time in year. The depth axis represents maturity (3-m to 30-y).

Then, we calculate rate of change $r_t^i = \left(y_t^i - y_{t-1}^i\right)/y_{t-1}^i$ to get historical yield curve change $\boldsymbol{R}$ (one less row than $\boldsymbol{Y}$).

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \boldsymbol{r}_3 \\ \vdots \\ \boldsymbol{r}_T \end{bmatrix} = \begin{bmatrix} r_1^1 & r_1^2 & \cdots & r_1^n \\ r_2^1 & r_2^2 & \cdots & r_2^n \\ r_3^1 & r_3^2 & \cdots & r_3^n \\ \vdots & \vdots & \ddots & \vdots \\ r_T^1 & r_T^2 & \cdots & r_T^n \end{bmatrix} = \begin{bmatrix} -0.23 & -0.13 & \cdots & 0.14 \\ -0.29 & -0.33 & \cdots & -0.24 \\ 0.13 & 0.14 & \cdots & 0.14 \\ \vdots & \vdots & \ddots & \vdots \\ -0.43 & 0.2 & \cdots & -0.31 \end{bmatrix} \tag{8}$$

We calculate covariance matrix $\boldsymbol{\Sigma}$ from the rate of change data $\boldsymbol{R}$, then decompose the covariance matrix by SVD (singular value decomposition). SVD produces three square matrices:

left singular vectors $\boldsymbol{U}$, right singular vectors $\boldsymbol{V}$, and singular values $\boldsymbol{D}$.

$$\boldsymbol{\Sigma} = \begin{bmatrix} u_{1,1} & \cdots & u_{1,n} \\ \vdots & \ddots & \vdots \\ u_{n,1} & \cdots & u_{n,n} \end{bmatrix} \begin{bmatrix} d_{1,1} & 0 & \cdots & 0 \\ 0 & d_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{n,n} \end{bmatrix} \begin{bmatrix} v_{1,1} & \cdots & v_{1,n} \\ \vdots & \ddots & \vdots \\ v_{n,1} & \cdots & v_{n,n} \end{bmatrix}^T \tag{9}$$

Since covariance matrices are positive definite and symmetric, so both singular vectors are equal $\boldsymbol{U} = \boldsymbol{V}$.

Each column of $\boldsymbol{U}$ is called PC (principal component). We choose how much PC we want by choosing how much first k columns from the singular vector $\boldsymbol{U}$ where ($k \le n$). We use $k = 3$ for this paper.

$$\boldsymbol{U}^3 = \begin{bmatrix} | & | & | \\ PC1 & PC2 & PC3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ \vdots & \cdots & \vdots \\ u_{n,1} & u_{n,2} & u_{n,3} \end{bmatrix} \tag{10}$$

Now we can get the PC score $\boldsymbol{C}^3$ by $\boldsymbol{C}^3 = \boldsymbol{R}\boldsymbol{U}^3$. Each column of $\boldsymbol{C}^3$ is the PC score. c_t^1, c_t^2, c_t^3 $t = 1, \dots, T$ is the 1st, 2nd, and 3rd PC score

$$\boldsymbol{C}^3 = \begin{bmatrix} c_1^1 & c_1^2 & c_1^3 \\ c_2^1 & c_2^2 & c_2^3 \\ c_3^1 & c_3^2 & c_3^3 \\ \vdots & \cdots & \vdots \\ c_T^1 & c_T^2 & c_T^3 \end{bmatrix} = \begin{bmatrix} r_1^1 & r_1^2 & \cdots & r_1^n \\ r_2^1 & r_2^2 & \cdots & r_2^n \\ r_3^1 & r_3^2 & \cdots & r_3^n \\ \vdots & \vdots & \ddots & \vdots \\ r_T^1 & r_T^2 & \cdots & r_T^n \end{bmatrix} \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ \vdots & \cdots & \vdots \\ u_{n,1} & u_{n,2} & u_{n,3} \end{bmatrix} \tag{11}$$

we can revert $\boldsymbol{C}^3$ into the recovered historical return $\widetilde{\boldsymbol{R}}_3$ which can be done by $\widetilde{\boldsymbol{R}}_3 = \boldsymbol{C}^3(\boldsymbol{U}^3)^T$.

$$\widetilde{\boldsymbol{R}}_3 = \begin{bmatrix} \tilde{r}_1^1 & \tilde{r}_1^2 & \cdots & \tilde{r}_1^n \\ \tilde{r}_2^1 & \tilde{r}_2^2 & \cdots & \tilde{r}_2^n \\ \tilde{r}_3^1 & \tilde{r}_3^2 & \cdots & \tilde{r}_3^n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}_T^1 & \tilde{r}_T^2 & \cdots & \tilde{r}_T^n \end{bmatrix} = \begin{bmatrix} c_1^1 & c_1^2 & c_1^3 \\ c_2^1 & c_2^2 & c_2^3 \\ c_3^1 & c_3^2 & c_3^3 \\ \vdots & \cdots & \vdots \\ c_T^1 & c_T^2 & c_T^3 \end{bmatrix} \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ \vdots & \cdots & \vdots \\ u_{n,1} & u_{n,2} & u_{n,3} \end{bmatrix} \tag{12}$$

We transform from only 3 columns of PC score $\boldsymbol{C}^k$ into full n dimension data of the yield curve dynamic which is the main idea of dimensionality reduction of PCA.

4.3.2 SIMULATING TRESURY YIELD CURVE SCENARIO

We will discuss how to compute a one-year ahead yield curve in this section. Let this year T and one year ahead $T + 1$. In one year ahead $T + 1$, risk factors of treasury yield curve are random variables of 3 PC score $(C^1, C^2, C^3) = (c_{T+1}^1, c_{T+1}^2, c_{T+1}^3)$ represent how much change of level, slope, and curvature of the yield curve in year $T + 1$. The yield curve change $\boldsymbol{r}_{T+1}$ can be retrieved by using (Equation 12) but with one row of PC score.

$$\begin{aligned} \boldsymbol{r}_{T+1} &= \begin{bmatrix} r_{T+1}^1 & r_{T+1}^2 & \cdots & r_{T+1}^n \end{bmatrix} \\ &= \begin{bmatrix} c_{T+1}^1 & c_{T+1}^2 & c_{T+1}^3 \end{bmatrix} \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ \vdots & \cdots & \vdots \\ u_{n,1} & u_{n,2} & u_{n,3} \end{bmatrix} \end{aligned} \tag{13}$$

We then can calculate the one-year ahead yield curve given this year yield curve data $\boldsymbol{y}_T$. The matrix calculation can be done by element-wise product $\boldsymbol{y}_{T+1} = \boldsymbol{y}_T \circ (1 + \boldsymbol{r}_{T+1})$.

$$\begin{aligned} \boldsymbol{y}_{T+1} &= \begin{bmatrix} y_T^1 & y_T^2 & \cdots & y_T^n \end{bmatrix} \circ \begin{bmatrix} 1 + r_{T+1}^1 & 1 + r_{T+1}^2 & \cdots & 1 + r_{T+1}^n \end{bmatrix} \\ &= \begin{bmatrix} y_T^1 \cdot (1 + r_{T+1}^1) & y_T^2 \cdot (1 + r_{T+1}^2) & \cdots & y_T^n \cdot (1 + r_{T+1}^n) \end{bmatrix} \\ &= \begin{bmatrix} y_{T+1}^1 & y_{T+1}^2 & \cdots & y_{T+1}^n \end{bmatrix} \end{aligned} \tag{14}$$

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4.4 CREDIT RATING MIGRATION AND DEFAULT EVENT

4.4.1 CREDIT RATINGS CHANGE

Credits in portfolios have initial credit ratings. The initial credit ratings are either good (rating: AA) or bad (rating: BB). In one year, credits can change its rating to $\{AAA, AA, A, BBB, BB, B, CCC/C\}$ where $\{AAA\}$ is the best credit rating and $\{CCC/C\}$ is the worst credit rating.

