รายงานการไปเพิ่มพูนความรู้ทางวิชาการ
ภายในประเทศ

จุฬาลงกรณ์มหาวิทยาลัย

ทุนงบประมาณแผ่นดิน ปี 2523

รายงานผลการวิจัย

การประมาณค่า normal probability integral
สำหรับการคำนวณด้วยเครื่องคิดเลข

โดย

สุชาดา ศรีพันธุ์

มิถุนายน 2526

Chulalongkorn University

The Thai Government's fiscal Budget

Report

An Approximation of the Normal Probability Integral for Desk Calculators

by

Suchada Siripant

June 1983

สารบัญ

การประมาณค่า normal probability integral

สำหรับการคำนวณด้วยเครื่องคิดเลข

สุชาดา ศิริพันธุ์

มิถุนายน 2526

บทคัดย่อ

งานวิจัยนี้ ได้เสนอวิธีการประมาณค่า normal probability integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$$

วิธีนี้สะดวกสำหรับนำไปใช้กับเครื่องคิดเลขทั้งชนิดธรรมดาและชนิดที่สามารถโปรแกรมได้ ซึ่งทำให้จำนวน keystrokes ที่ใช้ลดลง

ค่าของ probability Q(x) ประมาณได้โดย

$$Q(x) \approx a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$$

เมื่อ $a_0 = 0.0002743166$

$a_1 = 0.3943689450$

$a_2 = 0.0216086002$

$a_3 = -0.1083530025$

$a_4 = 0.0373481450$

$a_5 = -0.0040289291$

ซึ่งค่าประมาณที่ได้มีความคลาดเคลื่อนสัมพัทธ์น้อยกว่า 0.04% ในช่วง $0 \le x \le 3$

AN APPROXIMATION OF THE NORMAL PROBABILITY INTEGRAL FOR DESK CALCULATOS

Suchada Siripant

June 1983

Abstract

In this paper, we shall present a method of approximating the normal probability integral

$$Q(x) = \frac{1}{2\pi} \int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$$

This method is practical for use on electronic hand calculators and programable calculators. The method minimizes the number of keystrokes.

The probability $Q(x)$ is approximated by

$$Q(x) \approx a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

where

$a_0 = 0.0002743166,$

$a_1 = 0.3943689450,$

$a_2 = 0.0216086002,$

$a_3 = -0.1083530025,$

$a_4 = 0.0373481450,$

$$a_5 = -0.0040289291$$

with a relative of error less than 0.04 % in the range $0 \leq x \leq 3$.

Introduction

By the use of suitable approximations most functions can be conveniently evaluated on digital computers. In this paper we describe a method of approximating the normal probability integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt.$$

This approximation function is practical for use on hand calculators since we can not write a program to compute Q(x) on calculators. Moreover, we can not use the values from the normal probability integral table because many keystrokes are required for entering these values. Our method consists of three parts

1. selecting a suitable form for the approximation
2. fitting the approximation to the function
3. finding the best coefficients for the approximation

Method

1. Selecting a Suitable Form. Many electronic calculators are equipped with $\sqrt{x}$, sin x, ln x, e^x, x^y etc. keys. One should consider whether or not the function to be approximated resembles one of them.

The electronic calculators can evaluate polynomials as rapidly as the keys are depressed. This suggests that a polynomial

$$P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

should be used.

2. Fitting the Approximation to the Function. After a polynomial P(x) has been chosen, the unknown coefficients a_0 to a_n must be selected so that P(x) fits the function Q(x) to be approximated. The criterion of approximation that we shall use is the least-square approximation which requires that the summation of the square errors

$$S = \sum_{j=0}^{n} \left[P(x_j) - Q(x_j) \right]^2$$

be minimized.

3. Finding the Best Coefficients. After using the least-square method, we found that a polynomial of degree five is suitable. After finding a suitable polynomial of degree 5 we improved the coefficients by using small intervals of x_j and thus found the best approximation.

Result

The best coefficients a_k (k = 0,1,...,5) were obtained by using matrices and they are :

$$a_0 = 0.0002743166$$

$$a_1 = 0.3943689450$$

$$a_2 = 0.0216086002$$

$$a_3 = -0.1083530025$$

$$a_4 = 0.0373481450$$

$$a_5 = -0.0040289291$$

The approximations which yield the minimum errors described above are presented in Table I.

RESULT	TRUE RESULT	%ERROR
.0398226387052042	.0398	.0568811688547739
.0792040934491976	.0793	-.120941425980328
.117896972818897	.1179	-.00256758363273961
.155459534845194	.1554	.0383107111930502
.191525168961985	.1915	.0131430610887728
.225797561291136	.2257	.0432260926610545
.258045859927448	.258	.0177751656775194
.288099840223615	.2881	-.0000554586549809094
.315845070075181	.3159	-.0173883902560937
.341218075205509	.3413	-.0240037487521242
.364201504450732	.3643	-.0270369336447982
.38481929504472	.3849	-.0209677722213562
.403131837904039	.4032	-.0169052817363591
.419231142912912	.4192	.00742912998854962
.43323600420818	.4332	.00831122072483841
.445287165464263	.4452	.0195789452522462
.455542485178116	.4554	.0312879178998682
.4641721019542	.4641	.0155358660202543
.47135359978943	.4713	.0113727539635052
.477267173358148	.4772	.0140765628977368
.482090793297079	.4821	-.00190970813544908
.485995371490274	.4861	-.0215240711224028
.489139926354123	.4893	-.03271482646168
.491666748122232	.4918	-.0270947291110207
.493696564130482	.4938	-.0209469156577562
.495323704101905	.4953	.00478580696648496
.496611265431688	.4965	.0224099560298087
.49758627847213	.4974	.0374504366968235
.498234871817587	.4981	.0270772570943586
.498497437589458	.4987	-.0406180891401644

This approximation has a relative error of less than 0.04 % in the range $0 \leq x \leq 3$

Discussion

The polynomial approximation can be evaluated repidly by electronic calculators. The approximation presented here is useful for calculators (programmable and non-programmable) since it reduces the number of keystrokes.

References

1. Young, D.M. and Gregory, R.T. 1972. A Survey of Numerical Mathematics

2. Froberg, C.E. 1969. Introduction to Numerical Analysis

3. Hamming, R.W. 1971. Introdúction to Applied Numerical Analysis.

4. Ralston, A. 1965. A first Course in Numerical Analysis.

5. Theodore, C.A. 1975, Applied Mathematics.

6. Booth, Andrew D. Numerical Methods; Butterworth & Co. Ltd London, 1966.

7. Cheney, E.W. Introduction to Approximation Theory; McGraw-Hill Inc. New York, 1966.

8. Conte, S.D. and de Boor, Carl Elementary Numerical Analysis; McGraw-Hill Kogakusha Inc., Japan, 1965.

9. Grove, Wendell E., Brief Numerical Methods; Prentice Hall Inc. Englewood Cliffs, New Jersey, 1966.

10. Henrici, Peter, Elements of Numerical Analysis; John Wiley and Sons, Inc. New York, 1964.

11. Johnson Lee W. and Riess R. Dean, Numerical Analysis; Reading, Massachusetts, Addison - Wesley Publishing Company, 11977.

12. Scarborough, James B. Numerical Mathematical Analysis; Oxford & IBH Publishing Co. New Delhi, 1966.