The market usually values credit according to the ratings by **credit spreads** which are excess basis points over risk free rates. Good ratings will have low spreads. Bad ratings will have high spreads.

If a credit default $\{D\}$ which mean borrowers is unable to repay the debt in full amount, the value of the credit will be determined by stochastic recovery rate.

Rating change of a credit in the portfolios will be determined by the **credit quality factor model** (equation 15). The risk factors (Z, X, C^1, C^2, C^3) are systematic (affect all the credits in the portfolio). While individual risk ϵ_i is idiosyncratic (affect individual credit) and assumed i.i.d. standard normal distributed.

$$Q_i = \sqrt{\rho_z} Z + \rho_x X + \sum_{j=1}^{3} \rho_c^j C^j + \sqrt{1-\rho_z}\epsilon_i \tag{15}$$

The parameters $(\rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$ determine the sensitivities of credit to risk factors which are different given initial rating of the credit.

(Example) For portfolio with 100 credits, credit quality Q_i $(i = 1, \ldots, 100)$ for each credit will have their own value once risk factors $\left(Z, X, C^1, C^2, C^3, \epsilon_{i=1,\ldots,100}\right)$ and the parameters $(\rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$ are known. Credit i will change to new rating by the following conditions.

- Credit1: Change to rating $\{AAA\}$ if $Q_1 \in [q_{AAA}, +\infty)$
- Credit2: $\{AA\}$ if $Q_2 \in [q_{AA}, q_{AAA})$
- Credit3: $\{A\}$ if $Q_3 \in [q_A, q_{AA})$
- Credit4: $\{BBB\}$ if $Q_4 \in [q_{BBB}, q_A)$

Until…

- Credit100: Default $\{D\}$ if $Q_{100} \in (-\infty, q_{CCC/C})$,

If Q_i surpass q_{AA} threshold it will get AA credit ratings

The credit migration thresholds $(q_{AAA}, q_{AA}, \ldots, q_{CCC/C})$ can be estimated once we know the distribution of Q_i using historical credit

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rating transition matrix. The following picture shows an example of the distribution.

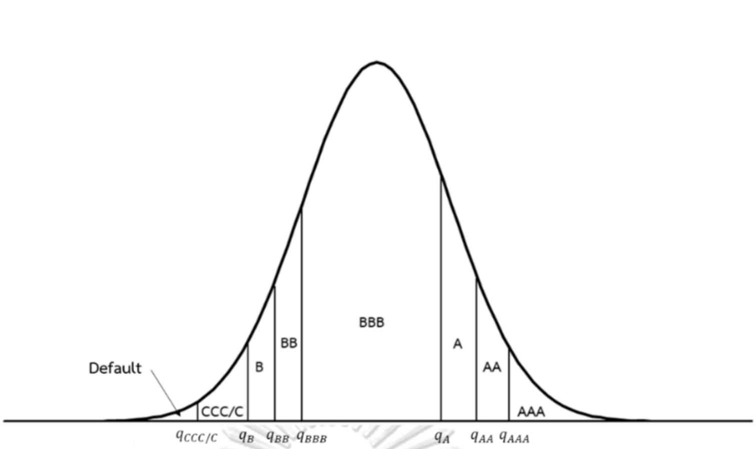

Figure 11
Distribution of credit rating with initial rating BBB.

For this paper, we have two set of Q_i distribution which are good initial credit rating $\{AA\}$ and bad initial credit rating $\{BB\}$. Both credit ratings have different sensitivity parameters $(\rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$ and different migration thresholds $(q_{AAA}, q_{AA}, \dots, q_{CCC/C})$.

4.4.2 SENSIVITES PARAMETERS ESTIMATION

We need to estimate the sensitivity parameters $(\rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$. The estimation can be done by maximizing the binomial loglikelihood function of historical default events.

Let $\theta = (R_k, \rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$, the loglikelihood function is

$$L(\theta) = \sum_{t=1}^{T} ln \int_{-\infty}^{+\infty} \binom{N_t}{d_t} q_t(z|\theta)^{d_t} \left(1 - q_t(z|\theta)\right)^{N_t - d_t} \phi(z)\ dz \tag{16}$$

Where

$$q_t(z|\theta) = \Phi\left(\frac{q_k - \sqrt{\rho_z} z - \rho_x x_t - \sum_{j=1}^{3} \rho_c^j c_t^j}{\sqrt{1 - \rho_z}}\right)$$

- x_t, c_t^j change in GDP and PC score
- N_t is the outstanding credit issued at time t
- d_t is the number of credit default at time t
- q_k is default migration.

The loglikelihood maximization problem to estimate θ

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$$\underset{\theta}{\text{maximize}}\, L(\theta) \qquad (17)$$
$$subject\ to\ 0 < \rho_z < 1$$

Note that we add constrain $0 < \rho_z < 1$ to prevent the loglikelihood function $L(\theta)$ to have division by zero and negative root.

4.5 CREDIT PORTFOLIO VALUATION

We will talk about portfolio valuation in this section. Prior to valuation date, a credit in the portfolio can be in 2 initial ratings: good $\{AA\}$ and bad $\{BB\}$. Each credit has a face value of one. The valuation date is one year in the future.

One year later, the value of a credit B_i will be valued by a risk-free treasury bond yield and a credit spread. The credit can change to new rating $\alpha = \{AAA, AA, A, BBB, BB, B, CCC/C\}$ with different credit spread S_i^{α}. Credit spread will be the highest for the worst rating $\alpha = \{CCC/C\}$ and the lowest for the best rating $\alpha = \{AAA\}$.

$$B_i = \exp(-(y^{\tau} + S_i^{\alpha}) \cdot \tau) \qquad (18)$$

Treasury bond yield y^{τ} has maturity τ at valuation date. The bond yield one year ahead can be calculated by using PCs (Principal components).

If default event $\alpha = \{D\}$ happens, the value of the credit is then

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$$B_i = \delta \cdot \exp(-(y^{\tau}) \cdot \tau)\ where\ 0 < \delta < 1 \qquad (19)$$

Where δ = beta-distributed recovery rate

Value of the credit portfolio value is the sum of N credits with identical maturity τ. For simplicity, we assume no addition credit is added or removed from N credits before the valuation date.

$$V(\tau) = \sum_{i=1}^{N} B_i(\tau) \qquad (20)$$

4.6 EXPERIMENTS SETUP

We discuss the experiments conducted on this paper. We would like to find how the stressed yield curves differ between exclusion/inclusion of the 3rd PC.

	What research question ask	Compare stressed yield curve
1.)	Medium term portfolio (2-year)	Good and Bad credit rating.
2.)	Long term portfolio (10-year)	Good and Bad credit rating.
3.)	Perturbing credit spread data	Shifting and Tilting

Table 1
List of experiments conducted to answer the established research questions.

We will use 2-year and 10-year to represent the portfolio with medium and long maturities. Initial credit ratings AA and BB will represents good and bad credit ratings.

For each condition, we will compute RST (Reverse stress test) twice: including and excluding the 3^{rd} PC. Then, we will compare the stressed yield curve shape by measuring the gap between stressed yield curves including and excluding the 3^{rd} PC.

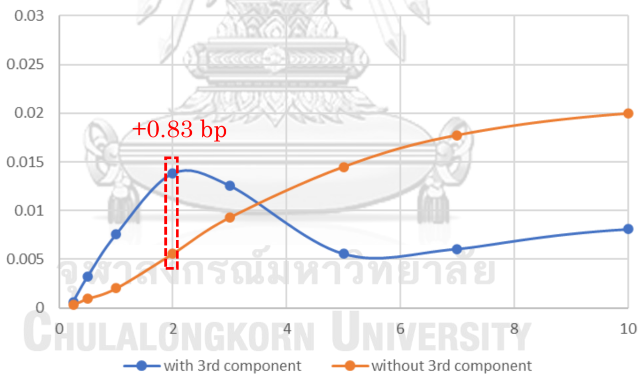

Figure 12
Show the stressed yield curve with portfolio with same maturity and rating. We can see how stressed yield curve is different between including and excluding the 3^{rd} PC.

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5. DATA

5.1 GDP: GROSS DOMESTIC PRODUCT DATA

We use end of year GDP of USA acquired from Federal Reserve Economic Data (Quote: GDP). The historical logarithmic change of U.S GDP from 1981-2014 are shown in (figure 11).

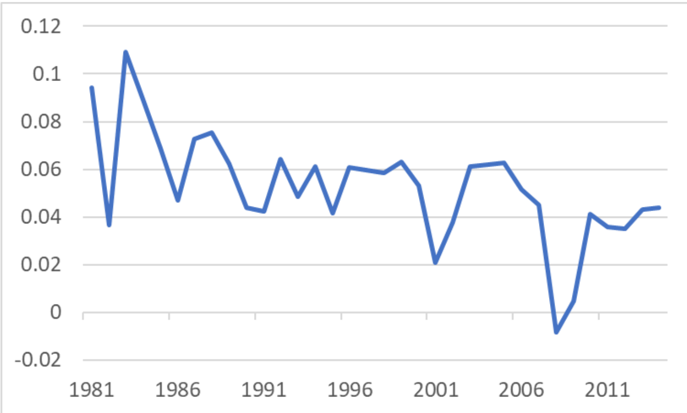

Figure 13
Annual GDP logarithmic change from year 1981-2014

5.2 TRESURY YIELD CURVE DATA

We use end of year treasury yield curve with 10 maturities. All the data are available in Federal Reserve Economic Data (Quote: TB3MS, TB6MS, DGS1, DGS2, DGS3, DGS5, DGS7, DGS10, DGS20, DGS30). The period of data is from 1980-2014.

5.3 HISTORICAL DEFAULT, TRANSITION MATRIX, CREDIT SPREAD, CREDIT RECOVERY RATE DATA

Historical defaults and credit rating transition matrices are acquired from rating agencies data, the data is from year 1981-2014. The data are from global gathering; however, most of the default data are in the USA.

The credit spread data are acquired from the Bank of America (BofA) Corporate Index Option-Adjusted Spread. The year of the data is 2014.

We acquired the senior unsecured recovery rate from rating agencies data which is an average data from the year 1981-2014. The mean recovery rate is 52.3% and standard deviation is 38.7%

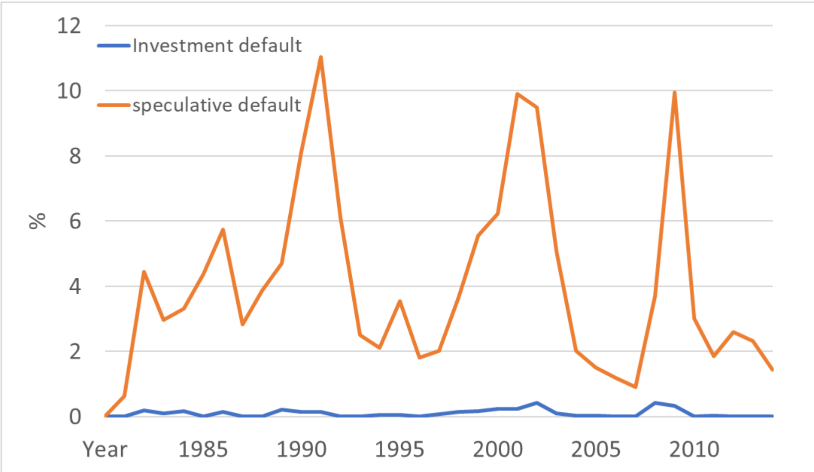

Figure 14

Global cooperate default rate from year 1981-2014. The vertical axis represents the default rate in percentage. The horizontal axis represents time in the year 1981-2014. Speculative grade usually has a higher default rate than investment in every year.

	AAA	AA	A	BBB	BB	B	CCC/C	D
AA	1.62%	93.15%	4.36%	0.66%	0.16%	0.01%	0.00%	0.04%
BB	0.02%	0.11%	0.64%	5.80%	88.99%	3.45%	0.29%	0.71%

Table 2

Credit rating transition matrix with AA and BB initial ratings

AAA	AA	A	BBB	BB	B	CCC/C
65	84	107	198	332	533	969

Table 3

Credit spread data from 2014 represent in bp (basis point)

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6. RESULT I: RISK FACTOR SENSITIVITY TO DEFAULT RATE

We first study how risk factors correlate with historical default rates. This section, we will tell see how sensitive the default rates to the GDP change, three principal component score (PC score) of yield curve and hidden risk factors.

The three PC scores and PCs already have their result shown in figure 5 in background section. The results are similar to what (Grundke and Pliszka 2015) have shown in their literature.

- Positive 1st PC score (C^1), the yield curve rises parallelly.
- Positive 2nd PC score (C^2), the yield curve steepening (short-term rates fall while long-term rates rise).
- Positive 3rd PC score (C^3), the yield curve develops hump shape (its medium-term rates, 0.5- to 3-year, rise while other rates fall).

We now focus on the credit quality model (equation 15) and its parameters $(\rho_z, \rho_x, \rho_c^1, \rho_c^2, \rho_c^3)$. The parameters represent how sensitive the default rates are to the risk factors (z, x, c^1, c^2, c^3). We estimate the parameters for both including and excluding the 3rd PC score.

We test with U.S corporate bond investment grade and speculative grade default data. The historical data that we use for estimation are as follows: U.S GDP log change, 1st, 2nd, 3rd PC score, and U.S corporate default rate (investment and speculative grade).

We first focus on the model that excludes the 3rd PC. From the result, we found that the sign of the estimated parameters have similar result with (Grundke and Pliszka 2015). We have positive signs on sensitivity of latent risk – if the risk factor is negative, default rate will rise.

GDPs have positive sensitivity – default rates rise if GDP changes are negative (both investment and speculative grade).

1st PC scores have positive sensitivity – default rates rise if treasury yield curves shift down (both investment and speculative grade).

2nd PC scores have negative sensitivity – default rates rise if treasury yield curves steepen (both investment and speculative grade).

	Latent ρ_z	GDP change ρ_x	1st PC score ρ_c^1	2nd PC score ρ_c^2	3rd PC score ρ_c^3
Excluding 3rd PC – similar to (Grundke and Pliszka 2015)					
Inv default	0.0625^{**}	2.7977^{**}	0.1366^{*}	-0.1673^{*}	-
Spec default	0.0659^{**}	2.8895^{**}	0.0628^{*}	-0.1202^{*}	-
Including 3rd PC					
Inv default	0.0630^{**}	2.8408^{**}	0.1365^{*}	-0.1681^{*}	-0.0983^{*}
Spec default	0.0647^{**}	2.7374^{**}	0.0643^{*}	-0.1215^{*}	0.1018^{*}

Estimated parameters are not to be confused with Pearson correlation coefficient

The superscript symbols *, ** denote Significance at 10% and 5% (bootstrapping method)

Positive parameter ρ = the risk factor (e.g., GDP change) is negative; default rate goes up.

Table 4
Estimated default rate sensitivity parameters for both credit quality model excluding and including the 3rd PC score.

However, 3rd PC scores have different signs for investment grade and speculative grade.

3rd PC scores have negative sensitivity – default rates rise if treasury yield curves develop hump shape (investment grade).

3rd PC scores have positive sensitivity – default rates fall if treasury yield curves develop hump shape (speculative grade).

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7. RESULT II: INCLUSION/EXCLUSION OF THE THIRD PC ON STRESED YIELD CURVE

7.1 EFFECT OF THE THIRD PC ON STRESSED YIELD CURVE SHAPE OF 2-YEAR CREDIT PORTFOLIO.

We first focus on the first research question. We would like to see how the shape of stressed yield curves for 2-year credit portfolio with good and bad initial credit ratings affected by inclusion/exclusion of the 3rd PC. All credit portfolios have 500 credits with a face value of 1 and have 2-year maturity at the date of valuation.

We conduct RST on 2-year good credit portfolios with 4 stress levels that lie at the very extreme left tail of portfolio distribution – less than 5th percentile.

7.1.1 2-YEAR, GOOD CREDIT PORTFOLIO

The stress levels (stressed portfolio values) are 460, 440, 420, 400. The shape of stressed yield curve with inclusion/exclusion can be shown compared together within the stress levels.

The stressed port values at 460, 440, 420 show us the hump shape on stressed yield curve. The hump is higher as stress level is higher. At the highest stress level of 400, the hump shape disappears and the shape of stressed yield curves with and without the 3rd PC are similar in shape.

To dig a little further, we show credit ratings distribution of 500 credits in the portfolio (figure 16). As stress level is higher, credits have lower chance changing into good ratings (Higher than BBB: AAA, AA, A) and higher changing into in bad ratings (Lower than BBB: BB, B, CCC/C, D).

Default rates are the chance of credits staying in rating D. Default rates increase linearly as stress level is higher for RST without the 3rd PC. The linear increase in default rates is not apparent for RST with the 3rd PC. Default rates jump only at the highest stress level for RST with the 3rd PC. For RST with 3rd PC, the result tells us that credit portfolio does not suffer credit downgrade and default as much compared to RST without 3rd PC. The portfolios are more likely to fall in value by rise in interest rates (the hump shape). If the stress levels are high enough to pass a certain threshold, the portfolios will be more likely to fall in value from credit downgrade and default rather than rise in interest rates (hump shape is less likely).

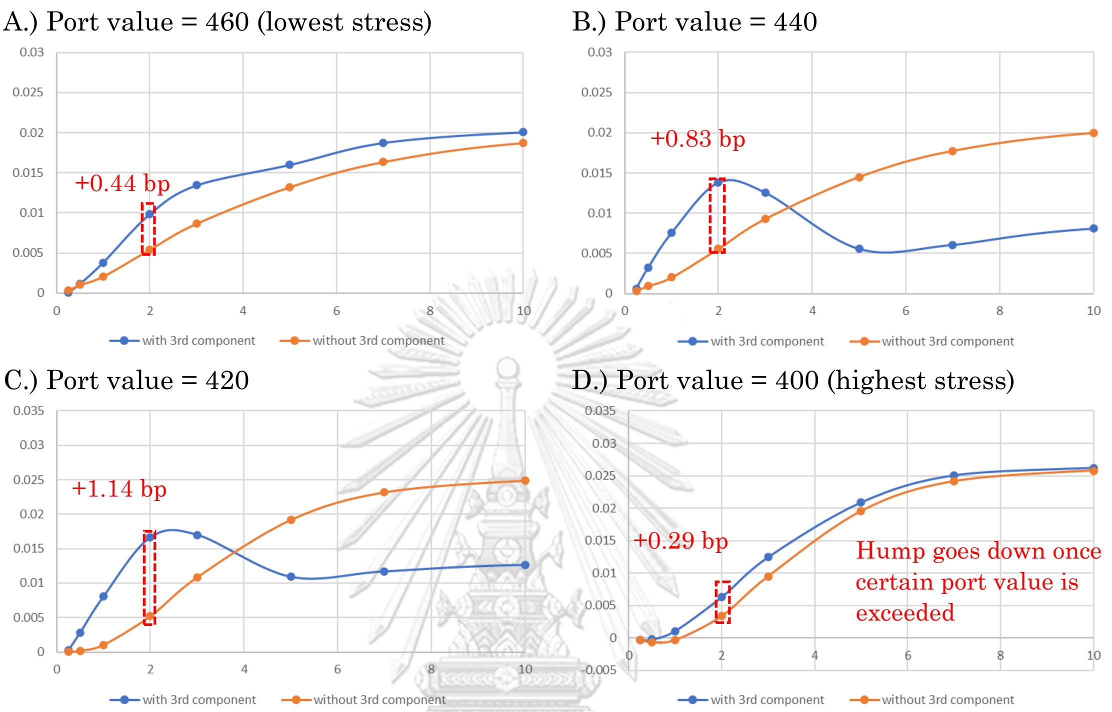

Figure 15

The stressed yield curves of the 2-year, good credit portfolio. Notice that the hump shapes appear with the stressed yield with the 3rd PC included. The gap length represented by bp (basis point) between 2-year yield for both yield curves is also shown.

The reasons our RST algorithm arrives at scenarios that prominently feature the 3rd PC scores may be explained as follows. Our historical data of U.S. treasury yield curve contains periods that see the 3rd PC scores. Histogram of the three PC scores can be shown in (figure 17) where there is a heavy positive tail of the 3rd PC scores (the rightmost figure). We also found that the 3rd PC score not only raises yields between 0.5-year to 3-year but also induces downgrade and default of bonds with good initial credits as well (see our result in section 6). Therefore, given an occurrence of high portfolio loss attributable to both an increase in yields and a downgrade of credits, we are likely to find the 3rd component at play. For this reason, high 3rd PC scores are chosen by our RST algorithm as the most likely stressed scenario for 2-year, good credit portfolios given that the thresholds of stress levels are high enough.

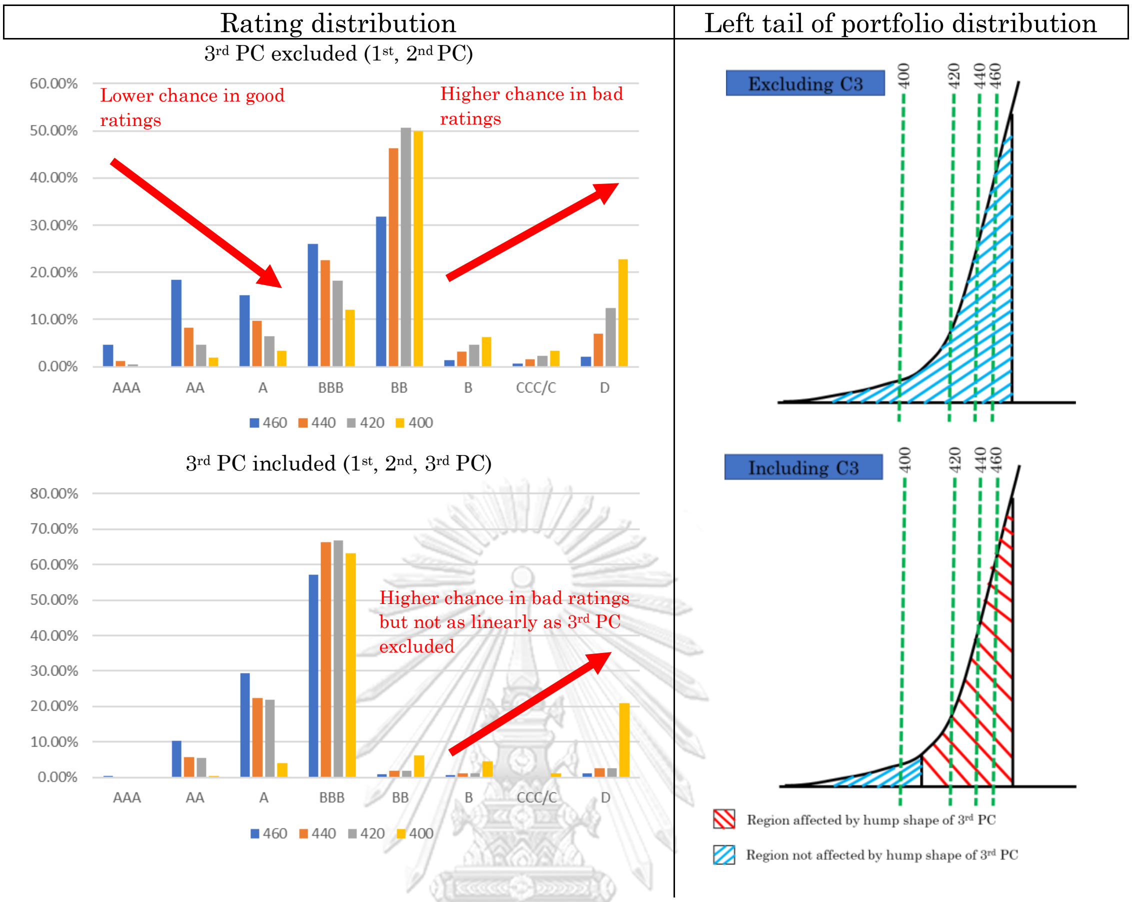

Figure 16

Rating distribution of credit portfolio at given stressed value from 460 to 400 (2-year, good credit portfolios). Notice that as stress level is higher (stress port value is lower), credit has a higher chance of changing into bad ratings (Lower than BBB).

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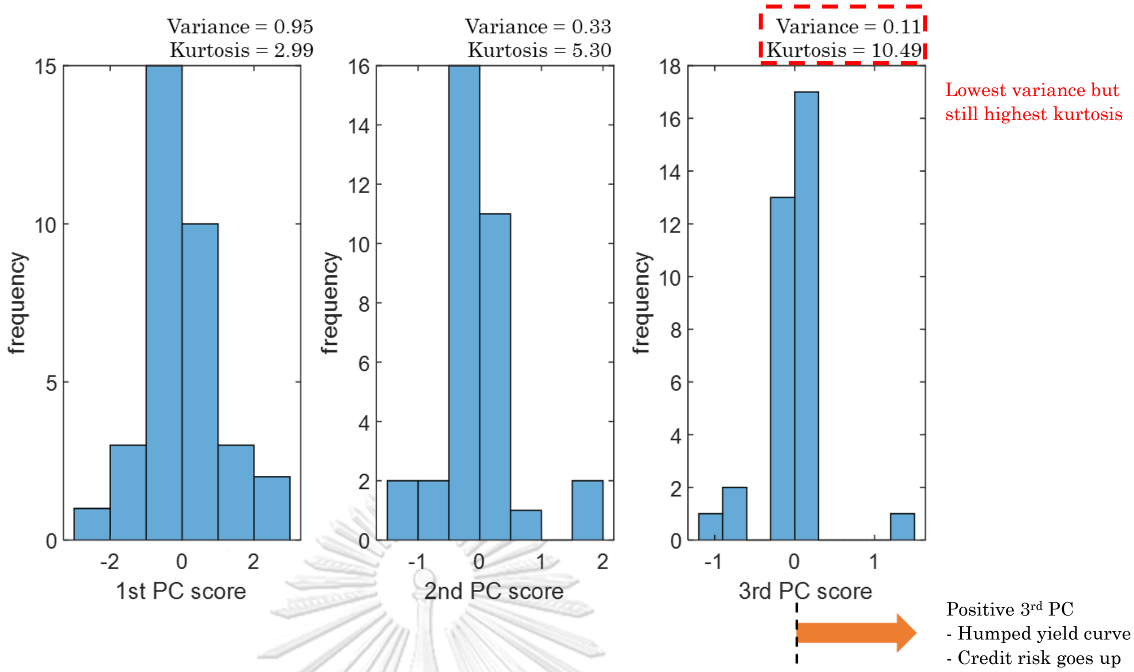

Figure 17

Show the historical PC (principal component) of U.S treasury yield curve from year 1981-2019. Even though the 3rd PC has the lowest variance compared to the first two components, it has heavy positive tail that contributes hump formed by medium maturity yield (6-month to 3-year).

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7.1.2 2-YEAR, BAD CREDIT PORTFOLIO

We now experiment on a 2-year, bad credit ratings portfolio to see whether the stressed yield curves get affected by the inclusion/exclusion of the 3rd PC. The stressed port values are 400, 370, 340, 310 which are less than 10th percentile of portfolio distribution.

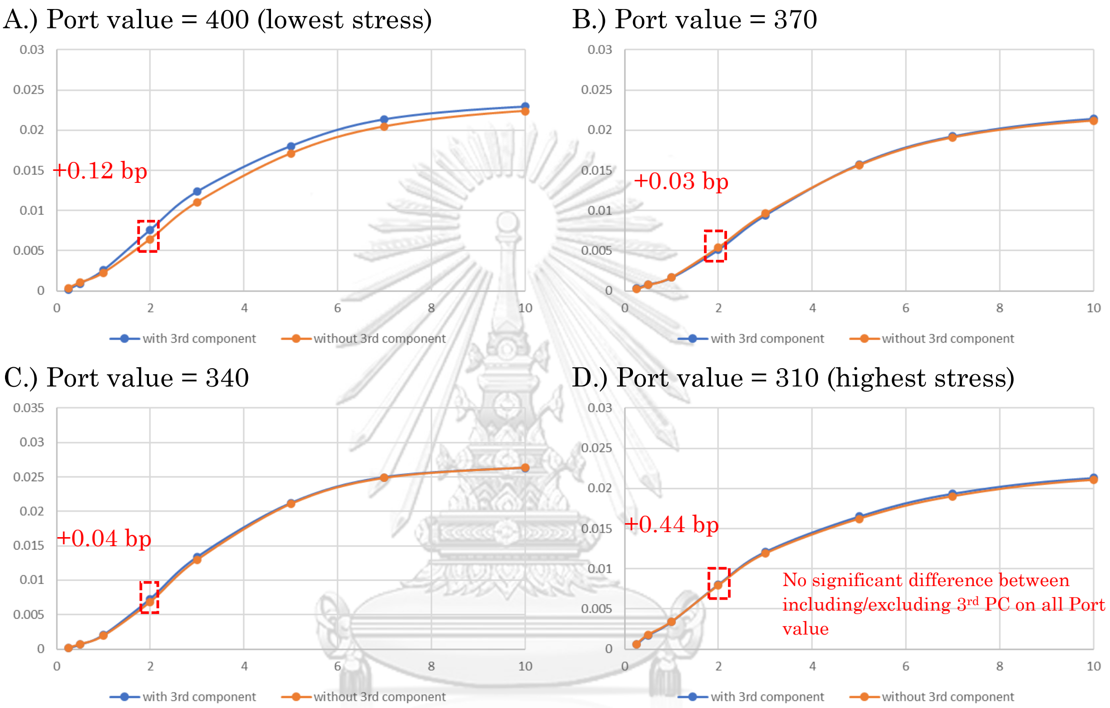

Figure 18

The stressed yield curves of the 2-year, bad credit portfolio. Notice that the hump shapes do not appear at any stressed portfolio value (Port value) and the shape of stressed yield curves do not differ so much.

We can see that the shape of stressed yield curves on the bad credit portfolio almost overlap. The result shows that the stressed yield curves do not get affected from the inclusion/exclusion of the 3rd PC.

We recall from our calibration results in section 6 that the 3rd component significantly induces the downgrade of bonds with an initial good credit rating. But for bad-credit bonds, which are already prone to credit downgrades and defaults, we do not find a significant relationship between the 3rd PC and their further downgrading. Therefore, for a bad credit portfolio, the 3rd component does not play a significant role in its incurring large loss - in other words, our RST does not choose high 3rd PC scores as a stressed scenario for portfolios with bad initial credit ratings.

7.2 EFFECT OF THE THIRD PC ON STRESSED YIELD CURVE SHAPE OF 10-YEAR CREDIT PORTFOLIO.

7.2.1 10-YEAR, GOOD CREDIT PORTFOLIO

We now experiment on a 10-year, bad credit ratings portfolio to see whether the stressed yield curves get affected by the inclusion/exclusion of the 3rd PC. The stressed port values are 310, 280, 250, 220 which are less than 5th percentile of portfolio distribution.

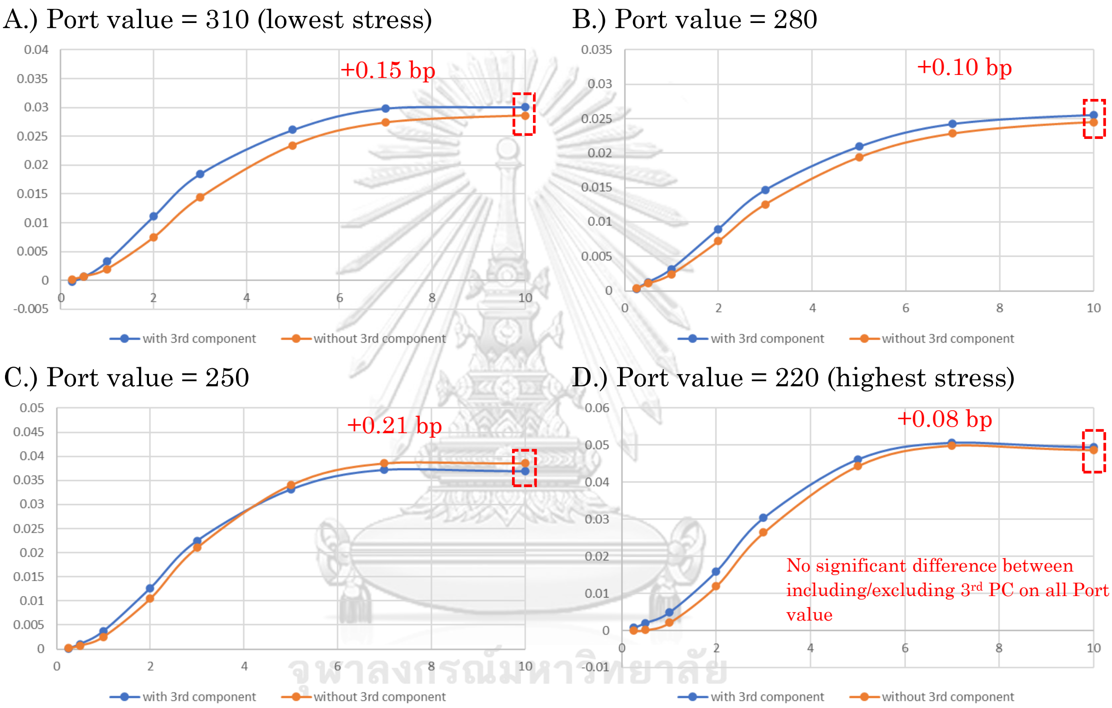

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Figure 19
The stressed yield curves of the 10-year, good credit portfolio.

There are some differences in shape of the stressed yield curve between inclusion/exclusion of the 3rd PC but not as apparent as 2-year good credit portfolio. Hump shape does not really in any stress level.

The reasons why high 3rd PC scores do not get chosen by our RST are as follows. While the 3rd PC score is responsible for raising the yields in between the 0.5-year to 30-year range, a 10-year portfolio falls far outside of this range. Therefore, the 3rd PC does not feature in the scenario selected by our RST algorithm.

7.2.2 10-YEAR, BAD CREDIT PORTFOLIO

We now experiment on a 10-year, bad credit ratings portfolio to see whether the stressed yield curves get affected by the inclusion/exclusion of the 3rd PC. The stressed port values are 280, 250, 220, 190 which are less than 5th percentile of portfolio distribution.

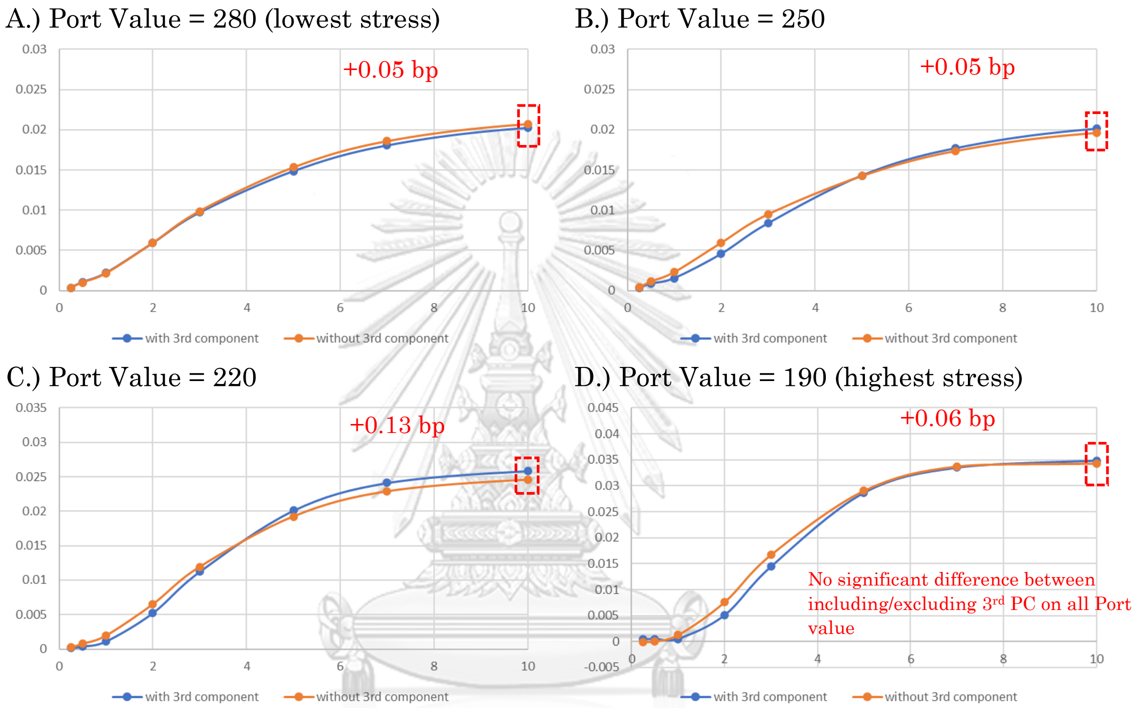

Figure 20
The stressed yield curves of the 10-year, bad credit portfolio.

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There are not many differences in shape between including and excluding the 3rd PC from our RST. The shape of stressed yield curves almost overlaps for 10-year, bad credit portfolios.

The reason high 3rd PC scores do not get chosen by our RST is the same reason as the portfolio from the previous section (10-year, good initial credit). While the 3rd PC is responsible for raising the yields in the 0.5-year to 3-year range, a 10-year portfolio falls far outside of this range. Therefore, the 3rd PC does not feature in the scenario selected by our RST algorithm.

7.3 2-YEAR GOOD CREDIT PORTFOLIO, PERTURBING THE CREDIT SPREAD DATA.

To answer our 3rd research question, we perturb the credit spread scales data from 2014 by 2 approaches: Shift (parallel shift) and Tilt. We try to perturb the credit spread as little in each step to see how it affects our original result from the 2-year, good credit portfolio on the 1st research question while holding the stress level at 460.

Shift – add same basis points on spreads with all ratings equally.

Tilt – add more basis points on spreads with bad credit ratings.

Per step (bp)	AAA	AA	A	BBB	BB	B	CCC/C
Shift	+10	+10	+10	+10	+10	+10	+10
Tilt	+10	+15	+18	+25	+40	+45	+80

Table 5
Increase of basis point per step for perturbing the credit spread scales.

From previous research questions, we found that a 2-year, good credit portfolio was stressed from either from the 3rd PC score (humped yield curve) or credit downgrade. As we increase steps of shift or tilting the credit spreads, the hump shape will start disappearing. Tilting the credit spreads will make the hump shape on stressed yield curve disappear faster than shifting the credit spreads

The reasons why perturbing the credit spread data, both by shifting and tilting, can make the effect of the 3rd PC scores on stressed yield curve disappear may be explained as follows. Recall that the 3rd PC not only raises some part of the yield curve but might also significantly induce downgrading of the bonds with good initial credit ratings (see result in section 6). However, as we increase the initial credit spread of the bonds in the portfolio, this effect dominates the effect of the 3rd PC and becomes the main culprit of portfolio loss. This is why the hump shape of the stressed yield curve becomes less prominent as we increase the initial credit spread of the bonds in the portfolio. The same reasons mentioned above can explain why tilting the credit spreads will make the humped yield curve disappear more noticeably than shifting the credit spreads, since this adds more credit loss on bonds with already bad ratings.

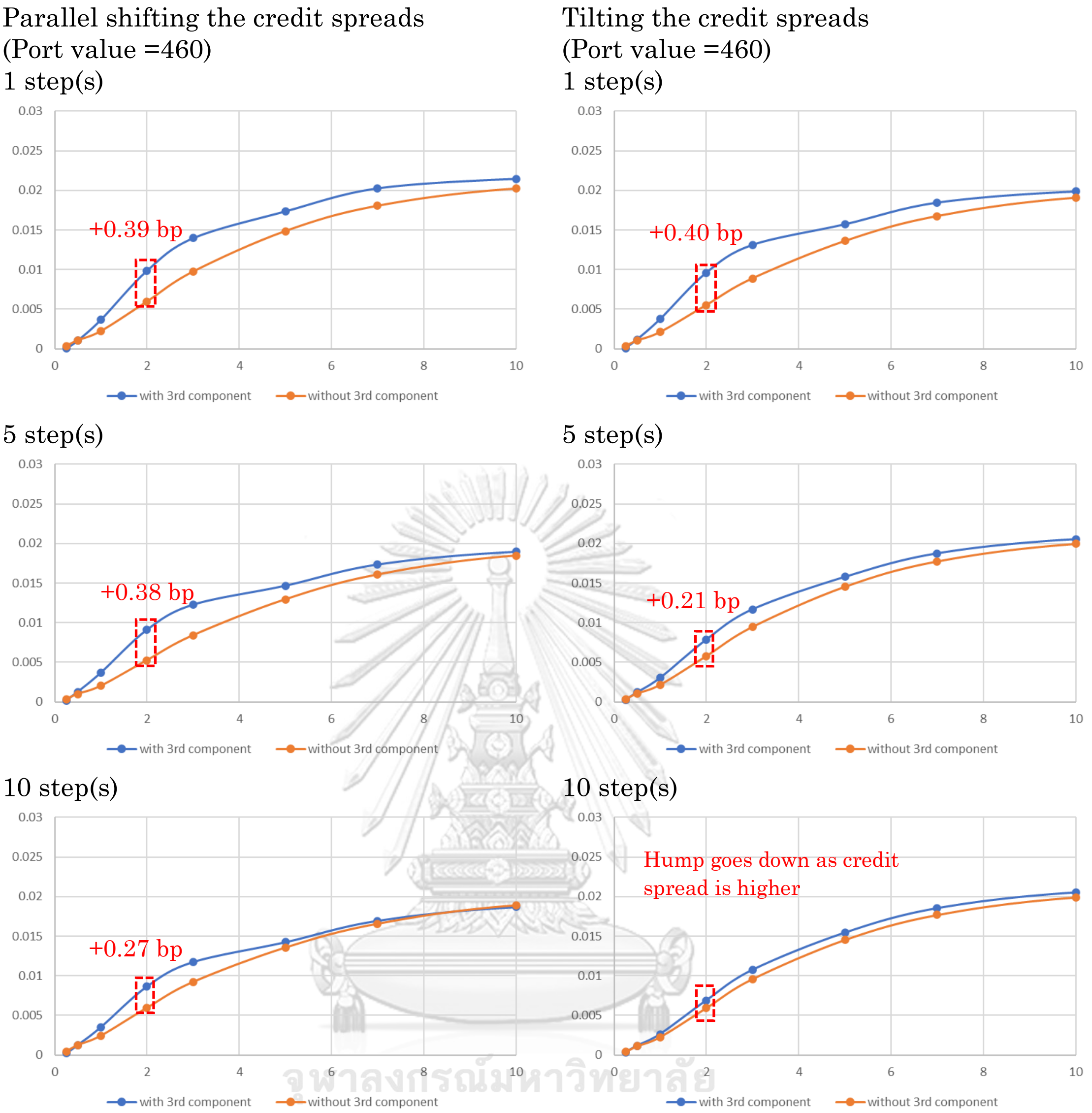

Figure 21

For the 3rd research question, we tinker with our credit spread data testing on 2-year, good credit portfolio.

8. CONCLUSION

In this paper, we would like to study how including and excluding the 3rd PC score (Principal component score) affects the shape of stressed yield curves of credit portfolios. For that, we use RST (Reverse stress test) on credit portfolios. Our RST finds the shape of stressed yield curve of credit portfolios given stress level (stressed portfolio value).

The three PC scores of yield curve are known movements of the yield curve – parallel shift, tilt, and bend. We focus on the 3^{rd} PC scores which we believe create hump shape on stressed yield curve of the credit portfolios. We asked three research questions and checked the shape of stressed yield curves for the following credit portfolios.

1. 2-year, good and bad credit portfolios (with several stress level)
2. 10-year, good and bad credit portfolios (with several stress level)
3. 2-year, good credit portfolios (perturbing the credit spread data)

We use the first research question to focus on the shape of stressed yield curves on 2-year, good and bad credit portfolios. As high 3^{rd} PC scores usually happens yields between 0.5-year to 3-year. Then, the second research question, we focus on longer maturity (10-year) to see whether the 3^{rd} PC gives hump shape on stressed yield curves. The third research question focused on how changing credit spread affects the hump shape of the stressed yield curves.

For 2-year credit portfolios with good credit, we found hump shape on the stressed yield curves. The hump shape becomes more obvious when we increase the stress level as the gap of stressed yield curves between including and excluding the 3^{rd} PC widen. However, at the highest stress level, the gap shrink makes the shape of stressed yield curves between including and excluding the 3^{rd} PC score similar. We then tried on the 2-year credit portfolios with bad credit and saw no significant hump shape. The results are consistent with our hypotheses that bad-rating credit portfolios, when stressed, are more likely suffer to credit risk rather than interest rate risk.

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We believe that the humped yield curve is the effect of how 3^{rd} PC contribute the heavy positive tail of the elevated medium yields (0.5-year to 3-year). The heavy tail makes humped yield curve more likely to be chosen by our RST. Also, our calibration showed that the 3^{rd} PC scores affect not only the hump shape of the yield curve but also investment grade default rate (higher 3^{rd} PC scores, higher investment default rate). The two reasons mentioned above led us to believe why our RST pick high 3^{rd} PC scores as stressed scenario for 2-year, good initial credit portfolios.

For a 10-year credit portfolio with good credit, we found that including the 3^{rd} PC does not make much difference on stressed yield curve shape. The hump shape from the 3^{rd} PC only happens at 0.5-year to 3-year interest rate and does not extend to higher maturities. Therefore, we conclude that stressed yield curve shape of 10-year credit portfolio does not get affected by including or excluding the 3^{rd} PC. The gap between stressed yield curve is even smaller if it is the bad credit rating 10-year portfolio make both stressed yield curves almost overlapped.

From the first two research questions, we concluded that 2-year good credit portfolio is the only portfolio that has its stress yield curve affected by the 3^{rd} PC. At higher stress level, the stressed yield curve will show higher hump shape represents how the portfolio is stressed by rise of interest rates created by positive 3^{rd} PC. The hump shape will be higher to the point where positive 3^{rd} PC become less likely and the portfolio is more likely stressed credit risk (credit downgrade and default) rather than interest risk that the hump shape disappears.

The third research question, we focus on widening the credit spread and hold other environment constant while we test on 2-year good credit portfolio. We found that hump shape on stressed yield curve start to disappear when the spread is widening. The hump shape disappears faster with tilting the credit spread (bad rating get more basis point add) than parallel shift the credit spread (all rating gets same basis point add). We then conclude that in the presence of severe widening credit spread, credit portfolios are more likely to suffer from credit loss rather than interest rate rise.

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